Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.3-Tangent/99-4.3.10-c+d-x-^m-a+b-tan-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [63]. This is test number [99].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (63)	0.00 (0)
Fricas	100.00 (63)	0.00 (0)
Rubi	98.41 (62)	1.59 (1)
Maple	92.06 (58)	7.94 (5)
Maxima	77.78 (49)	22.22 (14)
Giac	55.56 (35)	44.44 (28)
Mupad	50.79 (32)	49.21 (31)
Sympy	44.44 (28)	55.56 (35)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

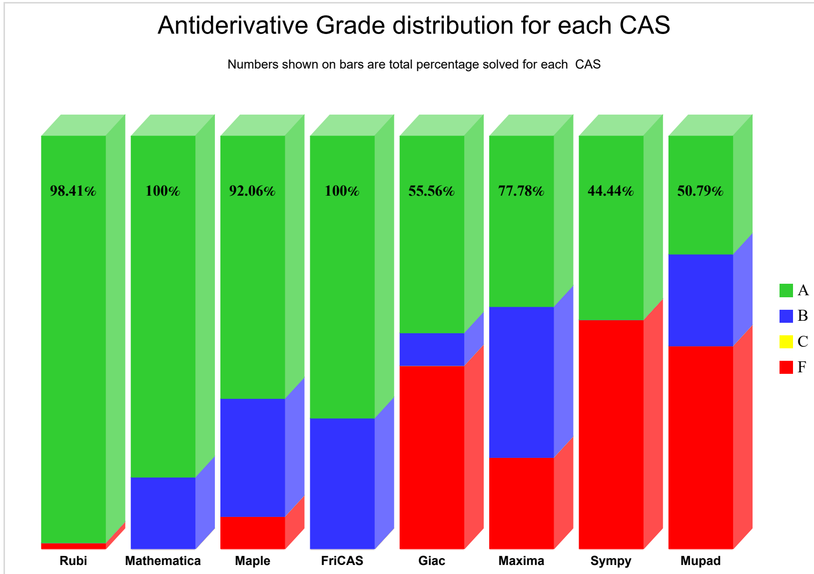
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

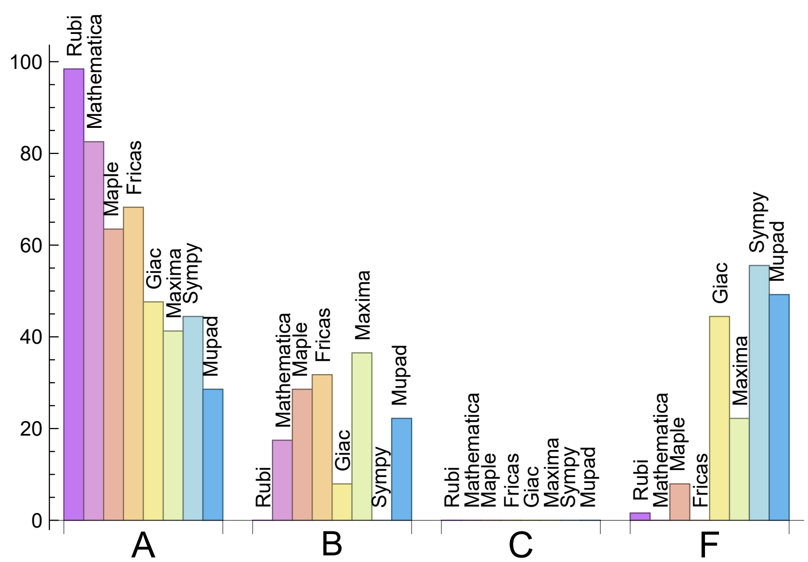
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.41	0.00	0.00	1.59
Mathematica	82.54	17.46	0.00	0.00
Fricas	68.25	31.75	0.00	0.00
Maple	63.49	28.57	0.00	7.94
Giac	47.62	7.94	0.00	44.44
Sympy	44.44	0.00	0.00	55.56
Maxima	41.27	36.51	0.00	22.22
Mupad	N/A	22.22	0.00	49.21

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	5	100.00 %	0.00 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
Giac	28	100.00 %	0.00 %	0.00 %
Maxima	14	35.71 %	0.00 %	64.29 %
Sympy	35	100.00 %	0.00 %	0.00 %
Mupad	31	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

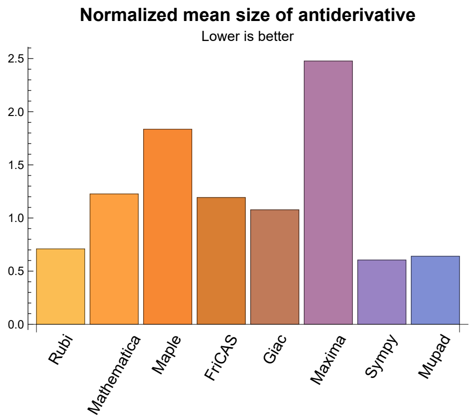
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

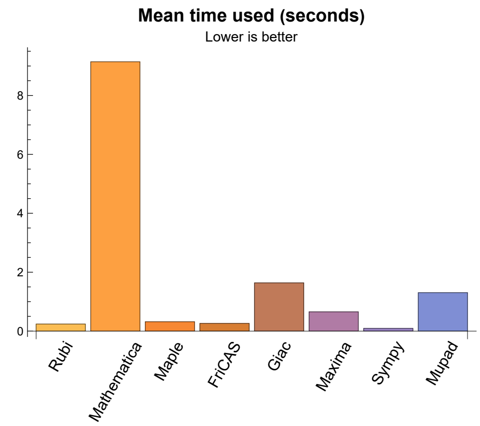
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.24	167.66	0.71	126.50	1.00
Mathematica	9.14	320.46	1.23	179.00	1.04
Maple	0.32	511.74	1.84	138.00	1.34
Maxima	0.65	665.63	2.48	208.00	0.76
Fricas	0.26	269.41	1.19	133.00	0.82
Sympy	0.09	132.46	0.60	0.00	0.00
Giac	1.64	305.20	1.08	0.00	0.00
Mupad	1.31	80.19	0.64	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{4, 5, 9, 10, 14, 15, 34, 35, 42, 43, 47, 48, 52, 53, 57, 58, 62, 63\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{33, 44, 45, 49, 50, 59, 61\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

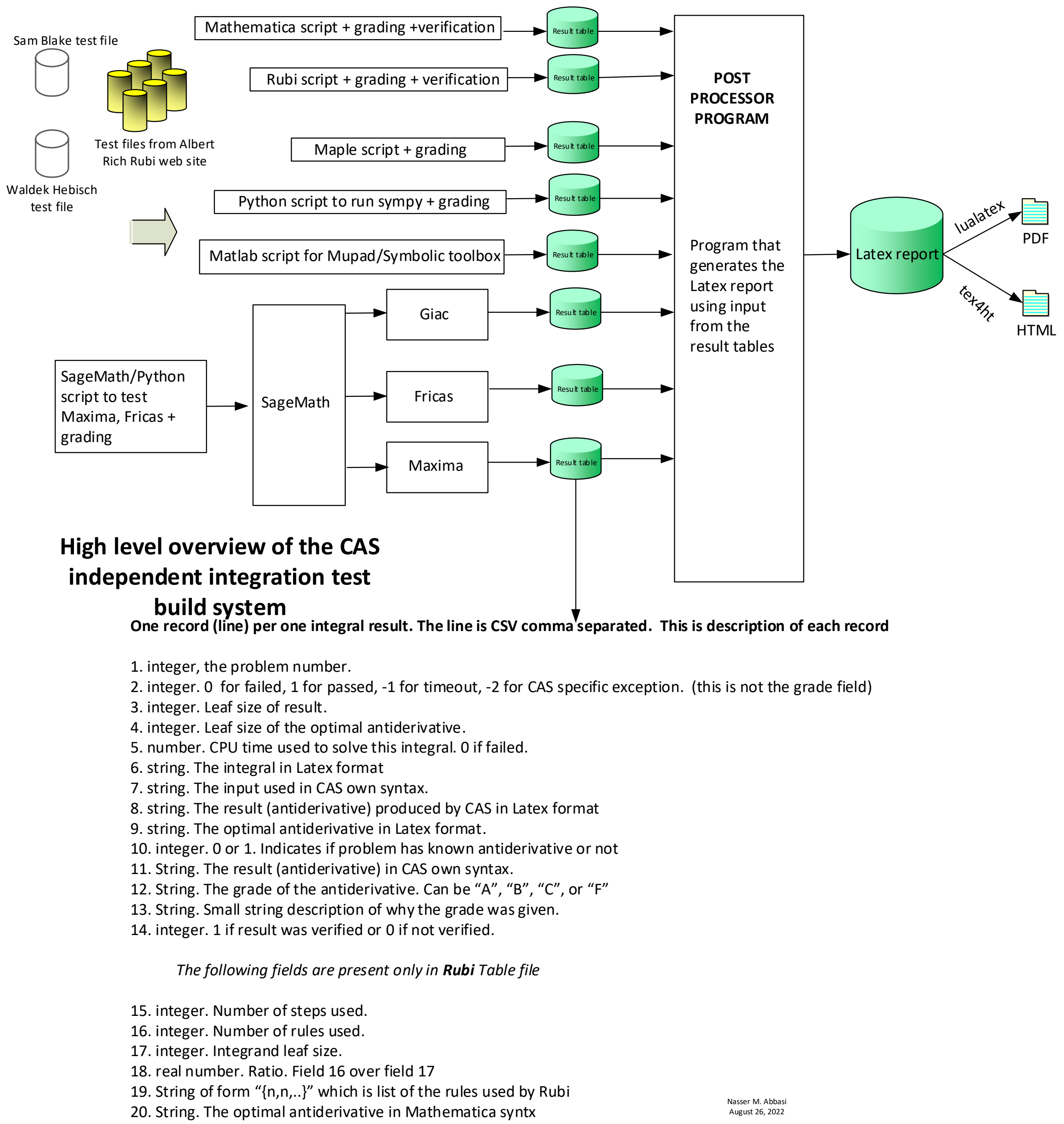
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63 }

B grade: { }

C grade: { }

F grade: { 17 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 60, 62, 63 }

B grade: { 7, 13, 36, 39, 40, 44, 45, 49, 50, 59, 61 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 26, 27, 28, 32, 33, 34, 35, 41, 42, 43, 46, 47, 48, 51, 52, 53, 57, 58, 62, 63 }

B grade: { 18, 24, 25, 29, 30, 31, 39, 40, 44, 45, 49, 50, 54, 55, 56, 59, 60, 61 }

C grade: { }

F grade: { 16, 17, 36, 37, 38 }

2.1.4 Maxima

A grade: { 4, 5, 9, 10, 14, 15, 21, 22, 23, 27, 28, 32, 33, 34, 35, 41, 42, 43, 47, 48, 52, 53, 57, 58, 62, 63 }

B grade: { 1, 2, 3, 6, 7, 8, 11, 12, 13, 39, 40, 44, 45, 46, 49, 50, 51, 54, 55, 56, 59, 60, 61 }

C grade: { }

F grade: { 16, 17, 18, 19, 20, 24, 25, 26, 29, 30, 31, 36, 37, 38 }

2.1.5 FriCAS

A grade: $\{$ 4, 5, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 42, 43, 46, 47, 48, 50, 51, 52, 53, 57, 58, 62, 63 $\}$

B grade: $\{$ 1, 2, 3, 6, 7, 11, 12, 13, 39, 40, 41, 44, 45, 49, 54, 55, 56, 59, 60, 61 $\}$

C grade: $\{\ \}$

F grade: $\{\ \}$

2.1.6 Sympy

A grade: $\{$ 4, 5, 8, 9, 10, 14, 15, 18, 19, 20, 24, 25, 26, 29, 30, 31, 34, 35, 42, 43, 47, 48, 52, 53, 57, 58, 62, 63 $\}$

B grade: $\{\ \}$

C grade: $\{\ \}$

F grade: $\{$ 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 21, 22, 23, 27, 28, 32, 33, 36, 37, 38, 39, 40, 41, 44, 45, 46, 49, 50, 51, 54, 55, 56, 59, 60, 61 $\}$

2.1.7 Giac

A grade: $\{$ 4, 5, 9, 10, 14, 15, 18, 19, 20, 21, 24, 25, 26, 27, 29, 30, 31, 32, 34, 35, 42, 43, 47, 48, 52, 53, 57, 58, 62, 63 $\}$

B grade: $\{$ 8, 22, 23, 28, 33 $\}$

C grade: $\{\ \}$

F grade: $\{$ 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 36, 37, 38, 39, 40, 41, 44, 45, 46, 49, 50, 51, 54, 55, 56, 59, 60, 61 $\}$

2.1.8 Mupad

A grade: $\{$ 4, 5, 9, 10, 14, 15, 34, 35, 42, 43, 47, 48, 52, 53, 57, 58, 62, 63 $\}$

B grade: $\{$ 3, 8, 16, 17, 18, 19, 20, 24, 25, 26, 29, 30, 31, 41 $\}$

C grade: $\{\ \}$

F grade: $\{$ 1, 2, 6, 7, 11, 12, 13, 21, 22, 23, 27, 28, 32, 33, 36, 37, 38, 39, 40, 44, 45, 46, 49, 50, 51, 54, 55, 56, 59, 60, 61 $\}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	125	243	286	0	0	-1
N.S.	1	1.00	1.00	1.18	2.29	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.014	0.155	0.533	0.380	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	103	163	200	0	0	-1
N.S.	1	1.00	1.00	1.34	2.12	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.013	0.063	0.555	0.373	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	78	92	122	0	0	129
N.S.	1	1.00	1.00	1.44	1.70	2.26	0.00	0.00	2.39
time (sec)	N/A	0.056	0.009	0.075	0.552	0.365	0.000	0.000	2.464

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.011	2.116	0.045	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.011	3.192	0.044	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	122	133	639	225	0	0	-1
N.S.	1	1.00	1.24	1.36	6.52	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.109	1.196	0.092	0.560	0.371	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	189	108	257	144	0	0	-1
N.S.	1	1.00	2.59	1.48	3.52	1.97	0.00	0.00	-0.01
time (sec)	N/A	0.078	6.159	0.087	0.610	0.398	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	43	40	214	38	41	182	35
N.S.	1	1.00	1.43	1.33	7.13	1.27	1.37	6.07	1.17
time (sec)	N/A	0.017	0.206	0.082	0.526	0.390	0.076	0.699	0.113

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.020	2.741	0.119	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.020	3.128	0.108	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	366	251	1205	344	0	0	-1
N.S.	1	1.00	1.79	1.22	5.88	1.68	0.00	0.00	-0.00
time (sec)	N/A	0.212	6.729	0.114	0.666	0.372	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	179	180	736	240	0	0	-1
N.S.	1	1.00	1.40	1.41	5.75	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.133	3.748	0.089	0.616	0.356	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	210	122	386	146	0	0	-1
N.S.	1	1.00	2.33	1.36	4.29	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.075	6.163	0.072	0.621	0.384	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.019	5.028	0.082	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.020	3.364	0.065	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	0	0	16	0	0	50
N.S.	1	1.00	1.00	0.00	0.00	0.89	0.00	0.00	2.78
time (sec)	N/A	1.928	0.697	0.362	0.000	0.413	0.000	0.000	3.123

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	A	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	17	0	0	15	0	0	45
N.S.	1	0.00	1.00	0.00	0.00	0.88	0.00	0.00	2.65
time (sec)	N/A	0.025	0.507	0.428	0.000	0.351	0.000	0.000	4.173

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	278	1061	0	166	258	193	423
N.S.	1	1.00	1.47	5.61	0.00	0.88	1.37	1.02	2.24
time (sec)	N/A	0.147	0.460	0.469	0.000	0.360	0.200	0.472	3.559

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	178	108	0	107	165	123	241
N.S.	1	1.00	1.30	0.79	0.00	0.78	1.20	0.90	1.76
time (sec)	N/A	0.090	0.302	0.558	0.000	0.352	0.165	0.524	3.078

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	96	50	0	57	92	65	105
N.S.	1	1.00	1.14	0.60	0.00	0.68	1.10	0.77	1.25
time (sec)	N/A	0.039	0.341	0.503	0.000	0.358	0.124	0.517	2.815

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	166	65	120	52	0	142	-1
N.S.	1	1.00	1.03	0.40	0.75	0.32	0.00	0.88	-0.01
time (sec)	N/A	0.208	0.385	0.463	0.359	0.367	0.000	0.527	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	224	285	127	86	0	1099	-1
N.S.	1	1.00	1.33	1.70	0.76	0.51	0.00	6.54	-0.01
time (sec)	N/A	0.168	0.884	0.370	0.384	0.366	0.000	2.767	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	285	358	167	133	0	540	-1
N.S.	1	1.00	1.26	1.58	0.74	0.59	0.00	2.38	-0.00
time (sec)	N/A	0.229	1.235	0.449	0.423	0.368	0.000	0.525	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	473	2046	0	270	665	383	289
N.S.	1	1.00	1.75	7.58	0.00	1.00	2.46	1.42	1.07
time (sec)	N/A	0.197	1.022	0.534	0.000	0.381	0.359	0.590	3.231

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	282	956	0	166	418	227	183
N.S.	1	1.00	1.40	4.73	0.00	0.82	2.07	1.12	0.91
time (sec)	N/A	0.138	0.692	0.486	0.000	0.377	0.277	0.586	2.987

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	130	82	0	82	226	107	103
N.S.	1	1.00	0.86	0.54	0.00	0.54	1.50	0.71	0.68
time (sec)	N/A	0.103	0.535	0.569	0.000	0.360	0.208	0.537	2.768

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	211	377	208	86	0	420	-1
N.S.	1	1.00	0.69	1.24	0.68	0.28	0.00	1.38	-0.00
time (sec)	N/A	0.546	0.572	0.442	0.355	0.369	0.000	0.549	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	467	536	225	147	0	2135	-1
N.S.	1	1.00	1.07	1.23	0.52	0.34	0.00	4.90	-0.00
time (sec)	N/A	0.505	1.727	0.481	0.438	0.380	0.000	13.086	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	667	3396	0	374	945	573	411
N.S.	1	1.00	1.68	8.58	0.00	0.94	2.39	1.45	1.04
time (sec)	N/A	0.268	2.427	0.620	0.000	0.360	0.505	0.749	3.851

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	405	1513	0	225	588	331	263
N.S.	1	1.00	1.38	5.15	0.00	0.77	2.00	1.13	0.89
time (sec)	N/A	0.189	1.619	0.541	0.000	0.361	0.387	0.725	3.375

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	205	523	0	109	311	151	146
N.S.	1	1.00	0.98	2.50	0.00	0.52	1.49	0.72	0.70
time (sec)	N/A	0.166	0.796	0.457	0.000	0.368	0.283	0.728	3.030

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	336	550	296	120	0	846	-1
N.S.	1	1.00	0.75	1.22	0.66	0.27	0.00	1.88	-0.00
time (sec)	N/A	1.281	0.847	0.524	0.386	0.350	0.000	0.682	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	833	787	317	204	0	3165	-1
N.S.	1	1.00	1.17	1.11	0.45	0.29	0.00	4.45	-0.00
time (sec)	N/A	1.252	3.285	0.573	0.475	0.367	0.000	33.043	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	30.536	0.201	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.021	14.605	0.319	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	205	0	0	86	0	0	-1
N.S.	1	1.00	2.09	0.00	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.084	1.399	0.298	0.000	0.119	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	205	0	0	146	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.135	151.497	0.267	0.000	0.100	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	269	0	0	201	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.179	119.593	0.298	0.000	0.108	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	546	500	710	520	0	0	-1
N.S.	1	1.00	3.59	3.29	4.67	3.42	0.00	0.00	-0.01
time (sec)	N/A	0.179	7.083	0.342	0.573	0.372	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	324	314	401	331	0	0	-1
N.S.	1	1.00	2.82	2.73	3.49	2.88	0.00	0.00	-0.01
time (sec)	N/A	0.141	6.325	0.207	0.546	0.396	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	87	143	136	168	0	0	161
N.S.	1	1.00	1.04	1.70	1.62	2.00	0.00	0.00	1.92
time (sec)	N/A	0.086	0.038	0.147	0.546	0.373	0.000	0.000	3.220

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	2.088	0.230	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.019	6.366	0.211	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	1347	952	2500	780	0	0	-1
N.S.	1	1.00	4.49	3.17	8.33	2.60	0.00	0.00	-0.00
time (sec)	N/A	0.366	6.966	0.374	1.262	0.395	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	656	575	1295	465	0	0	-1
N.S.	1	1.00	2.86	2.51	5.66	2.03	0.00	0.00	-0.00
time (sec)	N/A	0.256	6.949	0.267	0.723	0.388	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	200	238	559	228	0	0	-1
N.S.	1	1.00	1.47	1.75	4.11	1.68	0.00	0.00	-0.01
time (sec)	N/A	0.121	2.212	0.226	0.583	0.369	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	19.308	0.230	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	14.752	0.214	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	612	612	2607	1930	6424	1199	0	0	-1
N.S.	1	1.00	4.26	3.15	10.50	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.674	7.856	0.439	9.417	0.404	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	1860	1138	3327	704	0	0	-1
N.S.	1	1.00	4.27	2.61	7.63	1.61	0.00	0.00	-0.00
time (sec)	N/A	0.443	7.368	0.376	2.517	0.407	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	277	493	1384	337	0	0	-1
N.S.	1	1.00	1.00	1.78	5.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.234	3.505	0.257	0.878	0.367	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	14.888	0.222	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	20.659	0.218	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	418	1468	1038	1209	0	0	-1
N.S.	1	1.00	1.72	6.04	4.27	4.98	0.00	0.00	-0.00
time (sec)	N/A	0.245	2.268	0.540	0.748	0.469	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	321	940	755	858	0	0	-1
N.S.	1	1.00	1.77	5.19	4.17	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.196	1.702	0.394	0.647	0.391	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	113	462	422	549	0	0	-1
N.S.	1	1.00	0.90	3.70	3.38	4.39	0.00	0.00	-0.01
time (sec)	N/A	0.110	1.021	0.392	0.616	0.396	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	1.733	0.332	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	4.403	0.347	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	848	848	2857	3512	4521	2555	0	0	-1
N.S.	1	1.00	3.37	4.14	5.33	3.01	0.00	0.00	-0.00
time (sec)	N/A	1.383	11.026	0.661	2.809	0.507	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	654	654	535	2184	2556	1606	0	0	-1
N.S.	1	1.00	0.82	3.34	3.91	2.46	0.00	0.00	-0.00
time (sec)	N/A	1.016	7.757	0.560	1.243	0.431	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	745	999	1193	871	0	0	-1
N.S.	1	1.00	3.48	4.67	5.57	4.07	0.00	0.00	-0.00
time (sec)	N/A	0.194	6.867	0.517	0.885	0.442	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	16.294	0.426	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	16.814	0.471	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [16] had the largest ratio of [45]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	6	1.00	10	0.600
2	A	5	5	1.00	10	0.500
3	A	4	4	1.00	8	0.500
4	A	0	0	0.00	0	0.000
5	A	0	0	0.00	0	0.000
6	A	7	7	1.00	12	0.583
7	A	6	6	1.00	12	0.500
8	A	3	3	1.00	10	0.300
9	A	0	0	0.00	0	0.000
10	A	0	0	0.00	0	0.000
11	A	13	10	1.00	12	0.833
12	A	9	8	1.00	12	0.667
13	A	7	7	1.00	10	0.700
14	A	0	0	0.00	0	0.000
15	A	0	0	0.00	0	0.000
16	A	76	12	1.00	45	0.267
17	F	0	0	N/A	0.	N/A
18	A	5	3	1.00	23	0.130
19	A	4	3	1.00	23	0.130
20	A	3	3	1.00	21	0.143
21	A	7	4	1.00	23	0.174
22	A	7	4	1.00	23	0.174
23	A	8	5	1.00	23	0.217
24	A	10	3	1.00	23	0.130
25	A	8	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	7	3	1.00	21	0.143
27	A	21	5	1.00	23	0.217
28	A	24	7	1.00	23	0.304
29	A	14	3	1.00	23	0.130
30	A	11	3	1.00	23	0.130
31	A	11	3	1.00	21	0.143
32	A	53	7	1.00	23	0.304
33	A	60	9	1.00	23	0.391
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	2	2	1.00	23	0.087
37	A	4	2	1.00	23	0.087
38	A	5	2	1.00	23	0.087
39	A	8	7	1.00	18	0.389
40	A	7	6	1.00	18	0.333
41	A	6	5	1.00	16	0.312
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	15	9	1.00	20	0.450
45	A	13	10	1.00	20	0.500
46	A	9	7	1.00	18	0.389
47	A	0	0	0.00	0	0.000
48	A	0	0	0.00	0	0.000
49	A	28	11	1.00	20	0.550
50	A	22	11	1.00	20	0.550
51	A	16	9	1.00	18	0.500
52	A	0	0	0.00	0	0.000
53	A	0	0	0.00	0	0.000
54	A	6	6	1.00	20	0.300
55	A	5	5	1.00	20	0.250
56	A	4	4	1.00	18	0.222
57	A	0	0	0.00	0	0.000
58	A	0	0	0.00	0	0.000
59	A	21	9	1.00	20	0.450
60	A	18	10	1.00	20	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	5	1.00	18	0.278
62	A	0	0	0.00	0	0.000
63	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3 \tan(a+bx)\,dx$

Optimal. Leaf size=106

$$\frac{ix^4}{4}-\frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3ix^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3i\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}$$

```
[Out] 1/4*I*x^4-x^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*x^2*polylog(2,-exp(2*I*(b*x+a)
))/b^2-3/2*x*polylog(3,-exp(2*I*(b*x+a)))/b^3-3/4*I*polylog(4,-exp(2*I*(b*x
+a)))/b^4
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3800, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3i\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}-\frac{3x\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3ix^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{ix^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[a + b*x],x]
```

```
[Out] (I/4)*x^4 - (x^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3*I)/2)*x^2*PolyLog[2
, -E^((2*I)*(a + b*x))])/b^2 - (3*x*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^
3) - (((3*I)/4)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\tan(a+bx)\,dx &= \frac{ix^4}{4}-2i\int\frac{e^{2i(a+bx)}x^3}{1+e^{2i(a+bx)}}\,dx\\
&= \frac{ix^4}{4}-\frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3\int x^2\log\left(1+e^{2i(a+bx)}\right)\,dx}{b}\\
&= \frac{ix^4}{4}-\frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3ix^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{(3i)\int x\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)\,dx}{b^2}\\
&= \frac{ix^4}{4}-\frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3ix^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3x\mathrm{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3\int\mathrm{Li}_3\left(-\right.}{2}\\
&= \frac{ix^4}{4}-\frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3ix^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3x\mathrm{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{(3i)\mathrm{Subst}}{}\\
&= \frac{ix^4}{4}-\frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3ix^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3x\mathrm{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{3i\mathrm{Li}_4\left(-e^{2}\right.}{4b^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 106, normalized size = 1.00

$$\frac{ix^4}{4}-\frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3ix^2\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3x\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3i\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[a + b*x],x]
```

```
[Out] (I/4)*x^4 - (x^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3*I)/2)*x^2*PolyLog[2
, -E^((2*I)*(a + b*x))])/b^2 - (3*x*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^
3) - (((3*I)/4)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4
```

Maple [A]
time = 0.16, size = 125, normalized size = 1.18

method	result
risch	$\frac{ix^4}{4}+\frac{2ia^3x}{b^3}+\frac{3ia^4}{2b^4}-\frac{x^3\ln(\mathrm{e}^{2i(bx+a)}+1)}{b}+\frac{3ix^2\operatorname{polylog}(2,-\mathrm{e}^{2i(bx+a)})}{2b^2}-\frac{3x\operatorname{polylog}(3,-\mathrm{e}^{2i(bx+a)})}{2b^3}-\frac{3i\operatorname{polylog}(4,-\mathrm{e}^{2i(bx}}{4b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*x^4+2*I/b^3*a^3*x+3/2*I/b^4*a^4-x^3*ln(exp(2*I*(b*x+a))+1)/b+3/2*I*x^
2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*x*polylog(3,-exp(2*I*(b*x+a)))/b^3-3
/4*I*polylog(4,-exp(2*I*(b*x+a)))/b^4-2/b^4*a^3*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(83) = 166$.
time = 0.53, size = 243, normalized size = 2.29

$$-\frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2+12a^3\log(\sec(bx+a))-4\left(-4i(bx+a)^3+9i(bx+a)^2a-9i(bx+a)a^2\right)\arctan(\sin(2bx+2a),\cos(2bx+2a)+1)-6\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\mathrm{Li}_2(-e^{(2ibx+2ia)})+2\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)+6(4bx+a)\mathrm{Li}_3(-e^{(2ibx+2ia)})+12i\,\mathrm{Li}_4(-e^{(2ibx+2ia)})}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/12*(-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x + a)^2*a^2 + 12*a^
3*log(sec(b*x + a)) - 4*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x +
a)*a^2)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(4*I*(b*x + a)^
2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(4*(b*x + a)
^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(4*b*x + a)*polylog(3, -e^(2*I*b*x +
 2*I*a)) + 12*I*polylog(4, -e^(2*I*b*x + 2*I*a)))/b^4
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(83) = 166$.
time = 0.38, size = 286, normalized size = 2.70

$$-\frac{4b^3x^3\log\left(-\frac{2(i\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+4b^3x^3\log\left(-\frac{2(-i\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+6ib^2x^2\mathrm{Li}_2\left(\frac{2(i\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-6ib^2x^2\mathrm{Li}_2\left(\frac{2(-i\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+6bx\operatorname{polylog}\left(3,\frac{\tan(bx+a)^2+2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+6bx\operatorname{polylog}\left(3,\frac{\tan(bx+a)^2-2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)-3i\operatorname{polylog}\left(4,\frac{\tan(bx+a)^2+2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+3i\operatorname{polylog}\left(4,\frac{\tan(bx+a)^2-2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^3*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 4*b^3*x
^3*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*I*b^2*x^2*dilog(2
*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 6*I*b^2*x^2*dilog(2*(-I*t
an(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 6*b*x*polylog(3, (tan(b*x + a)
^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*b*x*polylog(3, (tan(b*
x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 3*I*polylog(4, (ta
n(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 3*I*polylog(4,
 (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)))/b^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \tan{(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(b*x+a),x)
```

```
[Out] Integral(x**3*tan(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*tan(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{tan}\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(a + b*x),x)
```

```
[Out] int(x^3*tan(a + b*x), x)
```

3.2 $\int x^2 \tan(a+bx)\,dx$

Optimal. Leaf size=77

$$\frac{ix^3}{3}-\frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{ix\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}$$

```
[Out] 1/3*I*x^3-x^2*ln(1+exp(2*I*(b*x+a)))/b+I*x*polylog(2,-exp(2*I*(b*x+a)))/b^2
-1/2*polylog(3,-exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {3800, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{ix\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{ix^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tan[a + b*x],x]
```

```
[Out] (I/3)*x^3 - (x^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*x*PolyLog[2, -E^((2*I
)*(a + b*x))])/b^2 - PolyLog[3, -E^((2*I)*(a + b*x))]/(2*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \tan(a+bx)\,dx &= \frac{ix^3}{3} - 2i\int \frac{e^{2i(a+bx)}x^2}{1+e^{2i(a+bx)}}\,dx \\
&= \frac{ix^3}{3} - \frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2\int x\log\left(1+e^{2i(a+bx)}\right)\,dx}{b} \\
&= \frac{ix^3}{3} - \frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{ix\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{i\int \mathrm{Li}_2\left(-e^{2i(a+bx)}\right)\,dx}{b^2} \\
&= \frac{ix^3}{3} - \frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{ix\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(-x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^3} \\
&= \frac{ix^3}{3} - \frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{ix\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{\mathrm{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 77, normalized size = 1.00

$$\frac{ix^3}{3} - \frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{ix\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tan[a + b*x],x]
```

```
[Out] (I/3)*x^3 - (x^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*x*PolyLog[2, -E^((2*I
)*(a + b*x))])/b^2 - PolyLog[3, -E^((2*I)*(a + b*x))]/(2*b^3)
```

Maple [A]
time = 0.06, size = 103, normalized size = 1.34

method	result	size
risch	$\frac{ix^3}{3}-\frac{2ia^2x}{b^2}-\frac{4ia^3}{3b^3}-\frac{x^2\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b}+\frac{ix\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(bx+a)}\right)}{b^2}-\frac{\mathrm{polylog}\left(3,-\mathrm{e}^{2i(bx+a)}\right)}{2b^3}+\frac{2a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*x^3-2*I/b^2*a^2*x-4/3*I/b^3*a^3-x^2*ln(exp(2*I*(b*x+a))+1)/b+I*x*poly
log(2,-exp(2*I*(b*x+a)))/b^2-1/2*polylog(3,-exp(2*I*(b*x+a)))/b^3+2/b^3*a^2
*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(62) = 124$.
time = 0.55, size = 163, normalized size = 2.12

$$-\frac{-2i(bx+a)^3+6i(bx+a)^2a-6i\,bx\mathrm{Li}_2(-e^{(2i\,bx+2i\,a)})-6a^2\log(\sec(bx+a))-6\left(-i(bx+a)^2+2i(bx+a)a\right)\arctan(\sin(2bx+2a),\cos(2bx+2a)+1)+3\left((bx+a)^2-2(bx+a)a\right)\log\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)+3\,\mathrm{Li}_3(-e^{(2i\,bx+2i\,a)})}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(-2*I*(b*x + a)^3 + 6*I*(b*x + a)^2*a - 6*I*b*x*dilog(-e^(2*I*b*x + 2*
I*a)) - 6*a^2*log(sec(b*x + a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arct
an2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*
a)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) +
3*polylog(3, -e^(2*I*b*x + 2*I*a)))/b^3
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(62) = 124$.
time = 0.37, size = 200, normalized size = 2.60

$$-\frac{2b^2x^2\log\left(-\frac{2(i\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+2b^2x^2\log\left(-\frac{2(-i\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+2i\,bx\mathrm{Li}_2\left(\frac{2(i\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-2i\,bx\mathrm{Li}_2\left(\frac{2(-i\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2+2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2-2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^2*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*b^2*x
^2*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*I*b*x*dilog(2*(I*
tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 2*I*b*x*dilog(2*(-I*tan(b*x +
 a) - 1)/(tan(b*x + a)^2 + 1) + 1) + polylog(3, (tan(b*x + a)^2 + 2*I*tan(b
*x + a) - 1)/(tan(b*x + a)^2 + 1)) + polylog(3, (tan(b*x + a)^2 - 2*I*tan(b
*x + a) - 1)/(tan(b*x + a)^2 + 1)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\tan(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tan(b*x+a),x)
```

```
[Out] Integral(x**2*tan(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*tan(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \tan(a + b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(a + b*x),x)
```

```
[Out] int(x^2*tan(a + b*x), x)
```

3.3 $\int x\tan(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{ix^2}{2}-\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}$$

```
[Out] 1/2*I*x^2-x*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*polylog(2,-exp(2*I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {3800, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[a + b*x],x]
```

```
[Out] (I/2)*x^2 - (x*Log[1 + E^((2*I)*(a + b*x))])/b + ((I/2)*PolyLog[2, -E^((2*I
)*(a + b*x))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
```

```
[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\tan(a+bx)\,dx &= \frac{ix^2}{2}-2i\int\frac{e^{2i(a+bx)}x}{1+e^{2i(a+bx)}}\,dx\\
&= \frac{ix^2}{2}-\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{\int\log\left(1+e^{2i(a+bx)}\right)\,dx}{b}\\
&= \frac{ix^2}{2}-\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{i\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2i(a+bx)}\right)}{2b^2}\\
&= \frac{ix^2}{2}-\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$\frac{ix^2}{2}-\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[a + b*x],x]
[Out] (I/2)*x^2 - (x*Log[1 + E^((2*I)*(a + b*x))])/b + ((I/2)*PolyLog[2, -E^((2*I
)*(a + b*x))])/b^2
```

Maple [A]
time = 0.08, size = 78, normalized size = 1.44

method	result	size
risch	$\frac{ix^2}{2}+\frac{2iax}{b}+\frac{ia^2}{b^2}-\frac{x\ln\left(e^{2i(bx+a)}+1\right)}{b}+\frac{i\,\mathrm{polylog}\left(2,-e^{2i(bx+a)}\right)}{2b^2}-\frac{2a\ln\left(e^{i(bx+a)}\right)}{b^2}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*I*x^2+2*I/b*a*x+I/b^2*a^2-x*ln(exp(2*I*(b*x+a))+1)/b+1/2*I*polylog(2,-e
xp(2*I*(b*x+a)))/b^2-2/b^2*a*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(41) = 82.
time = 0.55, size = 92, normalized size = 1.70

$$-\frac{-i\,b^2x^2+2i\,bx\arctan\left(\sin\left(2\,bx+2\,a\right),\cos\left(2\,bx+2\,a\right)+1\right)+bx\log\left(\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,bx+2\,a\right)^2+2\,\cos\left(2\,bx+2\,a\right)+1\right)-i\,\mathrm{Li}_2\left(-e^{(2i\,bx+2i\,a)}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*b^2*x^2 + 2*I*b*x*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1)
+ b*x*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
 - I*dilog(-e^(2*I*b*x + 2*I*a)))/b^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(41) = 82$.
time = 0.36, size = 122, normalized size = 2.26

$$-\frac{2\,bx\log\left(-\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+2\,bx\log\left(-\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+i\,\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-i\,\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*b*x*log(-
2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + I*dilog(2*(I*tan(b*x + a) -
 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x +
a)^2 + 1) + 1))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\tan\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a),x)
```

```
[Out] Integral(x*tan(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*tan(b*x + a), x)
```

Mupad [B]
time = 2.46, size = 129, normalized size = 2.39

$$-\frac{\pi\,\ln\left(\cos\left(b\,x\right)\right)+\mathrm{polylog}\left(2,-{\mathrm{e}}^{-a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-b\,x\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}-\pi\,\ln\left({\mathrm{e}}^{-a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-b\,x\,2{}\mathrm{i}}+1\right)+2\,a\,\ln\left({\mathrm{e}}^{-a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-b\,x\,2{}\mathrm{i}}+1\right)-\pi\,\ln\left({\mathrm{e}}^{b\,x\,2{}\mathrm{i}}+1\right)+b^2\,x^2\,1{}\mathrm{i}-\ln\left(\cos\left(a+b\,x\right)\right)\,\left(2\,a-\pi\right)+2\,b\,x\,\ln\left({\mathrm{e}}^{-a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-b\,x\,2{}\mathrm{i}}+1\right)+a\,b\,x\,2{}\mathrm{i}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(a + b*x),x)
```

```
[Out] -(polylog(2, -exp(-a*2i)*exp(-b*x*2i))*1i - pi*log(exp(b*x*2i) + 1) - pi*lo
g(exp(-a*2i)*exp(-b*x*2i) + 1) + 2*a*log(exp(-a*2i)*exp(-b*x*2i) + 1) + pi*
log(cos(b*x)) + b^2*x^2*1i - log(cos(a + b*x))*(2*a - pi) + 2*b*x*log(exp(-
a*2i)*exp(-b*x*2i) + 1) + a*b*x*2i)/(2*b^2)
```

3.4 $\int \frac{\tan(a+bx)}{x}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\tan(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan(a+bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]/x,x]
```

```
[Out] Defer[Int][Tan[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{\tan(a+bx)}{x}\,dx = \int \frac{\tan(a+bx)}{x}\,dx$$

Mathematica [A]
time = 2.12, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]/x,x]
```

```
[Out] Integrate[Tan[a + b*x]/x, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\tan(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)/x,x)
```

```
[Out] int(tan(b*x+a)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(tan(b*x + a)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(a + bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)/x,x)
```

```
[Out] Integral(tan(a + b*x)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\tan(a + b\,x)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/x,x)
```

```
[Out] int(tan(a + b*x)/x, x)
```

3.5 $\int \frac{\tan(a+bx)}{x^2} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\tan(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan(a+bx)}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]/x^2,x]
[Out] Defer[Int][Tan[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{\tan(a+bx)}{x^2} \, dx = \int \frac{\tan(a+bx)}{x^2} \, dx$$

Mathematica [A]
time = 3.19, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]/x^2,x]
[Out] Integrate[Tan[a + b*x]/x^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\tan(bx+a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)/x^2,x)
```

```
[Out] int(tan(b*x+a)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(tan(b*x + a)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{\left(a + bx \right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)/x**2,x)
```

```
[Out] Integral(tan(a + b*x)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\tan(a+b\,x)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/x^2,x)
```

```
[Out] int(tan(a + b*x)/x^2, x)
```

3.6 $\int x^3 \tan^2(a+bx)\,dx$

Optimal. Leaf size=98

$$-\frac{ix^3}{b}-\frac{x^4}{4}+\frac{3x^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{3ix\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{3\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4}+\frac{x^3\tan(a+bx)}{b}$$

```
[Out] -I*x^3/b-1/4*x^4+3*x^2*ln(1+exp(2*I*(b*x+a)))/b^2-3*I*x*polylog(2,-exp(2*I*
(b*x+a)))/b^3+3/2*polylog(3,-exp(2*I*(b*x+a)))/b^4+x^3*tan(b*x+a)/b
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3801, 3800, 2221, 2611, 2320, 6724, 30}

$$\frac{3\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3ix\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3x^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{x^3\tan(a+bx)}{b}-\frac{ix^3}{b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[a + b*x]^2,x]
```

```
[Out] ((-I)*x^3)/b - x^4/4 + (3*x^2*Log[1 + E^((2*I)*(a + b*x))])/b^2 - ((3*I)*x*
PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + (3*PolyLog[3, -E^((2*I)*(a + b*x))]
)/(2*b^4) + (x^3*Tan[a + b*x])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \tan^2(a+bx)\,dx &= \frac{x^3\tan(a+bx)}{b} - \frac{3\int x^2\tan(a+bx)\,dx}{b} - \int x^3\,dx \\
&= -\frac{ix^3}{b} - \frac{x^4}{4} + \frac{x^3\tan(a+bx)}{b} + \frac{(6i)\int \frac{e^{2i(a+bx)}x^2}{1+e^{2i(a+bx)}}\,dx}{b} \\
&= -\frac{ix^3}{b} - \frac{x^4}{4} + \frac{3x^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{x^3\tan(a+bx)}{b} - \frac{6\int x\log\left(1+e^{2i(a+bx)}\right)\,dx}{b^2} \\
&= -\frac{ix^3}{b} - \frac{x^4}{4} + \frac{3x^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3ix\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{x^3\tan(a+bx)}{b} + \frac{(3i)}{} \\
&= -\frac{ix^3}{b} - \frac{x^4}{4} + \frac{3x^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3ix\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{x^3\tan(a+bx)}{b} + \frac{3\mathrm{Sul}}{} \\
&= -\frac{ix^3}{b} - \frac{x^4}{4} + \frac{3x^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3ix\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{3\mathrm{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^4} + \frac{x^3}{}
\end{aligned}
$$

Mathematica [A]
time = 1.20, size = 122, normalized size = 1.24

$$-\frac{x^4}{4}+\frac{2b^2x^2\left(-\frac{2ibe^{2ia}x}{1+e^{2ia}}+3\log\left(1+e^{2i(a+bx)}\right)\right)-6ibx\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)+3\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4}+\frac{x^3\sec(a)\sec(a+bx)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[a + b*x]^2,x]
```

```
[Out] -1/4*x^4 + (2*b^2*x^2*(((-2*I)*b*E^((2*I)*a)*x)/(1 + E^((2*I)*a)) + 3*Log[1
 + E^((2*I)*(a + b*x))]) - (6*I)*b*x*PolyLog[2, -E^((2*I)*(a + b*x))] + 3*P
olyLog[3, -E^((2*I)*(a + b*x))])/(2*b^4) + (x^3*Sec[a]*Sec[a + b*x]*Sin[b*x
])/b
```

Maple [A]
time = 0.09, size = 133, normalized size = 1.36

method	result
risch	$-\frac{x^4}{4}+\frac{2ix^3}{b\left(e^{2i(bx+a)}+1\right)}-\frac{2ix^3}{b}+\frac{6ia^2x}{b^3}+\frac{4ia^3}{b^4}+\frac{3x^2\ln\left(e^{2i(bx+a)}+1\right)}{b^2}-\frac{3ix\,\mathrm{polylog}\left(2,-e^{2i(bx+a)}\right)}{b^3}+\frac{3\,\mathrm{polylog}\left(3,-e^{2i(bx}\right.}{2b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^4+2*I*x^3/b/(exp(2*I*(b*x+a))+1)-2*I/b*x^3+6*I/b^3*a^2*x+4*I/b^4*a^3
+3*x^2*ln(exp(2*I*(b*x+a))+1)/b^2-3*I*x*polylog(2,-exp(2*I*(b*x+a)))/b^3+3/
2*polylog(3,-exp(2*I*(b*x+a)))/b^4-6/b^4*a^2*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(83) = 166$.
time = 0.56, size = 639, normalized size = 6.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a - tan(b*x + a))*a^3 - 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (
b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^
2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x
+ a)*sin(2*b*x + 2*a))*a^2/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos
(2*b*x + 2*a) + 1) + 2*(I*(b*x + a)^4 - 4*I*(b*x + a)^3*a + 12*((b*x + a)^2
 - 2*(b*x + a)*a + ((b*x + a)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*a) - (-I*(b*
x + a)^2 + 2*I*(b*x + a)*a)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos
(2*b*x + 2*a) + 1) + (I*(b*x + a)^4 - 4*(b*x + a)^3*(I*a + 2) + 24*(b*x + a
```

```
)^2*a)*cos(2*b*x + 2*a) - 12*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x + 2*a)
 + b*x)*dilog(-e^(2*I*b*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a +
(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*cos(2*b*x + 2*a) - ((b*x + a)^2 - 2*(b*x
+ a)*a)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c
os(2*b*x + 2*a) + 1) - 6*(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + I)*polylo
g(3, -e^(2*I*b*x + 2*I*a)) - ((b*x + a)^4 - 4*(b*x + a)^3*(a - 2*I) - 24*I*
(b*x + a)^2*a)*sin(2*b*x + 2*a))/(-4*I*cos(2*b*x + 2*a) + 4*sin(2*b*x + 2*a
) - 4*I))/b^4
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(83) = 166$.
time = 0.37, size = 225, normalized size = 2.30

$$-\frac{b^4x^4 - 4\,b^3x^3\tan\left(bx+a\right) - 6\,b^2x^2\log\left(-\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right) - 6\,b^2x^2\log\left(-\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right) - 6i\,bx\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right) + 6i\,bx\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right) - 3\,\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) - 3\,\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 - 4*b^3*x^3*tan(b*x + a) - 6*b^2*x^2*log(-2*(I*tan(b*x + a) -
 1)/(tan(b*x + a)^2 + 1)) - 6*b^2*x^2*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x
 + a)^2 + 1)) - 6*I*b*x*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) +
 1) + 6*I*b*x*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 3*p
olylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 3
*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)))/
b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\tan^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(b*x+a)**2,x)
```

```
[Out] Integral(x**3*tan(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*tan(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \tan(a + b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(a + b*x)^2,x)
```

```
[Out] int(x^3*tan(a + b*x)^2, x)
```

3.7 $\int x^2 \tan^2(a+bx)\,dx$

Optimal. Leaf size=73

$$-\frac{ix^2}{b}-\frac{x^3}{3}+\frac{2x\log\left(1+e^{2i(a+bx)}\right)}{b^2}-\frac{i\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{x^2\tan(a+bx)}{b}$$

```
[Out] -I*x^2/b-1/3*x^3+2*x*ln(1+exp(2*I*(b*x+a)))/b^2-I*polylog(2,-exp(2*I*(b*x+a
)))/b^3+x^2*tan(b*x+a)/b
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3800, 2221, 2317, 2438, 30}

$$-\frac{i\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{2x\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{x^2\tan(a+bx)}{b}-\frac{ix^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tan[a + b*x]^2,x]
```

```
[Out] ((-I)*x^2)/b - x^3/3 + (2*x*Log[1 + E^((2*I)*(a + b*x))])/b^2 - (I*PolyLog[
2, -E^((2*I)*(a + b*x))])/b^3 + (x^2*Tan[a + b*x])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tan^2(a+bx)\,dx &= \frac{x^2\tan(a+bx)}{b} - \frac{2\int x\tan(a+bx)\,dx}{b} - \int x^2\,dx \\
&= -\frac{ix^2}{b} - \frac{x^3}{3} + \frac{x^2\tan(a+bx)}{b} + \frac{(4i)\int \frac{e^{2i(a+bx)}x}{1+e^{2i(a+bx)}}\,dx}{b} \\
&= -\frac{ix^2}{b} - \frac{x^3}{3} + \frac{2x\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{x^2\tan(a+bx)}{b} - \frac{2\int \log\left(1+e^{2i(a+bx)}\right)\,dx}{b^2} \\
&= -\frac{ix^2}{b} - \frac{x^3}{3} + \frac{2x\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{x^2\tan(a+bx)}{b} + \frac{i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i(a+}\right.}{b^3} \\
&= -\frac{ix^2}{b} - \frac{x^3}{3} + \frac{2x\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{i\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{x^2\tan(a+bx)}{b}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 189 vs. 2(73) = 146.
time = 6.16, size = 189, normalized size = 2.59

$$
-\frac{x^3}{3} + \frac{\csc(a)\left(b^2 e^{-i\text{ArcTan}(\cot(a))}x^2 - \frac{\cot(a)\left(ibx(-\pi-2\text{ArcTan}(\cot(a))) - \pi\log\left(1+e^{-2ibx}\right) - 2(bx-\text{ArcTan}(\cot(a)))\log\left(1-e^{2i(bx-\text{ArcTan}(\cot(a)))}\right) + \pi\log(\cos(bx)) - 2\text{ArcTan}(\cot(a))\log(\sin(bx-\text{ArcTan}(\cot(a)))) + i\text{PolyLog}\left(2, e^{2i(bx-\text{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{1+\cot^2(a)}}\right)\sec(a)}{b^3\sqrt{\csc^2(a)\left(\cos^2(a)+\sin^2(a)\right)}} + \frac{x^2\sec(a)\sec(a+bx)\sin(bx)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tan[a + b*x]^2,x]
```

```
[Out] -1/3*x^3 + (Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi -
2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*L
og[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[
a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Co
```

```
t[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a
]^2)]) + (x^2*Sec[a]*Sec[a + b*x]*Sin[b*x])/b
```

Maple [A]
time = 0.09, size = 108, normalized size = 1.48

method	result	siz
risch	$-\frac{x^3}{3}+\frac{2ix^2}{b\left(e^{2i(bx+a)}+1\right)}-\frac{2ix^2}{b}-\frac{4iax}{b^2}-\frac{2ia^2}{b^3}+\frac{2x\ln\left(e^{2i(bx+a)}+1\right)}{b^2}-\frac{i\,\mathrm{polylog}\left(2,-e^{2i(bx+a)}\right)}{b^3}+\frac{4a\ln\left(e^{i(bx+a)}\right)}{b^3}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3+2*I*x^2/b/(exp(2*I*(b*x+a))+1)-2*I/b*x^2-4*I/b^2*a*x-2*I/b^3*a^2+2
*x*ln(exp(2*I*(b*x+a))+1)/b^2-I*polylog(2,-exp(2*I*(b*x+a)))/b^3+4/b^3*a*ln
(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(62) = 124$.
time = 0.61, size = 257, normalized size = 3.52

$$\frac{i\,b^3x^3+6\,(bx\cos(2\,bx+2\,a)+i\,bx\sin(2\,bx+2\,a)+bx)\arctan(\sin(2\,bx+2\,a),\cos(2\,bx+2\,a)+1)+(i\,b^3x^3-6\,b^2x^2)\cos(2\,bx+2\,a)-3\,(\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a)+1)\mathrm{Li}_2(-e^{(2i\,bx+2i\,a)})-3\,(i\,bx\cos(2\,bx+2\,a)-bx\sin(2\,bx+2\,a)+i\,bx)\log(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1)-(b^3x^3+6i\,b^2x^2)\sin(2\,bx+2\,a)}{-3i\,b^3\cos(2\,bx+2\,a)+3\,b^3\sin(2\,bx+2\,a)-3i\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (I*b^3*x^3 + 6*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x + 2*a) + b*x)*arctan
2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (I*b^3*x^3 - 6*b^2*x^2)*cos(2*b
*x + 2*a) - 3*(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*dilog(-e^(2*I*b*x
 + 2*I*a)) - 3*(I*b*x*cos(2*b*x + 2*a) - b*x*sin(2*b*x + 2*a) + I*b*x)*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b^3*x^
3 + 6*I*b^2*x^2)*sin(2*b*x + 2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2
*b*x + 2*a) - 3*I*b^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(62) = 124$.
time = 0.40, size = 144, normalized size = 1.97

$$-\frac{2\,b^3x^3-6\,b^2x^2\tan(bx+a)-6\,bx\log\left(-\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)-6\,bx\log\left(-\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)-3i\,\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+3i\,\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*x^3 - 6*b^2*x^2*tan(b*x + a) - 6*b*x*log(-2*(I*tan(b*x + a) - 1
)/(tan(b*x + a)^2 + 1)) - 6*b*x*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^
```

```
2 + 1)) - 3*I*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 3*I*
dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \tan^2 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tan(b*x+a)**2,x)
```

```
[Out] Integral(x**2*tan(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*tan(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \tan(a + b\, x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(a + b*x)^2,x)
```

```
[Out] int(x^2*tan(a + b*x)^2, x)
```

3.8 $\int x \tan^2(a+bx)\, dx$

Optimal. Leaf size=30

$$-\frac{x^2}{2}+\frac{\log(\cos(a+bx))}{b^2}+\frac{x\tan(a+bx)}{b}$$

```
[Out] -1/2*x^2+ln(cos(b*x+a))/b^2+x*tan(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3801, 3556, 30}

$$\frac{\log(\cos(a+bx))}{b^2}+\frac{x\tan(a+bx)}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[a + b*x]^2,x]
```

```
[Out] -1/2*x^2 + Log[Cos[a + b*x]]/b^2 + (x*Tan[a + b*x])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\tan^2(a+bx)\,dx &= \frac{x\tan(a+bx)}{b}-\frac{\int \tan(a+bx)\,dx}{b}-\int x\,dx\\ &= -\frac{x^2}{2}+\frac{\log(\cos(a+bx))}{b^2}+\frac{x\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 43, normalized size = 1.43

$$-\frac{x^2}{2}+\frac{\log(\cos(a+bx))}{b^2}+\frac{x\sec(a)\sec(a+bx)\sin(bx)}{b}+\frac{x\tan(a)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[a + b*x]^2,x]
```

```
[Out] -1/2*x^2 + Log[Cos[a + b*x]]/b^2 + (x*Sec[a]*Sec[a + b*x]*Sin[b*x])/b + (x*
Tan[a])/b
```

Maple [A]
time = 0.08, size = 40, normalized size = 1.33

method	result	size
norman	$\frac{x\tan(bx+a)}{b}-\frac{x^2}{2}-\frac{\ln(1+\tan^2(bx+a))}{2b^2}$	34
default	$-\frac{x^2}{2}+\frac{(bx+a)\tan(bx+a)+\ln(\cos(bx+a))-a\tan(bx+a)}{b^2}$	40
risch	$-\frac{x^2}{2}-\frac{2ix}{b}-\frac{2ia}{b^2}+\frac{2ix}{b(\mathrm{e}^{2i(bx+a)}+1)}+\frac{\ln(\mathrm{e}^{2i(bx+a)}+1)}{b^2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2+1/b^2*((b*x+a)*tan(b*x+a)+ln(cos(b*x+a))-a*tan(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(28) = 56$.
time = 0.53, size = 214, normalized size = 7.13

$$\frac{2\,(bx+a-\tan(bx+a))a-\frac{(bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2+2\,(bx+a)^2\cos(2\,bx+2\,a)+(bx+a)^2-\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)\log\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)-4\,(bx+a)\sin(2\,bx+2\,a)}{\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a - tan(b*x + a))*a - ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x
 + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 -
(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)
*sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
 2*a) + 1))/b^2
```

Fricas [A]
time = 0.39, size = 38, normalized size = 1.27

$$-\frac{b^2x^2-2\,bx\tan(bx+a)-\log\left(\frac{1}{\tan(bx+a)^2+1}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - 2*b*x*tan(b*x + a) - log(1/(tan(b*x + a)^2 + 1)))/b^2
```

Sympy [A]
time = 0.08, size = 41, normalized size = 1.37

$$\begin{cases} -\frac{x^{2}}{2} + \frac{x \tan{\left(a + b x \right)}}{b} - \frac{\log{\left(\tan^{2}{\left(a + b x \right)} + 1 \right)}}{2 b^{2}} & \text{for}\: b \neq 0 \\ \frac{x^{2} \tan^{2}{\left(a \right)}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a)**2,x)
```

```
[Out] Piecewise((-x**2/2 + x*tan(a + b*x)/b - log(tan(a + b*x)**2 + 1)/(2*b**2),
Ne(b, 0)), (x**2*tan(a)**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(28) = 56.
time = 0.70, size = 182, normalized size = 6.07

$$-\frac{b^{2} x^{2} \tan\left(b x\right) \tan\left(a\right) - b^{2} x^{2} + 2 \, b x \tan\left(b x\right) + 2 \, b x \tan\left(a\right) - \log\left(\frac{4 \, {\left(\tan\left(b x\right)^{4} \tan\left(a\right)^{2} - 2 \, \tan\left(b x\right)^{3} \tan\left(a\right) + \tan\left(b x\right)^{2} \tan\left(a\right)^{2} + \tan\left(b x\right)^{2} - 2 \, \tan\left(b x\right) \tan\left(a\right) + 1\right)}}{\tan\left(a\right)^{2} + 1}\right) \tan\left(b x\right) \tan\left(a\right) + \log\left(\frac{4 \, {\left(\tan\left(b x\right)^{4} \tan\left(a\right)^{2} - 2 \, \tan\left(b x\right)^{3} \tan\left(a\right) + \tan\left(b x\right)^{2} \tan\left(a\right)^{2} + \tan\left(b x\right)^{2} - 2 \, \tan\left(b x\right) \tan\left(a\right) + 1\right)}}{\tan\left(a\right)^{2} + 1}\right)}{2 \, {\left(b^{2} \tan\left(b x\right) \tan\left(a\right) - b^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*x^2*tan(b*x)*tan(a) - b^2*x^2 + 2*b*x*tan(b*x) + 2*b*x*tan(a) - l
og(4*(tan(b*x)^4*tan(a)^2 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + tan
(b*x)^2 - 2*tan(b*x)*tan(a) + 1)/(tan(a)^2 + 1))*tan(b*x)*tan(a) + log(4*(t
an(b*x)^4*tan(a)^2 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + tan(b*x)^2
 - 2*tan(b*x)*tan(a) + 1)/(tan(a)^2 + 1)))/(b^2*tan(b*x)*tan(a) - b^2)
```

Mupad [B]
time = 0.11, size = 35, normalized size = 1.17

$$-\frac{\frac{\ln\left({\mathrm{tan}\left(a+b\,x\right)}^2+1\right)}{2}-b\,x\,\mathrm{tan}\left(a+b\,x\right)}{b^2}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(a + b*x)^2,x)
```

```
[Out] - (log(tan(a + b*x)^2 + 1)/2 - b*x*tan(a + b*x))/b^2 - x^2/2
```

3.9 $\int \frac{\tan^2(a+bx)}{x}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\tan^2(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^2/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][Tan[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\tan^2(a+bx)}{x}\,dx = \int \frac{\tan^2(a+bx)}{x}\,dx$$

Mathematica [A]
time = 2.74, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^2/x,x]
```

```
[Out] Integrate[Tan[a + b*x]^2/x, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^2/x,x)
```

```
[Out] int(tan(b*x+a)^2/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -(b*x*cos(2*b*x + 2*a)^2*log(x) + b*x*log(x)*sin(2*b*x + 2*a)^2 + 2*b*x*cos
(2*b*x + 2*a)*log(x) + b*x*log(x) - 2*(b^2*x*cos(2*b*x + 2*a)^2 + b^2*x*sin
(2*b*x + 2*a)^2 + 2*b^2*x*cos(2*b*x + 2*a) + b^2*x)*integrate(sin(2*b*x + 2
*a)/(b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(2*b*x + 2*a)^2 + 2*b^2*x^2*co
s(2*b*x + 2*a) + b^2*x^2), x) - 2*sin(2*b*x + 2*a))/(b*x*cos(2*b*x + 2*a)^2
 + b*x*sin(2*b*x + 2*a)^2 + 2*b*x*cos(2*b*x + 2*a) + b*x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^2/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a + bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**2/x,x)
```

```
[Out] Integral(tan(a + b*x)**2/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^2/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\tan(a+b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/x,x)
```

```
[Out] int(tan(a + b*x)^2/x, x)
```

3.10 $\int \frac{\tan^2(a+bx)}{x^2} \, dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(\frac{\tan^2(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^2/x^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][Tan[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\tan^2(a+bx)}{x^2} \, dx = \int \frac{\tan^2(a+bx)}{x^2} \, dx$$

Mathematica [A]
time = 3.13, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[Tan[a + b*x]^2/x^2, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(bx+a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^2/x^2,x)
```

```
[Out] int(tan(b*x+a)^2/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] (b*x*cos(2*b*x + 2*a)^2 + b*x*sin(2*b*x + 2*a)^2 + 2*b*x*cos(2*b*x + 2*a) +
 b*x + 4*(b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(2*b*x + 2*a)^2 + 2*b^2*x
^2*cos(2*b*x + 2*a) + b^2*x^2)*integrate(sin(2*b*x + 2*a)/(b^2*x^3*cos(2*b*
x + 2*a)^2 + b^2*x^3*sin(2*b*x + 2*a)^2 + 2*b^2*x^3*cos(2*b*x + 2*a) + b^2*
x^3), x) + 2*sin(2*b*x + 2*a))/(b*x^2*cos(2*b*x + 2*a)^2 + b*x^2*sin(2*b*x
+ 2*a)^2 + 2*b*x^2*cos(2*b*x + 2*a) + b*x^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^2/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**2/x**2,x)
```

```
[Out] Integral(tan(a + b*x)**2/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^2/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\tan(a+b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/x^2,x)
```

```
[Out] int(tan(a + b*x)^2/x^2, x)
```

3.11 $\int x^3 \tan^3(a+bx)\,dx$

Optimal. Leaf size=205

$$\frac{3ix^2}{2b^2}+\frac{x^3}{2b}-\frac{ix^4}{4}-\frac{3x\log\left(1+e^{2i(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{3i\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4}-\frac{3ix^2\text{PolyLog}\left(2,-e^{2\right.}}{2b^2}$$

```
[Out] 3/2*I*x^2/b^2+1/2*x^3/b-1/4*I*x^4-3*x*ln(1+exp(2*I*(b*x+a)))/b^3+x^3*ln(1+e
xp(2*I*(b*x+a)))/b+3/2*I*polylog(2,-exp(2*I*(b*x+a)))/b^4-3/2*I*x^2*polylog
(2,-exp(2*I*(b*x+a)))/b^2+3/2*x*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*poly
log(4,-exp(2*I*(b*x+a)))/b^4-3/2*x^2*tan(b*x+a)/b^2+1/2*x^3*tan(b*x+a)^2/b
```

Rubi [A]
time = 0.21, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3801, 3800, 2221, 2317, 2438, 30, 2611, 6744, 2320, 6724}

$$\frac{3i\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}+\frac{3i\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3x\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{3x\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3ix^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{3x^2\tan(a+bx)}{2b^2}+\frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{x^3\tan^2(a+bx)}{2b}+\frac{3ix^2}{2b^2}+\frac{x^3}{2b}-\frac{ix^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*x^2)/b^2 + x^3/(2*b) - (I/4)*x^4 - (3*x*Log[1 + E^((2*I)*(a + b*
x))])/b^3 + (x^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3*I)/2)*PolyLog[2, -E
^((2*I)*(a + b*x))])/b^4 - (((3*I)/2)*x^2*PolyLog[2, -E^((2*I)*(a + b*x))])
/b^2 + (3*x*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*PolyLog[
4, -E^((2*I)*(a + b*x))])/b^4 - (3*x^2*Tan[a + b*x])/(2*b^2) + (x^3*Tan[a +
 b*x]^2)/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\tan^3(a+bx)\,dx &= \frac{x^3\tan^2(a+bx)}{2b} - \frac{3\int x^2\tan^2(a+bx)\,dx}{2b} - \int x^3\tan(a+bx)\,dx \\
&= -\frac{ix^4}{4} - \frac{3x^2\tan(a+bx)}{2b^2} + \frac{x^3\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}x^3}{1+e^{2i(a+bx)}}\,dx + \frac{3\int x\tan(a+b}{b^2} \\
&= \frac{3ix^2}{2b^2} + \frac{x^3}{2b} - \frac{ix^4}{4} + \frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3x^2\tan(a+bx)}{2b^2} + \frac{x^3\tan^2(a+bx)}{2b} - \frac{(6i}{} \\
&= \frac{3ix^2}{2b^2} + \frac{x^3}{2b} - \frac{ix^4}{4} - \frac{3x\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3ix^2\mathrm{Li}_2\left(-e^{2i(a+b}\right.}{2b^2} \\
&= \frac{3ix^2}{2b^2} + \frac{x^3}{2b} - \frac{ix^4}{4} - \frac{3x\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3ix^2\mathrm{Li}_2\left(-e^{2i(a+b}\right.}{2b^2} \\
&= \frac{3ix^2}{2b^2} + \frac{x^3}{2b} - \frac{ix^4}{4} - \frac{3x\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3i\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4} \\
&= \frac{3ix^2}{2b^2} + \frac{x^3}{2b} - \frac{ix^4}{4} - \frac{3x\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3i\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}
\end{aligned}$$

Mathematica [A]
time = 6.73, size = 366, normalized size = 1.79

$$\frac{1}{4}ie^{ia}\left(-x^4+\left(1+e^{-2ia}\right)x^4-\frac{e^{-2ia}\left(1+e^{2ia}\right)\left(2b^4x^4+4ib^3x^3\log\left(1+e^{2i(a+bx)}\right)+6b^2x^2\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)+6ibx\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)-3\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)\right)}{2b^4}\right)\sec(a)+\frac{x^3\sec^2(a+bx)}{2b}-\frac{3\csc(a)\left(b^2e^{-i\mathrm{ArcTan}(\cot(a))}x^2-\frac{\cot(a)\left(ibx(-\pi-2\mathrm{ArcTan}(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\mathrm{ArcTan}(\cot(a)))\log\left(1-e^{2i(bx-\mathrm{ArcTan}(\cot(a)))}\right)+\pi\log(\cos(bx))-2\mathrm{ArcTan}(\cot(a))\log(\sin(bx-\mathrm{ArcTan}(\cot(a))))+i\mathrm{PolyLog}\left(2,e^{2i(bx-\mathrm{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{1+\cot^2(a)}}\right)\sec(a)}{2b^4\sqrt{\csc^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}-\frac{3x^2\sec(a)\sec(a+bx)\sin(bx)}{2b^2}-\frac{1}{4}x^4\tan(a)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[a + b*x]^3,x]
```

```
[Out] (I/4)*E^(I*a)*(-x^4 + (1 + E^((-2*I)*a))*x^4 - ((1 + E^((2*I)*a))*(2*b^4*x^
4 + (4*I)*b^3*x^3*Log[1 + E^((2*I)*(a + b*x))] + 6*b^2*x^2*PolyLog[2, -E^((
2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[3, -E^((2*I)*(a + b*x))] - 3*PolyLog[4
, -E^((2*I)*(a + b*x))]))/(2*b^4*E^((2*I)*a)))*Sec[a] + (x^3*Sec[a + b*x]^2
)/(2*b) - (3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi -
 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*
Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot
[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[C
```

```
ot[a]]))]]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Si
n[a]^2)]) - (3*x^2*Sec[a]*Sec[a + b*x]*Sin[b*x])/(2*b^2) - (x^4*Tan[a])/4
```

Maple [A]
time = 0.11, size = 251, normalized size = 1.22

method	result
risch	$\frac{3ix^2}{b^2}+\frac{x^2\left(2bx\,\mathrm{e}^{2i(bx+a)}-3i\mathrm{e}^{2i(bx+a)}-3i\right)}{b^2\left(\mathrm{e}^{2i(bx+a)}+1\right)^2}+\frac{3i\operatorname{polylog}\left(2,-\mathrm{e}^{2i(bx+a)}\right)}{2b^4}-\frac{2ia^3x}{b^3}+\frac{3i\operatorname{polylog}\left(4,-\mathrm{e}^{2i(bx+a)}\right)}{4b^4}-\frac{3ix^2\operatorname{polylog}(}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*I/b^2*x^2+x^2*(2*b*x*exp(2*I*(b*x+a))-3*I*exp(2*I*(b*x+a))-3*I)/b^2/(exp(
2*I*(b*x+a))+1)^2+3/2*I*polylog(2,-exp(2*I*(b*x+a)))/b^4-2*I/b^3*a^3*x+3/4*
I*polylog(4,-exp(2*I*(b*x+a)))/b^4-3/2*I*x^2*polylog(2,-exp(2*I*(b*x+a)))/b
^2+3*I/b^4*a^2-1/4*I*x^4+6*I/b^3*a*x+2/b^4*a^3*ln(exp(I*(b*x+a)))-3*x*ln(ex
p(2*I*(b*x+a))+1)/b^3+3/2*x*polylog(3,-exp(2*I*(b*x+a)))/b^3-3/2*I/b^4*a^4+
x^3*ln(exp(2*I*(b*x+a))+1)/b-6/b^4*a*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1205 vs. 2(163) = 326.
time = 0.67, size = 1205, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^3*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 2*(3*(b*x + a
)^4 - 12*(b*x + a)^3*a + 18*(b*x + a)^2*a^2 + 36*a^2 - 4*(4*(b*x + a)^3 - 9
*(b*x + a)^2*a + 9*(a^2 - 1)*(b*x + a) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a +
 9*(a^2 - 1)*(b*x + a) + 9*a)*cos(4*b*x + 4*a) + 2*(4*(b*x + a)^3 - 9*(b*x
+ a)^2*a + 9*(a^2 - 1)*(b*x + a) + 9*a)*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^
3 + 9*I*(b*x + a)^2*a + 9*(-I*a^2 + I)*(b*x + a) - 9*I*a)*sin(4*b*x + 4*a)
- 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a + 9*(-I*a^2 + I)*(b*x + a) - 9*I*
a)*sin(2*b*x + 2*a) + 9*a)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1)
+ 3*((b*x + a)^4 - 4*(b*x + a)^3*a + 6*(a^2 - 2)*(b*x + a)^2 + 24*(b*x + a)
*a)*cos(4*b*x + 4*a) + 6*((b*x + a)^4 - 4*(b*x + a)^3*(a - I) + 6*(a^2 - 2*
I*a - 1)*(b*x + a)^2 + 12*(I*a^2 + a)*(b*x + a) + 6*a^2)*cos(2*b*x + 2*a) +
 6*(4*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + (4*(b*x + a)^2 - 6*(b*x + a)*a
+ 3*a^2 - 3)*cos(4*b*x + 4*a) + 2*(4*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 -
3)*cos(2*b*x + 2*a) + (4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2 - 3*I)*s
in(4*b*x + 4*a) + 2*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2 - 3*I)*sin
(2*b*x + 2*a) - 3)*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(4*I*(b*x + a)^3 - 9*I*(
```

```
b*x + a)^2*a + 9*(I*a^2 - I)*(b*x + a) + (4*I*(b*x + a)^3 - 9*I*(b*x + a)^2
*a + 9*(I*a^2 - I)*(b*x + a) + 9*I*a)*cos(4*b*x + 4*a) + 2*(4*I*(b*x + a)^3
 - 9*I*(b*x + a)^2*a + 9*(I*a^2 - I)*(b*x + a) + 9*I*a)*cos(2*b*x + 2*a) -
(4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(a^2 - 1)*(b*x + a) + 9*a)*sin(4*b*x +
 4*a) - 2*(4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(a^2 - 1)*(b*x + a) + 9*a)*s
in(2*b*x + 2*a) + 9*I*a)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*co
s(2*b*x + 2*a) + 1) - 12*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) + I*sin(4*b
*x + 4*a) + 2*I*sin(2*b*x + 2*a) + 1)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 6*
(4*I*b*x + (4*I*b*x + I*a)*cos(4*b*x + 4*a) + 2*(4*I*b*x + I*a)*cos(2*b*x +
 2*a) - (4*b*x + a)*sin(4*b*x + 4*a) - 2*(4*b*x + a)*sin(2*b*x + 2*a) + I*a
)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 3*(I*(b*x + a)^4 - 4*I*(b*x + a)^3*a +
 6*(I*a^2 - 2*I)*(b*x + a)^2 + 24*I*(b*x + a)*a)*sin(4*b*x + 4*a) + 6*(I*(b
*x + a)^4 + 4*(b*x + a)^3*(-I*a - 1) + 6*(I*a^2 + 2*a - I)*(b*x + a)^2 - 12
*(a^2 - I*a)*(b*x + a) + 6*I*a^2)*sin(2*b*x + 2*a))/(-12*I*cos(4*b*x + 4*a)
 - 24*I*cos(2*b*x + 2*a) + 12*sin(4*b*x + 4*a) + 24*sin(2*b*x + 2*a) - 12*I
))/b^4
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(163) = 326$.
time = 0.37, size = 344, normalized size = 1.68

$$\frac{4\,b^3x^3\tan\left(bx+a\right)^2+4\,b^3x^3-12\,b^2x^2\tan\left(bx+a\right)+6\,bx\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2+2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+6\,bx\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2-2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)-6\left(-i\,b^2x^2+i\right)\mathrm{Li}_2\left(\frac{2\,(i\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-6\left(i\,b^2x^2-i\right)\mathrm{Li}_2\left(\frac{2\,(-i\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+4\left(b^3x^3-3\,bx\right)\log\left(-\frac{2\,(i\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+4\left(b^3x^3-3\,bx\right)\log\left(-\frac{2\,(-i\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)-3i\,\mathrm{polylog}\left(4,\frac{\tan(bx+a)^2+2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+3i\,\mathrm{polylog}\left(4,\frac{\tan(bx+a)^2-2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^3*x^3*tan(b*x + a)^2 + 4*b^3*x^3 - 12*b^2*x^2*tan(b*x + a) + 6*b*x
*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) +
 6*b*x*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 +
 1)) - 6*(-I*b^2*x^2 + I)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)
 + 1) - 6*(I*b^2*x^2 - I)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1
) + 1) + 4*(b^3*x^3 - 3*b*x)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 +
1)) + 4*(b^3*x^3 - 3*b*x)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)
) - 3*I*polylog(4, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2
+ 1)) + 3*I*polylog(4, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a
)^2 + 1)))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\tan^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(b*x+a)**3,x)
```

```
[Out] Integral(x**3*tan(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^3*tan(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\tan(a+b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(a + b*x)^3,x)
[Out] int(x^3*tan(a + b*x)^3, x)
```

3.12 $\int x^2 \tan^3(a+bx)\,dx$

Optimal. Leaf size=128

$$\frac{x^2}{2b}-\frac{ix^3}{3}+\frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{\log(\cos(a+bx))}{b^3}-\frac{ix\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{x\tan$$

```
[Out] 1/2*x^2/b-1/3*I*x^3+x^2*ln(1+exp(2*I*(b*x+a)))/b-ln(cos(b*x+a))/b^3-I*x*pol
ylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*polylog(3,-exp(2*I*(b*x+a)))/b^3-x*tan(b*
x+a)/b^2+1/2*x^2*tan(b*x+a)^2/b
```

Rubi [A]
time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3801, 3556, 30, 3800, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{\log(\cos(a+bx))}{b^3}-\frac{ix\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{x\tan(a+bx)}{b^2}+\frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{x^2\tan^2(a+bx)}{2b}+\frac{x^2}{2b}-\frac{ix^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tan[a + b*x]^3,x]
```

```
[Out] x^2/(2*b) - (I/3)*x^3 + (x^2*Log[1 + E^((2*I)*(a + b*x))])/b - Log[Cos[a +
b*x]]/b^3 - (I*x*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + PolyLog[3, -E^((2*
I)*(a + b*x))]/(2*b^3) - (x*Tan[a + b*x])/b^2 + (x^2*Tan[a + b*x]^2)/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\tan^3(a+bx)\,dx = \frac{x^2\tan^2(a+bx)}{2b} - \frac{\int x\tan^2(a+bx)\,dx}{b} - \int x^2\tan(a+bx)\,dx$$
$$= -\frac{ix^3}{3} - \frac{x\tan(a+bx)}{b^2} + \frac{x^2\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}x^2}{1+e^{2i(a+bx)}}\,dx + \frac{\int \tan(a+bx)\,dx}{b^2}$$
$$= \frac{x^2}{2b} - \frac{ix^3}{3} + \frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{\log(\cos(a+bx))}{b^3} - \frac{x\tan(a+bx)}{b^2} + \frac{x^2\tan^2(a+}{2b}$$
$$= \frac{x^2}{2b} - \frac{ix^3}{3} + \frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{\log(\cos(a+bx))}{b^3} - \frac{ix\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{x\tan(a}{b^2}$$
$$= \frac{x^2}{2b} - \frac{ix^3}{3} + \frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{\log(\cos(a+bx))}{b^3} - \frac{ix\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{x\tan(a}{b^2}$$
$$= \frac{x^2}{2b} - \frac{ix^3}{3} + \frac{x^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{\log(\cos(a+bx))}{b^3} - \frac{ix\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} + \frac{\text{Li}_3\left(-e^2\right.}{2b}$$

Mathematica [A]
time = 3.75, size = 179, normalized size = 1.40

$$\frac{e^{-ia}\left(2b^2x^2\left(-2ibe^{2ia}x+3(1+e^{2ia})\log\left(1+e^{2i(a+bx)}\right)\right)-6ib(1+e^{2ia})\,x\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)+3(1+e^{2ia})\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)\right)\sec(a)+6b^2x^2\sec^2(a+bx)-12bx\sec(a)\sec(a+bx)\sin(bx)-4b^3x^3\tan(a)-12(\log(\cos(a+bx))+bx\tan(a))}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tan[a + b*x]^3,x]
```

```
[Out] (((2*b^2*x^2*((-2*I)*b*E^((2*I)*a)*x + 3*(1 + E^((2*I)*a))*Log[1 + E^((2*I)
*(a + b*x))]) - (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((2*I)*(a + b*x))
] + 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((2*I)*(a + b*x))])*Sec[a])/E^(I*a) +
 6*b^2*x^2*Sec[a + b*x]^2 - 12*b*x*Sec[a]*Sec[a + b*x]*Sin[b*x] - 4*b^3*x^3
*Tan[a] - 12*(Log[Cos[a + b*x]] + b*x*Tan[a]))/(12*b^3)
```

Maple [A]
time = 0.09, size = 180, normalized size = 1.41

method	result
risch	$-\frac{ix^3}{3}+\frac{2x\left(bx\,\mathrm{e}^{2i(bx+a)}-i\mathrm{e}^{2i(bx+a)}-i\right)}{b^2\left(\mathrm{e}^{2i(bx+a)}+1\right)^2}-\frac{2a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}+\frac{2ia^2x}{b^2}+\frac{4ia^3}{3b^3}+\frac{x^2\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b}-\frac{ix\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(bx+a}\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*x^3+2*x*(b*x*exp(2*I*(b*x+a))-I*exp(2*I*(b*x+a))-I)/b^2/(exp(2*I*(b*
x+a))+1)^2-2/b^3*a^2*ln(exp(I*(b*x+a)))+2*I/b^2*a^2*x+4/3*I/b^3*a^3+x^2*ln(
exp(2*I*(b*x+a))+1)/b-I*x*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*polylog(3,-e
xp(2*I*(b*x+a)))/b^3-1/b^3*ln(exp(2*I*(b*x+a))+1)+2/b^3*ln(exp(I*(b*x+a)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(109) = 218$.
time = 0.62, size = 736, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) + 2*(2*(b*x +
a)^3 - 6*(b*x + a)^2*a - 6*((b*x + a)^2 - 2*(b*x + a)*a + ((b*x + a)^2 - 2*
(b*x + a)*a - 1)*cos(4*b*x + 4*a) + 2*((b*x + a)^2 - 2*(b*x + a)*a - 1)*cos
(2*b*x + 2*a) - (-I*(b*x + a)^2 + 2*I*(b*x + a)*a + I)*sin(4*b*x + 4*a) - 2
*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a + I)*sin(2*b*x + 2*a) - 1)*arctan2(sin(2
*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 2*((b*x + a)^3 - 3*(b*x + a)^2*a - 6*b
*x - 6*a)*cos(4*b*x + 4*a) + 4*((b*x + a)^3 - 3*(b*x + a)^2*(a - I) + 3*(b*
x + a)*(-2*I*a - 1) - 3*a)*cos(2*b*x + 2*a) + 6*(b*x*cos(4*b*x + 4*a) + 2*b
*x*cos(2*b*x + 2*a) + I*b*x*sin(4*b*x + 4*a) + 2*I*b*x*sin(2*b*x + 2*a) + b
*x)*dilog(-e^(2*I*b*x + 2*I*a)) + 3*(I*(b*x + a)^2 - 2*I*(b*x + a)*a + (I*(
b*x + a)^2 - 2*I*(b*x + a)*a - I)*cos(4*b*x + 4*a) + 2*(I*(b*x + a)^2 - 2*I
*(b*x + a)*a - I)*cos(2*b*x + 2*a) - ((b*x + a)^2 - 2*(b*x + a)*a - 1)*sin(
4*b*x + 4*a) - 2*((b*x + a)^2 - 2*(b*x + a)*a - 1)*sin(2*b*x + 2*a) - I)*lo
g(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 3*(I*
cos(4*b*x + 4*a) + 2*I*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x +
2*a) + I)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 2*(I*(b*x + a)^3 - 3*I*(b*x +
a)^2*a - 6*I*b*x - 6*I*a)*sin(4*b*x + 4*a) + 4*(I*(b*x + a)^3 + 3*(b*x + a)
^2*(-I*a - 1) + 3*(b*x + a)*(2*a - I) - 3*I*a)*sin(2*b*x + 2*a) - 12*a)/(-6
*I*cos(4*b*x + 4*a) - 12*I*cos(2*b*x + 2*a) + 6*sin(4*b*x + 4*a) + 12*sin(2
*b*x + 2*a) - 6*I))/b^3
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(109) = 218$.
time = 0.36, size = 240, normalized size = 1.88

$$\frac{2\,b^2x^2\tan\left(bx+a\right)^2+2\,b^2x^2+2i\,bx\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-2i\,bx\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-4\,bx\tan\left(bx+a\right)+2\,(b^2x^2-1)\log\left(-\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+2\,(b^2x^2-1)\log\left(-\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*tan(b*x + a)^2 + 2*b^2*x^2 + 2*I*b*x*dilog(2*(I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1) + 1) - 2*I*b*x*dilog(2*(-I*tan(b*x + a) - 1)/(ta
n(b*x + a)^2 + 1) + 1) - 4*b*x*tan(b*x + a) + 2*(b^2*x^2 - 1)*log(-2*(I*tan
(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*x^2 - 1)*log(-2*(-I*tan(b*x +
 a) - 1)/(tan(b*x + a)^2 + 1)) + polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x +
 a) - 1)/(tan(b*x + a)^2 + 1)) + polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x +
 a) - 1)/(tan(b*x + a)^2 + 1)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \tan^3{(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tan(b*x+a)**3,x)
```

```
[Out] Integral(x**2*tan(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*tan(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\mathrm{tan}\left(a+b\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(a + b*x)^3,x)
```

```
[Out] int(x^2*tan(a + b*x)^3, x)
```

3.13 $\int x \tan^3(a+bx)\,dx$

Optimal. Leaf size=90

$$\frac{x}{2b}-\frac{ix^2}{2}+\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{i\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{\tan(a+bx)}{2b^2}+\frac{x\tan^2(a+bx)}{2b}$$

```
[Out] 1/2*x/b-1/2*I*x^2+x*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*polylog(2,-exp(2*I*(b*x+
a)))/b^2-1/2*tan(b*x+a)/b^2+1/2*x*tan(b*x+a)^2/b
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3801, 3554, 8, 3800, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{\tan(a+bx)}{2b^2}+\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{x\tan^2(a+bx)}{2b}+\frac{x}{2b}-\frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[a + b*x]^3,x]
```

```
[Out] x/(2*b) - (I/2)*x^2 + (x*Log[1 + E^((2*I)*(a + b*x))])/b - ((I/2)*PolyLog[2
, -E^((2*I)*(a + b*x))])/b^2 - Tan[a + b*x]/(2*b^2) + (x*Tan[a + b*x]^2)/(2
*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\tan^3(a+bx)\,dx &= \frac{x\tan^2(a+bx)}{2b}-\frac{\int \tan^2(a+bx)\,dx}{2b}-\int x\tan(a+bx)\,dx\\
&= -\frac{ix^2}{2}-\frac{\tan(a+bx)}{2b^2}+\frac{x\tan^2(a+bx)}{2b}+2i\int \frac{e^{2i(a+bx)}x}{1+e^{2i(a+bx)}}\,dx+\frac{\int 1\,dx}{2b}\\
&= \frac{x}{2b}-\frac{ix^2}{2}+\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{\tan(a+bx)}{2b^2}+\frac{x\tan^2(a+bx)}{2b}-\frac{\int \log\left(1+e^{2i(a+b}\right.}{b}\\
&= \frac{x}{2b}-\frac{ix^2}{2}+\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{\tan(a+bx)}{2b^2}+\frac{x\tan^2(a+bx)}{2b}+\frac{i\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\right.}{2b^{\cdot}}\\
&= \frac{x}{2b}-\frac{ix^2}{2}+\frac{x\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{i\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{\tan(a+bx)}{2b^2}+\frac{x\tan^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 210 vs. 2(90) = 180.
time = 6.16, size = 210, normalized size = 2.33

$$
\frac{x\sec^2(a+bx)}{2b}+\frac{\csc(a)\left(b^2e^{-i\mathrm{ArcTan}(\cot(a))}x^2-\frac{\cot(a)\left(ibx(-\pi-2\mathrm{ArcTan}(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\mathrm{ArcTan}(\cot(a)))\log\left(1-e^{2i(bx-\mathrm{ArcTan}(\cot(a)))}\right)+\pi\log(\cos(bx))-2\mathrm{ArcTan}(\cot(a))\log(\sin(bx-\mathrm{ArcTan}(\cot(a))))+i\mathrm{PolyLog}\left(2,e^{2i(bx-\mathrm{ArcTan}(\cot(a)))}\right)\right)}{\sqrt{1+\cot^2(a)}}\right)\sec(a)}{2b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}-\frac{\sec(a)\sec(a+bx)\sin(bx)}{2b^2}-\frac{1}{2}x^2\tan(a)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[a + b*x]^3,x]
```

```
[Out] (x*Sec[a + b*x]^2)/(2*b) + (Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a
]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - A
rcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]]
 - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*
(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2
*(Cos[a]^2 + Sin[a]^2)]) - (Sec[a]*Sec[a + b*x]*Sin[b*x])/(2*b^2) - (x^2*Ta
n[a])/2
```

Maple [A]
time = 0.07, size = 122, normalized size = 1.36

method	result	s
risch	$-\frac{i x^{2}}{2}+\frac{2 b x \,{\mathrm e}^{2 i \left(b x +a \right)}-i {\mathrm e}^{2 i \left(b x +a \right)}-i}{b^{2} \left({\mathrm e}^{2 i \left(b x +a \right)}+1\right)^{2}}-\frac{2 i a x}{b}-\frac{i a^{2}}{b^{2}}+\frac{x \ln \left({\mathrm e}^{2 i \left(b x +a \right)}+1\right)}{b}-\frac{i \operatorname{polylog}\left(2,-{\mathrm e}^{2 i \left(b x +a \right)}\right)}{2 b^{2}}+\frac{2 a \ln \left({\mathrm e}^{i \left(b x +a \right)}\right)}{b^{2}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*x^2+(2*b*x*exp(2*I*(b*x+a))-I*exp(2*I*(b*x+a))-I)/b^2/(exp(2*I*(b*x+
a))+1)^2-2*I/b*a*x-I/b^2*a^2+x*ln(exp(2*I*(b*x+a))+1)/b-1/2*I*polylog(2,-ex
p(2*I*(b*x+a)))/b^2+2/b^2*a*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(71) = 142$.
time = 0.62, size = 386, normalized size = 4.29

$$-\frac{b^{2} x^{2} \cos\left(4 \, b x + 4 \, a\right) + i \, b^{2} x^{2} \sin\left(4 \, b x + 4 \, a\right) + b^{2} x^{2} - 2 \, {\left(b x \cos\left(4 \, b x + 4 \, a\right) + 2 \, b x \cos\left(2 \, b x + 2 \, a\right) + i \, b x \sin\left(4 \, b x + 4 \, a\right) + 2 i \, b x \sin\left(2 \, b x + 2 \, a\right) + b x\right)} \arctan\left(\sin\left(2 \, b x + 2 \, a\right), \cos\left(2 \, b x + 2 \, a\right) + 1\right) + 2 \, {\left(b^{2} x^{2} + 2 i \, b x + 1\right)} \cos\left(2 \, b x + 2 \, a\right) + {\left(\cos\left(4 \, b x + 4 \, a\right) + 2 \, \cos\left(2 \, b x + 2 \, a\right) + i \, \sin\left(4 \, b x + 4 \, a\right) + 2 i \, \sin\left(2 \, b x + 2 \, a\right) + 1\right)} {\rm Li}_2\left(-e^{\left(2 i \, b x + 2 i \, a\right)}\right) - {\left(-i \, b x \cos\left(4 \, b x + 4 \, a\right) - 2 i \, b x \cos\left(2 \, b x + 2 \, a\right) + b x \sin\left(4 \, b x + 4 \, a\right) + 2 \, b x \sin\left(2 \, b x + 2 \, a\right) - i \, b x\right)} \log\left(\cos\left(2 \, b x + 2 \, a\right)^{2} + \sin\left(2 \, b x + 2 \, a\right)^{2} + 2 \, \cos\left(2 \, b x + 2 \, a\right) + 1\right) + 2 \, {\left(i \, b^{2} x^{2} - 2 \, b x + i\right)} \sin\left(2 \, b x + 2 \, a\right) + 2}{-2 i \, b^{2} \cos\left(4 \, b x + 4 \, a\right) - 4 i \, b^{2} \cos\left(2 \, b x + 2 \, a\right) + 2 \, b^{2} \sin\left(4 \, b x + 4 \, a\right) + 4 \, b^{2} \sin\left(2 \, b x + 2 \, a\right) - 2 i \, b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(b^2*x^2*cos(4*b*x + 4*a) + I*b^2*x^2*sin(4*b*x + 4*a) + b^2*x^2 - 2*(b*x*
cos(4*b*x + 4*a) + 2*b*x*cos(2*b*x + 2*a) + I*b*x*sin(4*b*x + 4*a) + 2*I*b*
x*sin(2*b*x + 2*a) + b*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) +
 2*(b^2*x^2 + 2*I*b*x + 1)*cos(2*b*x + 2*a) + (cos(4*b*x + 4*a) + 2*cos(2*b
*x + 2*a) + I*sin(4*b*x + 4*a) + 2*I*sin(2*b*x + 2*a) + 1)*dilog(-e^(2*I*b*
x + 2*I*a)) - (-I*b*x*cos(4*b*x + 4*a) - 2*I*b*x*cos(2*b*x + 2*a) + b*x*sin
(4*b*x + 4*a) + 2*b*x*sin(2*b*x + 2*a) - I*b*x)*log(cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 2*(I*b^2*x^2 - 2*b*x + I)*sin(
2*b*x + 2*a) + 2)/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x + 2*a) + 2
*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(71) = 142$.

time = 0.38, size = 146, normalized size = 1.62

$$\frac{2\,bx\tan\left(bx+a\right)^2+2\,bx\log\left(-\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+2\,bx\log\left(-\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right)+2\,bx+i\,\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-i\,\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-2\,\tan\left(bx+a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x*tan(b*x + a)^2 + 2*b*x*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)
^2 + 1)) + 2*b*x*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*b*x
 + I*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(2*(-I
*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 2*tan(b*x + a))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\tan^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a)**3,x)
```

```
[Out] Integral(x*tan(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*tan(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\tan\left(a+b\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(a + b*x)^3,x)
```

```
[Out] int(x*tan(a + b*x)^3, x)
```

3.14 $\int \frac{\tan^3(a+bx)}{x} dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\frac{\tan^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^3/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan^3(a+bx)}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]^3/x,x]
[Out] Defer[Int][Tan[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\tan^3(a+bx)}{x} dx = \int \frac{\tan^3(a+bx)}{x} dx$$

Mathematica [A]
time = 5.03, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^3/x,x]
[Out] Integrate[Tan[a + b*x]^3/x, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(bx+a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^3/x,x)
```

```
[Out] int(tan(b*x+a)^3/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] (4*b*x*cos(2*b*x + 2*a)^2 + 4*b*x*sin(2*b*x + 2*a)^2 + 2*b*x*cos(2*b*x + 2*
a) + (2*b*x*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (b^2*x^
2*cos(4*b*x + 4*a)^2 + 4*b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(4*b*x + 4
*a)^2 + 4*b^2*x^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^2*sin(2*b*x +
 2*a)^2 + 4*b^2*x^2*cos(2*b*x + 2*a) + b^2*x^2 + 2*(2*b^2*x^2*cos(2*b*x + 2
*a) + b^2*x^2)*cos(4*b*x + 4*a))*integrate(2*(b^2*x^2 - 1)*sin(2*b*x + 2*a)
/(b^2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(2*b*x + 2*a)^2 + 2*b^2*x^3*cos(2
*b*x + 2*a) + b^2*x^3), x) + (2*b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1
)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))/(b^2*x^2*cos(4*b*x + 4*a)^2 + 4*b^2*
x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 + 4*b^2*x^2*sin(4*b*x +
 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^2*sin(2*b*x + 2*a)^2 + 4*b^2*x^2*cos(2*b*x
 + 2*a) + b^2*x^2 + 2*(2*b^2*x^2*cos(2*b*x + 2*a) + b^2*x^2)*cos(4*b*x + 4*
a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^3/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/x,x)
```

```
[Out] Integral(tan(a + b*x)**3/x, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^3/x, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\tan(a+b\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3/x,x)
```

```
[Out] int(tan(a + b*x)^3/x, x)
```

3.15 $\int \frac{\tan^3(a+bx)}{x^2} \, dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(\frac{\tan^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^3/x^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan^3(a+bx)}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[a + b*x]^3/x^2,x]
```

```
[Out] Defer[Int][Tan[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\tan^3(a+bx)}{x^2} \, dx = \int \frac{\tan^3(a+bx)}{x^2} \, dx$$

Mathematica [A]
time = 3.36, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^3/x^2,x]
```

```
[Out] Integrate[Tan[a + b*x]^3/x^2, x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(bx+a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^3/x^2,x)
```

```
[Out] int(tan(b*x+a)^3/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] (4*b*x*cos(2*b*x + 2*a)^2 + 4*b*x*sin(2*b*x + 2*a)^2 + 2*b*x*cos(2*b*x + 2*
a) + 2*(b*x*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (b^2*x^
3*cos(4*b*x + 4*a)^2 + 4*b^2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(4*b*x + 4
*a)^2 + 4*b^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^3*sin(2*b*x +
 2*a)^2 + 4*b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 + 2*(2*b^2*x^3*cos(2*b*x + 2
*a) + b^2*x^3)*cos(4*b*x + 4*a))*integrate(2*(b^2*x^2 - 3)*sin(2*b*x + 2*a)
/(b^2*x^4*cos(2*b*x + 2*a)^2 + b^2*x^4*sin(2*b*x + 2*a)^2 + 2*b^2*x^4*cos(2
*b*x + 2*a) + b^2*x^4), x) + 2*(b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1
)*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))/(b^2*x^3*cos(4*b*x + 4*a)^2 + 4*b^
2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 + 4*b^2*x^3*sin(4*b*x
 + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^3*sin(2*b*x + 2*a)^2 + 4*b^2*x^3*cos(2*b
*x + 2*a) + b^2*x^3 + 2*(2*b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3)*cos(4*b*x +
4*a))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^3/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/x**2,x)
```

```
[Out] Integral(tan(a + b*x)**3/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^3/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\tan(a+b\,x)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3/x^2,x)
```

```
[Out] int(tan(a + b*x)^3/x^2, x)
```

3.16 $\int \left(\frac{x^2}{\tan^{\frac{3}{2}}(a+bx)} - \frac{4x}{b\sqrt{\tan(a+bx)}} + x^2\sqrt{\tan(a+bx)}\right)$

Optimal. Leaf size=18

$$-\frac{2x^2}{b\sqrt{\tan(a+bx)}}$$

```
[Out] -2*x^2/b/tan(b*x+a)^(1/2)
```

Rubi [A]
time = 1.93, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 76, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3802, 3819, 209, 281, 6857, 6139, 6057, 2449, 2352, 2497, 6131, 6055}

$$-\frac{2x^2}{b\sqrt{\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Tan[a + b*x]^(3/2) - (4*x)/(b*Sqrt[Tan[a + b*x]]) + x^2*Sqrt[Tan[a
+ b*x]],x]
```

```
[Out] (-2*x^2)/(b*Sqrt[Tan[a + b*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 3802

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(c + d*x)^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n + 1))), x] + (-Di
st[d*(m/(b*f*(n + 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n + 1), x],
 x] - Dist[1/b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n + 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 0]
```

Rule 3819

```
Int[((c_.) + (d_.)*(x_))/Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*((c + d*x)/(f*Rt[a - I*b, 2]))*ArcTanh[Sqrt[a + b*Tan[e +
 f*x]]/Rt[a - I*b, 2]], x] + (Dist[I*(d/(f*Rt[a - I*b, 2])), Int[ArcTanh[Sq
rt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Dist[I*(d/(f*Rt[a + I*b, 2
])), Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x], x] + Simp[I*
((c + d*x)/(f*Rt[a + I*b, 2]))*ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b,
 2]], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x^2}{\tan^{\frac{3}{2}}(a+bx)}-\frac{4x}{b\sqrt{\tan(a+bx)}}+x^2\sqrt{\tan(a+bx)}\right)dx &= -\frac{4\int \frac{x}{\sqrt{\tan(a+bx)}}\,dx}{b}+\int \frac{x^2}{\tan^{\frac{3}{2}}(a+b\end{aligned}$$

$$= -\frac{2x^2}{b\sqrt{\tan(a+bx)}}$$

Mathematica [A]
time = 0.70, size = 18, normalized size = 1.00

$$-\frac{2x^2}{b\sqrt{\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Tan[a + b*x]^(3/2) - (4*x)/(b*Sqrt[Tan[a + b*x]]) + x^2*Sqrt[
Tan[a + b*x]],x]
```

```
[Out] (-2*x^2)/(b*Sqrt[Tan[a + b*x]])
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int -\frac{4x}{b\sqrt{\tan\left(bx+a\right)}}+x^2\left(\sqrt{\tan}\left(bx+a\right)\right)+\frac{x^2}{\tan\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-4*x/b/tan(b*x+a)^(1/2)+x^2*tan(b*x+a)^(1/2)+x^2/tan(b*x+a)^(3/2),x)
```

```
[Out] int(-4*x/b/tan(b*x+a)^(1/2)+x^2*tan(b*x+a)^(1/2)+x^2/tan(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-4*x/b/tan(b*x+a)^(1/2)+x^2*tan(b*x+a)^(1/2)+x^2/tan(b*x+a)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(tan(b*x + a)) + x^2/tan(b*x + a)^(3/2) - 4*x/(b*sqrt(tan
(b*x + a))), x)
```

Fricas [A]
time = 0.41, size = 16, normalized size = 0.89

$$-\frac{2\,x^2}{b\sqrt{\tan\left(bx+a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-4*x/b/tan(b*x+a)^(1/2)+x^2*tan(b*x+a)^(1/2)+x^2/tan(b*x+a)^(3/2)
,x, algorithm="fricas")
```

```
[Out] -2*x^2/(b*sqrt(tan(b*x + a)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-\frac{4x}{\sqrt{\tan\left(a+bx\right)}}\right)dx+\int\frac{bx^2}{\tan^{\frac{3}{2}}\left(a+bx\right)}\,dx+\int bx^2\sqrt{\tan\left(a+bx\right)}\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-4*x/b/tan(b*x+a)**(1/2)+x**2*tan(b*x+a)**(1/2)+x**2/tan(b*x+a)**
(3/2),x)
```

```
[Out] (Integral(-4*x/sqrt(tan(a + b*x)), x) + Integral(b*x**2/tan(a + b*x)**(3/2)
, x) + Integral(b*x**2*sqrt(tan(a + b*x)), x))/b
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-4*x/b/tan(b*x+a)^(1/2)+x^2*tan(b*x+a)^(1/2)+x^2/tan(b*x+a)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(tan(b*x + a)) + x^2/tan(b*x + a)^(3/2) - 4*x/(b*sqrt(tan
(b*x + a))), x)
```

Mupad [B]
time = 3.12, size = 50, normalized size = 2.78

$$-\frac{x^2\,\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{\sin\left(2\,a+2\,b\,x\right)}{\cos\left(2\,a+2\,b\,x\right)+1}}}{b\,{\sin\left(a+b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tan(a + b*x)^(1/2) + x^2/tan(a + b*x)^(3/2) - (4*x)/(b*tan(a + b*x)
^(1/2)),x)
```

```
[Out] -(x^2*sin(2*a + 2*b*x)*(sin(2*a + 2*b*x)/(cos(2*a + 2*b*x) + 1))^(1/2))/(b*
sin(a + b*x)^2)
```

3.17 $\int \left(\frac{x^2}{\sqrt{\tan(a+bx^2)}} + \frac{\sqrt{\tan(a+bx^2)}}{b} + x^2 \tan^{\frac{3}{2}}(a+$

Optimal. Leaf size=17

$$\frac{x\sqrt{\tan(a+bx^2)}}{b}$$

[Out] x*tan(b*x^2+a)^(1/2)/b

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\frac{x^2}{\sqrt{\tan(a+bx^2)}} + \frac{\sqrt{\tan(a+bx^2)}}{b} + x^2 \tan^{\frac{3}{2}}(a+bx^2) \right) dx$$

Verification is not applicable to the result.

[In] Int[x^2/Sqrt[Tan[a + b*x^2]] + Sqrt[Tan[a + b*x^2]]/b + x^2*Tan[a + b*x^2]^(3/2),x]

[Out] Defer[Int][x^2/Sqrt[Tan[a + b*x^2]], x] + Defer[Int][Sqrt[Tan[a + b*x^2]], x]/b + Defer[Int][x^2*Tan[a + b*x^2]^(3/2), x]

Rubi steps

$$\int \left(\frac{x^2}{\sqrt{\tan(a+bx^2)}} + \frac{\sqrt{\tan(a+bx^2)}}{b} + x^2 \tan^{\frac{3}{2}}(a+bx^2) \right) dx = \frac{\int \sqrt{\tan(a+bx^2)}\, dx}{b} + \int \frac{x^2}{\sqrt{\tan(a+}}$$

Mathematica [A]
time = 0.51, size = 17, normalized size = 1.00

$$\frac{x\sqrt{\tan(a+bx^2)}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[Tan[a + b*x^2]] + Sqrt[Tan[a + b*x^2]]/b + x^2*Tan[a + b*x^2]^(3/2),x]

[Out] (x*Sqrt[Tan[a + b*x^2]])/b

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\tan \left(b\,x^2+a\right)}}+\frac{\sqrt{\tan}\left(b\,x^2+a\right)}{b}+x^2\left(\tan^{\frac{3}{2}}\left(b\,x^2+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/tan(b*x^2+a)^(1/2)+tan(b*x^2+a)^(1/2)/b+x^2*tan(b*x^2+a)^(3/2),x)
```

```
[Out] int(x^2/tan(b*x^2+a)^(1/2)+tan(b*x^2+a)^(1/2)/b+x^2*tan(b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/tan(b*x^2+a)^(1/2)+tan(b*x^2+a)^(1/2)/b+x^2*tan(b*x^2+a)^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate(x^2*tan(b*x^2 + a)^(3/2) + x^2/sqrt(tan(b*x^2 + a)) + sqrt(tan(b*
x^2 + a))/b, x)
```

Fricas [A]
time = 0.35, size = 15, normalized size = 0.88

$$\frac{x\sqrt{\tan \left(bx^2+a\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/tan(b*x^2+a)^(1/2)+tan(b*x^2+a)^(1/2)/b+x^2*tan(b*x^2+a)^(3/2
),x, algorithm="fricas")
```

```
[Out] x*sqrt(tan(b*x^2 + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{bx^2}{\sqrt{\tan \left(a+bx^2\right)}}\,dx+\int bx^2\tan^{\frac{3}{2}}\left(a+bx^2\right)dx+\int \sqrt{\tan \left(a+bx^2\right)}\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/tan(b*x**2+a)**(1/2)+tan(b*x**2+a)**(1/2)/b+x**2*tan(b*x**2+
a)**(3/2),x)
```

```
[Out] (Integral(b*x**2/sqrt(tan(a + b*x**2)), x) + Integral(b*x**2*tan(a + b*x**2
)**(3/2), x) + Integral(sqrt(tan(a + b*x**2)), x))/b
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/tan(b*x^2+a)^(1/2)+tan(b*x^2+a)^(1/2)/b+x^2*tan(b*x^2+a)^(3/2
),x, algorithm="giac")
```

```
[Out] integrate(x^2*tan(b*x^2 + a)^(3/2) + x^2/sqrt(tan(b*x^2 + a)) + sqrt(tan(b*
x^2 + a))/b, x)
```

Mupad [B]
time = 4.17, size = 45, normalized size = 2.65

$$\frac{x\sqrt{-\frac{\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}}{\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}+1}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x^2)^(1/2)/b + x^2/tan(a + b*x^2)^(1/2) + x^2*tan(a + b*x^2)^
(3/2),x)
```

```
[Out] (x*(-(exp(a*2i + b*x^2*2i)*1i - 1i)/(exp(a*2i + b*x^2*2i) + 1))^(1/2))/b
```

3.18 $\int \frac{(c+dx)^3}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=189

$$\frac{3id^3x}{8af^3}-\frac{3d(c+dx)^2}{8af^2}-\frac{i(c+dx)^3}{4af}+\frac{(c+dx)^4}{8ad}-\frac{3d^3}{8f^4(a+ia\tan(e+fx))}-\frac{3id^2(c+dx)}{4f^3(a+ia\tan(e+fx))}+\frac{3d}{4f^2(a+}$$

```
[Out] 3/8*I*d^3*x/a/f^3-3/8*d*(d*x+c)^2/a/f^2-1/4*I*(d*x+c)^3/a/f+1/8*(d*x+c)^4/a
/d-3/8*d^3/f^4/(a+I*a*tan(f*x+e))-3/4*I*d^2*(d*x+c)/f^3/(a+I*a*tan(f*x+e))+
3/4*d*(d*x+c)^2/f^2/(a+I*a*tan(f*x+e))+1/2*I*(d*x+c)^3/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3804, 3560, 8}

$$-\frac{3id^2(c+dx)}{4f^3(a+ia\tan(e+fx))}+\frac{3d(c+dx)^2}{4f^2(a+ia\tan(e+fx))}+\frac{i(c+dx)^3}{2f(a+ia\tan(e+fx))}-\frac{3d(c+dx)^2}{8af^2}-\frac{i(c+dx)^3}{4af}+\frac{(c+dx)^4}{8ad}-\frac{3d^3}{8f^4(a+ia\tan(e+fx))}+\frac{3id^3x}{8af^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((3*I)/8)*d^3*x)/(a*f^3) - (3*d*(c + d*x)^2)/(8*a*f^2) - ((I/4)*(c + d*x)^
3)/(a*f) + (c + d*x)^4/(8*a*d) - (3*d^3)/(8*f^4*(a + I*a*Tan[e + f*x])) - (
((3*I)/4)*d^2*(c + d*x))/(f^3*(a + I*a*Tan[e + f*x])) + (3*d*(c + d*x)^2)/(
4*f^2*(a + I*a*Tan[e + f*x])) + ((I/2)*(c + d*x)^3)/(f*(a + I*a*Tan[e + f*x
]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3804

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[a*d*(m/(2*b*f))
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m/
(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+ia\tan(e+fx)}\,dx = \frac{(c+dx)^4}{8ad} + \frac{i(c+dx)^3}{2f(a+ia\tan(e+fx))} - \frac{(3id)\int \frac{(c+dx)^2}{a+ia\tan(e+fx)}\,dx}{2f}$$

$$= -\frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} + \frac{3d(c+dx)^2}{4f^2(a+ia\tan(e+fx))} + \frac{i(c+dx)^3}{2f(a+ia\tan(e+fx))} -$$

$$= -\frac{3d(c+dx)^2}{8af^2} - \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3id^2(c+dx)}{4f^3(a+ia\tan(e+fx))} +$$

$$= -\frac{3d(c+dx)^2}{8af^2} - \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+ia\tan(e+fx))} -$$

$$= \frac{3id^3x}{8af^3} - \frac{3d(c+dx)^2}{8af^2} - \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+ia\tan(e+fx))} -$$

Mathematica [A]
time = 0.46, size = 278, normalized size = 1.47

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left((4ic^3f^3+6c^2df^2(1+2ifx)+6cd^2f(-i+2fx+2if^2x^2)+d^3(-3-6ifx+6f^2x^2+4if^3x^3))\cos(2fx)(\cos(e)-i\sin(e))+2f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)(\cos(e)+i\sin(e))+(4c^3f^3+6c^2df^2(-i+2fx)+6cd^2f(-1-2ifx+2f^2x^2)+d^3(3i-6fx-6if^2x^2+4f^3x^3))(\cos(e)-i\sin(e))\sin(2fx)\right)}{16f^4(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*(((4*I)*c^3*f^3 + 6*c^2*d*f^2*(1 + (2
*I)*f*x) + 6*c*d^2*f*(-I + 2*f*x + (2*I)*f^2*x^2) + d^3*(-3 - (6*I)*f*x + 6
*f^2*x^2 + (4*I)*f^3*x^3))*Cos[2*f*x]*(Cos[e] - I*Sin[e]) + 2*f^4*x*(4*c^3
+ 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(Cos[e] + I*Sin[e]) + (4*c^3*f^3 + 6*c
^2*d*f^2*(-I + 2*f*x) + 6*c*d^2*f*(-1 - (2*I)*f*x + 2*f^2*x^2) + d^3*(3*I -
 6*f*x - (6*I)*f^2*x^2 + 4*f^3*x^3))*(Cos[e] - I*Sin[e])*Sin[2*f*x]))/(16*f
^4*(a + I*a*Tan[e + f*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(165) = 330$.
time = 0.47, size = 1061, normalized size = 5.61

method	result
risch	$\frac{d^3x^4}{8a} + \frac{d^2cx^3}{2a} + \frac{3dc^2x^2}{4a} + \frac{c^3x}{2a} + \frac{c^4}{8da} + \frac{i(4d^3x^3f^3+12cd^2f^3x^2-6id^3f^2x^2+12c^2df^3x-12icd^2f^2x+4c^3f^3-6ic^2df^2-6d^3fx}{16af^4}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/f*(-I/f^3*d^3*(-1/2*(f*x+e)^3*cos(f*x+e)^2+3/2*(f*x+e)^2*(1/2*cos(f*x+e
)*sin(f*x+e)+1/2*f*x+1/2*e)+3/4*(f*x+e)*cos(f*x+e)^2-3/8*cos(f*x+e)*sin(f*x
```

```
+e)-3/8*f*x-3/8*e-1/2*(f*x+e)^3)+3*I/f^3*d^3*e*(-1/2*(f*x+e)^2*cos(f*x+e)^2
+(f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*
x+e)^2)+1/2*I*c^3*cos(f*x+e)^2-1/2*I/f^3*d^3*e^3*cos(f*x+e)^2-3*I/f^3*d^3*e
^2*(-1/2*(f*x+e)*cos(f*x+e)^2+1/4*cos(f*x+e)*sin(f*x+e)+1/4*f*x+1/4*e)+3/2*
I/f^2*c*d^2*e^2*cos(f*x+e)^2-3*I/f^2*c*d^2*(-1/2*(f*x+e)^2*cos(f*x+e)^2+(f*
x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)
^2)-3*I/f*c^2*d*(-1/2*(f*x+e)*cos(f*x+e)^2+1/4*cos(f*x+e)*sin(f*x+e)+1/4*f*
x+1/4*e)-3/2*I/f*c^2*d*e*cos(f*x+e)^2+6*I/f^2*c*d^2*e*(-1/2*(f*x+e)*cos(f*x
+e)^2+1/4*cos(f*x+e)*sin(f*x+e)+1/4*f*x+1/4*e)+c^3*(1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)-3/f*c^2*d*e*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+3/f
*c^2*d*((f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4
*sin(f*x+e)^2)+3/f^2*c*d^2*e^2*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-6/
f^2*c*d^2*e*((f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^
2-1/4*sin(f*x+e)^2)+3/f^2*c*d^2*((f*x+e)^2*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f
*x+1/2*e)+1/2*(f*x+e)*cos(f*x+e)^2-1/4*cos(f*x+e)*sin(f*x+e)-1/4*f*x-1/4*e-
1/3*(f*x+e)^3)-1/f^3*d^3*e^3*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+3/f^
3*d^3*e^2*((f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-
1/4*sin(f*x+e)^2)-3/f^3*d^3*e*((f*x+e)^2*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x
+1/2*e)+1/2*(f*x+e)*cos(f*x+e)^2-1/4*cos(f*x+e)*sin(f*x+e)-1/4*f*x-1/4*e-1/
3*(f*x+e)^3)+1/f^3*d^3*((f*x+e)^3*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)
+3/4*(f*x+e)^2*cos(f*x+e)^2-3/2*(f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+
1/2*e)+3/8*(f*x+e)^2+3/8*sin(f*x+e)^2-3/8*(f*x+e)^4))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 166, normalized size = 0.88

$$\frac{\left(4i\,d^3f^3x^3+4i\,c^3f^3+6\,c^2df^2-6i\,cd^2f-3\,d^3-6\,(-2i\,cd^2f^3-d^3f^2)x^2-6\,(-2i\,c^2df^3-2\,cd^2f^2+i\,d^3f)x+2\,(d^3f^4x^4+4\,cd^2f^4x^3+6\,c^2df^4x^2+4\,c^3f^4x)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{16\,af^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*I*d^3*f^3*x^3 + 4*I*c^3*f^3 + 6*c^2*d*f^2 - 6*I*c*d^2*f - 3*d^3 - 6
*(-2*I*c*d^2*f^3 - d^3*f^2)*x^2 - 6*(-2*I*c^2*d*f^3 - 2*c*d^2*f^2 + I*d^3*f
)*x + 2*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2 + 4*c^3*f^4*x)*e^(
2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a*f^4)
```

Sympy [A]
time = 0.20, size = 258, normalized size = 1.37

$$\begin{cases} \frac{(4ic^3f^3+12ic^2df^3x+6c^2df^2+12icd^2f^3x^2+12cd^2f^2x-6icd^2f+4id^3f^3x^3+6d^3f^2x^2-6id^3fx-3d^3)e^{-2ie}e^{-2ifx}}{16af^4} & \text{for } af^4e^{2ie} \neq 0 \\ \frac{c^3xe^{-2ie}}{2a} + \frac{3c^2dx^2e^{-2ie}}{4a} + \frac{cd^2x^3e^{-2ie}}{2a} + \frac{d^3x^4e^{-2ie}}{8a} & \text{otherwise} \end{cases} + \frac{c^3x}{2a} + \frac{3c^2dx^2}{4a} + \frac{cd^2x^3}{2a} + \frac{d^3x^4}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*tan(f*x+e)),x)
```

```
[Out] Piecewise(((4*I*c**3*f**3 + 12*I*c**2*d*f**3*x + 6*c**2*d*f**2 + 12*I*c*d**
2*f**3*x**2 + 12*c*d**2*f**2*x - 6*I*c*d**2*f + 4*I*d**3*f**3*x**3 + 6*d**3
*f**2*x**2 - 6*I*d**3*f*x - 3*d**3)*exp(-2*I*e)*exp(-2*I*f*x)/(16*a*f**4),
Ne(a*f**4*exp(2*I*e), 0)), (c**3*x*exp(-2*I*e)/(2*a) + 3*c**2*d*x**2*exp(-2
*I*e)/(4*a) + c*d**2*x**3*exp(-2*I*e)/(2*a) + d**3*x**4*exp(-2*I*e)/(8*a),
True)) + c**3*x/(2*a) + 3*c**2*d*x**2/(4*a) + c*d**2*x**3/(2*a) + d**3*x**4
/(8*a)
```

Giac [A]
time = 0.47, size = 193, normalized size = 1.02

$$\frac{\left(2\,d^3f^4x^4e^{(2ifx+2ie)} + 8\,cd^2f^4x^3e^{(2ifx+2ie)} + 12\,c^2df^4x^2e^{(2ifx+2ie)} + 4i\,d^3f^3x^3 + 8\,c^3f^4xe^{(2ifx+2ie)} + 12i\,cd^2f^3x^2 + 12i\,c^2df^3x + 6\,d^3f^2x^2 + 4i\,c^3f^3 + 12\,cd^2f^2x + 6\,c^2df^2 - 6i\,d^3fx - 6i\,cd^2f - 3\,d^3\right)e^{(-2ifx-2ie)}}{16\,af^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/16*(2*d^3*f^4*x^4*e^(2*I*f*x + 2*I*e) + 8*c*d^2*f^4*x^3*e^(2*I*f*x + 2*I*
e) + 12*c^2*d*f^4*x^2*e^(2*I*f*x + 2*I*e) + 4*I*d^3*f^3*x^3 + 8*c^3*f^4*x*e
^(2*I*f*x + 2*I*e) + 12*I*c*d^2*f^3*x^2 + 12*I*c^2*d*f^3*x + 6*d^3*f^2*x^2
+ 4*I*c^3*f^3 + 12*c*d^2*f^2*x + 6*c^2*d*f^2 - 6*I*d^3*f*x - 6*I*c*d^2*f -
3*d^3)*e^(-2*I*f*x - 2*I*e)/(a*f^4)
```

Mupad [B]
time = 3.56, size = 423, normalized size = 2.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (d^3*sin(2*e + 2*f*x)*3i - 3*d^3*cos(2*e + 2*f*x) + 8*c^3*f^4*x + c^3*f^3*c
os(2*e + 2*f*x)*4i + 4*c^3*f^3*sin(2*e + 2*f*x) + 2*d^3*f^4*x^4 + 6*c^2*d*f
^2*cos(2*e + 2*f*x) - c^2*d*f^2*sin(2*e + 2*f*x)*6i + 12*c^2*d*f^4*x^2 + 8*
c*d^2*f^4*x^3 + 6*d^3*f^2*x^2*cos(2*e + 2*f*x) + d^3*f^3*x^3*cos(2*e + 2*f*
x)*4i - d^3*f^2*x^2*sin(2*e + 2*f*x)*6i + 4*d^3*f^3*x^3*sin(2*e + 2*f*x) -
c*d^2*f*cos(2*e + 2*f*x)*6i - 6*c*d^2*f*sin(2*e + 2*f*x) - d^3*f*x*cos(2*e
+ 2*f*x)*6i - 6*d^3*f*x*sin(2*e + 2*f*x) + 12*c*d^2*f^2*x*cos(2*e + 2*f*x)
+ c^2*d*f^3*x*cos(2*e + 2*f*x)*12i - c*d^2*f^2*x*sin(2*e + 2*f*x)*12i + 12*
c^2*d*f^3*x*sin(2*e + 2*f*x) + c*d^2*f^3*x^2*cos(2*e + 2*f*x)*12i + 12*c*d^
2*f^3*x^2*sin(2*e + 2*f*x))/(16*a*f^4)
```

3.19 $\int \frac{(c+dx)^2}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=137

$$-\frac{d^2x}{4af^2}-\frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{id^2}{4f^3(a+ia\tan(e+fx))}+\frac{d(c+dx)}{2f^2(a+ia\tan(e+fx))}+\frac{i(c+dx)^2}{2f(a+ia\tan(e+fx))}$$

```
[Out] -1/4*d^2*x/a/f^2-1/4*I*(d*x+c)^2/a/f+1/6*(d*x+c)^3/a/d-1/4*I*d^2/f^3/(a+I*a
*tan(f*x+e))+1/2*d*(d*x+c)/f^2/(a+I*a*tan(f*x+e))+1/2*I*(d*x+c)^2/f/(a+I*a*
tan(f*x+e))
```

Rubi [A]
time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3804, 3560, 8}

$$\frac{d(c+dx)}{2f^2(a+ia\tan(e+fx))}+\frac{i(c+dx)^2}{2f(a+ia\tan(e+fx))}-\frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{id^2}{4f^3(a+ia\tan(e+fx))}-\frac{d^2x}{4af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] -1/4*(d^2*x)/(a*f^2) - ((I/4)*(c + d*x)^2)/(a*f) + (c + d*x)^3/(6*a*d) - ((
I/4)*d^2)/(f^3*(a + I*a*Tan[e + f*x])) + (d*(c + d*x))/(2*f^2*(a + I*a*Tan[
e + f*x])) + ((I/2)*(c + d*x)^2)/(f*(a + I*a*Tan[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3804

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[a*d*(m/(2*b*f))
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m/
(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^2}{a+ia\tan(e+fx)}\,dx &= \frac{(c+dx)^3}{6ad}+\frac{i(c+dx)^2}{2f(a+ia\tan(e+fx))}-\frac{(id)\int \frac{c+dx}{a+ia\tan(e+fx)}\,dx}{f} \\
&= -\frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}+\frac{d(c+dx)}{2f^2(a+ia\tan(e+fx))}+\frac{i(c+dx)^2}{2f(a+ia\tan(e+fx))}- \\
&= -\frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{id^2}{4f^3(a+ia\tan(e+fx))}+\frac{d(c+dx)}{2f^2(a+ia\tan(e+fx))} \\
&= -\frac{d^2x}{4af^2}-\frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{id^2}{4f^3(a+ia\tan(e+fx))}+\frac{d(c+dx)}{2f^2(a+ia\tan(e}
\end{aligned}
$$

Mathematica [**A**]
time = 0.30, size = 178, normalized size = 1.30

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))\left((d+(1+i)cf+(1+i)dfx)((1+i)cf+d(-i+(1+i)fx))\cos(2fx)(\cos(e)-i\sin(e))+\frac{4}{3}f^3x(3c^2+3cdx+d^2x^2)\,(\cos(e)+i\sin(e))-i(d+(1+i)cf+(1+i)dfx)((1+i)cf+d(-i+(1+i)fx))(\cos(e)-i\sin(e))\sin(2fx)\right)}{8f^3(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])*((d + (1 + I)*c*f + (1 + I)*d*f*x)*((
1 + I)*c*f + d*(-I + (1 + I)*f*x))*Cos[2*f*x]*(Cos[e] - I*Sin[e]) + (4*f^3*
x*(3*c^2 + 3*c*d*x + d^2*x^2)*(Cos[e] + I*Sin[e]))/3 - I*(d + (1 + I)*c*f +
 (1 + I)*d*f*x)*((1 + I)*c*f + d*(-I + (1 + I)*f*x))*(Cos[e] - I*Sin[e])*Si
n[2*f*x]))/(8*f^3*(a + I*a*Tan[e + f*x]))
```

Maple [**A**]
time = 0.56, size = 108, normalized size = 0.79

method	result	size
risch	$\frac{d^2x^3}{6a}+\frac{dc\,x^2}{2a}+\frac{c^2x}{2a}+\frac{c^3}{6da}+\frac{i\left(2d^2x^2f^2+4cd\,f^2x-2id^2fx+2c^2f^2-2icdf-d^2\right)\mathrm{e}^{-2i(fx+e)}}{8a\,f^3}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*d^2/a*x^3+1/2*d/a*c*x^2+1/2/a*c^2*x+1/6/d/a*c^3+1/8*I*(2*d^2*x^2*f^2+4*
c*d*f^2*x+2*c^2*f^2-2*I*d^2*f*x-d^2-2*I*c*d*f)/a/f^3*exp(-2*I*(f*x+e))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.35, size = 107, normalized size = 0.78

$$\frac{\left(6i\, d^2 f^2 x^2 + 6i\, c^2 f^2 + 6\, cdf - 3i\, d^2 - 6\left(-2i\, cdf^2 - d^2 f\right)x + 4\left(d^2 f^3 x^3 + 3\, cdf^3 x^2 + 3\, c^2 f^3 x\right)e^{(2i\, fx + 2i\, e)}\right)e^{(-2i\, fx - 2i\, e)}}{24\, af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*I*d^2*f^2*x^2 + 6*I*c^2*f^2 + 6*c*d*f - 3*I*d^2 - 6*(-2*I*c*d*f^2 -
 d^2*f)*x + 4*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x)*e^(2*I*f*x + 2*I*
e))*e^(-2*I*f*x - 2*I*e)/(a*f^3)
```

Sympy [A]
time = 0.17, size = 165, normalized size = 1.20

$$\begin{cases} \frac{\left(2ic^2f^2+4icdf^2x+2cdf+2id^2f^2x^2+2d^2fx-id^2\right)e^{-2ie}e^{-2ifx}}{8af^3} & \text{for}\ af^3e^{2ie} \neq 0 \\ \frac{c^2xe^{-2ie}}{2a} + \frac{cdx^2e^{-2ie}}{2a} + \frac{d^2x^3e^{-2ie}}{6a} & \text{otherwise} \end{cases} + \frac{c^2x}{2a} + \frac{cdx^2}{2a} + \frac{d^2x^3}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*tan(f*x+e)),x)
```

```
[Out] Piecewise(((2*I*c**2*f**2 + 4*I*c*d*f**2*x + 2*c*d*f + 2*I*d**2*f**2*x**2 +
 2*d**2*f*x - I*d**2)*exp(-2*I*e)*exp(-2*I*f*x)/(8*a*f**3), Ne(a*f**3*exp(2
*I*e), 0)), (c**2*x*exp(-2*I*e)/(2*a) + c*d*x**2*exp(-2*I*e)/(2*a) + d**2*x
**3*exp(-2*I*e)/(6*a), True)) + c**2*x/(2*a) + c*d*x**2/(2*a) + d**2*x**3/(
6*a)
```

Giac [A]
time = 0.52, size = 123, normalized size = 0.90

$$\frac{\left(4\, d^2 f^3 x^3 e^{(2i\, fx + 2i\, e)} + 12\, cdf^3 x^2 e^{(2i\, fx + 2i\, e)} + 12\, c^2 f^3 x e^{(2i\, fx + 2i\, e)} + 6i\, d^2 f^2 x^2 + 12i\, cdf^2 x + 6i\, c^2 f^2 + 6\, d^2 fx + 6\, cdf - 3i\, d^2\right)e^{(-2i\, fx - 2i\, e)}}{24\, af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/24*(4*d^2*f^3*x^3*e^(2*I*f*x + 2*I*e) + 12*c*d*f^3*x^2*e^(2*I*f*x + 2*I*e
) + 12*c^2*f^3*x*e^(2*I*f*x + 2*I*e) + 6*I*d^2*f^2*x^2 + 12*I*c*d*f^2*x + 6
*I*c^2*f^2 + 6*d^2*f*x + 6*c*d*f - 3*I*d^2)*e^(-2*I*f*x - 2*I*e)/(a*f^3)
```

Mupad [B]
time = 3.08, size = 241, normalized size = 1.76

$$\frac{12\,c^2\,f^3\,x-3\,d^2\,\sin\left(2\,e+2\,f\,x\right)+6\,c^2\,f^2\,\sin\left(2\,e+2\,f\,x\right)+4\,d^2\,f^3\,x^3+6\,c\,d\,f\,\cos\left(2\,e+2\,f\,x\right)+6\,d^2\,f^2\,x^2\,\sin\left(2\,e+2\,f\,x\right)+12\,c\,d\,f^3\,x^2+6\,d^2\,f\,x\,\cos\left(2\,e+2\,f\,x\right)+12\,c\,d\,f^2\,x\,\sin\left(2\,e+2\,f\,x\right)-d^2\,\cos\left(2\,e+2\,f\,x\right)\,3{}\mathrm{i}+c^2\,f^2\,\cos\left(2\,e+2\,f\,x\right)\,6{}\mathrm{i}-c\,d\,f\,\sin\left(2\,e+2\,f\,x\right)\,6{}\mathrm{i}+d^2\,f^2\,x^2\,\cos\left(2\,e+2\,f\,x\right)\,6{}\mathrm{i}-d^2\,f\,x\,\sin\left(2\,e+2\,f\,x\right)\,6{}\mathrm{i}+c\,d\,f^2\,x\,\cos\left(2\,e+2\,f\,x\right)\,12{}\mathrm{i}}{24\,a\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (12*c^2*f^3*x - 3*d^2*sin(2*e + 2*f*x) - d^2*cos(2*e + 2*f*x)*3i + c^2*f^2*
cos(2*e + 2*f*x)*6i + 6*c^2*f^2*sin(2*e + 2*f*x) + 4*d^2*f^3*x^3 + 6*c*d*f*
cos(2*e + 2*f*x) - c*d*f*sin(2*e + 2*f*x)*6i + d^2*f^2*x^2*cos(2*e + 2*f*x)
*6i + 6*d^2*f^2*x^2*sin(2*e + 2*f*x) + 12*c*d*f^3*x^2 + 6*d^2*f*x*cos(2*e +
 2*f*x) - d^2*f*x*sin(2*e + 2*f*x)*6i + c*d*f^2*x*cos(2*e + 2*f*x)*12i + 12
*c*d*f^2*x*sin(2*e + 2*f*x))/(24*a*f^3)
```

3.20 $\int \frac{c+dx}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{idx}{4af}+\frac{(c+dx)^2}{4ad}+\frac{d}{4f^2(a+ia\tan(e+fx))}+\frac{i(c+dx)}{2f(a+ia\tan(e+fx))}$$

```
[Out] -1/4*I*d*x/a/f+1/4*(d*x+c)^2/a/d+1/4*d/f^2/(a+I*a*tan(f*x+e))+1/2*I*(d*x+c)
/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3804, 3560, 8}

$$\frac{i(c+dx)}{2f(a+ia\tan(e+fx))}+\frac{(c+dx)^2}{4ad}+\frac{d}{4f^2(a+ia\tan(e+fx))}-\frac{idx}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-1/4*I)*d*x)/(a*f) + (c + d*x)^2/(4*a*d) + d/(4*f^2*(a + I*a*Tan[e + f*x]
)) + ((I/2)*(c + d*x))/(f*(a + I*a*Tan[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3804

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[a*d*(m/(2*b*f))
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m/
(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{a+ia\tan(e+fx)}\,dx &= \frac{(c+dx)^2}{4ad}+\frac{i(c+dx)}{2f(a+ia\tan(e+fx))}-\frac{(id)\int \frac{1}{a+ia\tan(e+fx)}\,dx}{2f} \\
&= \frac{(c+dx)^2}{4ad}+\frac{d}{4f^2(a+ia\tan(e+fx))}+\frac{i(c+dx)}{2f(a+ia\tan(e+fx))}-\frac{(id)\int 1\,dx}{4af} \\
&= -\frac{idx}{4af}+\frac{(c+dx)^2}{4ad}+\frac{d}{4f^2(a+ia\tan(e+fx))}+\frac{i(c+dx)}{2f(a+ia\tan(e+fx))}
\end{aligned}
$$

Mathematica [A]
time = 0.34, size = 96, normalized size = 1.14

$$
\frac{-i(2cf(i+2fx)+d(1+2ifx+2f^2x^2))+(2cf(-i+2fx)+d(-1-2ifx+2f^2x^2))\tan(e+fx)}{8af^2(-i+\tan(e+fx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((-I)*(2*c*f*(I + 2*f*x) + d*(1 + (2*I)*f*x + 2*f^2*x^2)) + (2*c*f*(-I + 2*
f*x) + d*(-1 - (2*I)*f*x + 2*f^2*x^2))*Tan[e + f*x])/(8*a*f^2*(-I + Tan[e +
 f*x]))
```

Maple [A]
time = 0.50, size = 50, normalized size = 0.60

method	result	size
risch	$\frac{d\,x^2}{4a}+\frac{cx}{2a}+\frac{i(2dxf+2cf-id)\mathrm{e}^{-2i(fx+e)}}{8a\,f^2}$	50
norman	$\frac{\frac{d\,x^2}{4a}+\frac{2icf+d}{4a\,f^2}+\frac{d\,x^2\left(\tan^2(fx+e)\right)}{4a}+\frac{(2cf-id)\tan(fx+e)}{4a\,f^2}+\frac{(2cf+id)x}{4af}+\frac{dx\tan(fx+e)}{2af}+\frac{(2cf-id)x\left(\tan^2(fx+e)\right)}{4fa}}{1+\tan^2(fx+e)}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*d*x^2+1/2/a*c*x+1/8*I*(2*d*x*f+2*c*f-I*d)/a/f^2*exp(-2*I*(f*x+e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 57, normalized size = 0.68

$$\frac{\left(2i\,dfx + 2i\,cf + 2\left(df^2x^2 + 2\,cf^2x\right)e^{(2i\,fx+2i\,e)} + d\right)e^{(-2i\,fx-2i\,e)}}{8\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*I*d*f*x + 2*I*c*f + 2*(d*f^2*x^2 + 2*c*f^2*x)*e^(2*I*f*x + 2*I*e) +
d)*e^(-2*I*f*x - 2*I*e)/(a*f^2)
```

Sympy [A]
time = 0.12, size = 92, normalized size = 1.10

$$\begin{cases} \frac{(2icf+2idfx+d)e^{-2ie}e^{-2ifx}}{8af^2} & \text{for}\ af^2e^{2ie} \neq 0 \\ \frac{cxe^{-2ie}}{2a} + \frac{dx^2e^{-2ie}}{4a} & \text{otherwise} \end{cases} + \frac{cx}{2a} + \frac{dx^2}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] Piecewise(((2*I*c*f + 2*I*d*f*x + d)*exp(-2*I*e)*exp(-2*I*f*x)/(8*a*f**2),
Ne(a*f**2*exp(2*I*e), 0)), (c*x*exp(-2*I*e)/(2*a) + d*x**2*exp(-2*I*e)/(4*a
), True)) + c*x/(2*a) + d*x**2/(4*a)
```

Giac [A]
time = 0.52, size = 65, normalized size = 0.77

$$\frac{\left(2\,df^2x^2e^{(2i\,fx+2i\,e)} + 4\,cf^2xe^{(2i\,fx+2i\,e)} + 2i\,dfx + 2i\,cf + d\right)e^{(-2i\,fx-2i\,e)}}{8\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/8*(2*d*f^2*x^2*e^(2*I*f*x + 2*I*e) + 4*c*f^2*x*e^(2*I*f*x + 2*I*e) + 2*I*
d*f*x + 2*I*c*f + d)*e^(-2*I*f*x - 2*I*e)/(a*f^2)
```

Mupad [B]
time = 2.81, size = 105, normalized size = 1.25

$$\frac{d\,\cos\left(2\,e+2\,f\,x\right)+2\,d\,f^2\,x^2+2\,c\,f\,\sin\left(2\,e+2\,f\,x\right)+4\,c\,f^2\,x+2\,d\,f\,x\,\sin\left(2\,e+2\,f\,x\right)-d\,\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}+c\,f\,\cos\left(2\,e+2\,f\,x\right)\,2{}\mathrm{i}+d\,f\,x\,\cos\left(2\,e+2\,f\,x\right)\,2{}\mathrm{i}}{8\,a\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] (d*cos(2*e + 2*f*x) - d*sin(2*e + 2*f*x)*1i + 2*d*f^2*x^2 + c*f*cos(2*e + 2
*f*x)*2i + 2*c*f*sin(2*e + 2*f*x) + 4*c*f^2*x + d*f*x*cos(2*e + 2*f*x)*2i +
 2*d*f*x*sin(2*e + 2*f*x))/(8*a*f^2)
```

3.21 $\int \frac{1}{(c+dx)(a+ia\tan(e+fx))}\,dx$

Optimal. Leaf size=161

$$\frac{\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{2ad}+\frac{\log(c+dx)}{2ad}-\frac{i\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{2ad}-\frac{i\cos\left(2e-\frac{2c}{a}\right.}{}$$

```
[Out] 1/2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a/d+1/2*ln(d*x+c)/a/d-1/2*I*cos(-2*
e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a/d+1/2*I*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/
a/d+1/2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d
```

Rubi [A]
time = 0.21, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3807, 3384, 3380, 3383}

$$-\frac{i\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{2ad}+\frac{\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2ad}-\frac{\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2ad}-\frac{i\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2ad}+\frac{\log(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*a*d) + Log[c + d*x
]/(2*a*d) - ((I/2)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a*
d) - ((I/2)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a*d) - (S
in[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*a*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3807

```
Int[1/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Symb
ol] :> Simp[Log[c + d*x]/(2*a*d), x] + (Dist[1/(2*a), Int[Cos[2*e + 2*f*x]/
(c + d*x), x], x] + Dist[1/(2*b), Int[Sin[2*e + 2*f*x]/(c + d*x), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+dx)(a+ia\tan(e+fx))}\,dx &= \frac{\log(c+dx)}{2ad}-\frac{i\int \frac{\sin(2e+2fx)}{c+dx}\,dx}{2a}+\frac{\int \frac{\cos(2e+2fx)}{c+dx}\,dx}{2a}\\ &= \frac{\log(c+dx)}{2ad}-\frac{\left(i\cos\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\sin\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{2a}+\frac{\cos\left(2e-\frac{2cf}{d}\right)\int}{2a}\\ &= \frac{\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{2ad}+\frac{\log(c+dx)}{2ad}-\frac{i\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e\right.}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 166, normalized size = 1.03

$$\frac{\sec(e+fx)\left(-i\cos\left(f\left(\frac{c}{d}+x\right)\right)+\sin\left(f\left(\frac{c}{d}+x\right)\right)\right)\left(\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\left(\cos\left(e-\frac{cf}{d}\right)-i\sin\left(e-\frac{cf}{d}\right)\right)+\log(f(c+dx))\left(\cos\left(e-\frac{cf}{d}\right)+i\sin\left(e-\frac{cf}{d}\right)\right)+\left(-i\cos\left(e-\frac{cf}{d}\right)-\sin\left(e-\frac{cf}{d}\right)\right)\operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{2ad(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (Sec[e + f*x]*((-I)*Cos[f*(c/d + x)] + Sin[f*(c/d + x)])*(CosIntegral[(2*f*
(c + d*x))/d]*(Cos[e - (c*f)/d] - I*Sin[e - (c*f)/d]) + Log[f*(c + d*x)]*(C
os[e - (c*f)/d] + I*Sin[e - (c*f)/d]) + ((-I)*Cos[e - (c*f)/d] - Sin[e - (c
*f)/d])*SinIntegral[(2*f*(c + d*x))/d]))/(2*a*d*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.46, size = 65, normalized size = 0.40

method	result	size
risch	$\frac{\ln(dx+c)}{2da}-\frac{\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\text{expIntegral}\left(1,2ifx+2ie+\frac{2i(cf-de)}{d}\right)}{2ad}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(d*x+c)/d/a-1/2/a/d*exp(2*I*(c*f-d*e)/d)*Ei(1,2*I*f*x+2*I*e+2*I*(c*f-
d*e)/d)
```

Maxima [A]
time = 0.36, size = 120, normalized size = 0.75

$$-\frac{f\cos\left(\frac{2(cf-de)}{d}\right)E_1\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)+ifE_1\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right)\sin\left(\frac{2(cf-de)}{d}\right)-f\log\left((fx+e)d+cf-de\right)}{2\,adf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(f*cos(2*(c*f - d*e)/d)*exp_integral_e(1, -2*(-I*(f*x + e)*d - I*c*f +
 I*d*e)/d) + I*f*exp_integral_e(1, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d)*s
in(2*(c*f - d*e)/d) - f*log((f*x + e)*d + c*f - d*e))/(a*d*f)
```

Fricas [A]
time = 0.37, size = 52, normalized size = 0.32

$$\frac{\operatorname{Ei}\left(-\frac{2(idfx+icf)}{d}\right)e^{\left(-\frac{2(-icf+ide)}{d}\right)}+\log\left(\frac{dx+c}{d}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(Ei(-2*(I*d*f*x + I*c*f)/d)*e^(-2*(-I*c*f + I*d*e)/d) + log((d*x + c)/d
))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{c\tan(e+fx)-ic+dx\tan(e+fx)-idx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(1/(c*tan(e + f*x) - I*c + d*x*tan(e + f*x) - I*d*x), x)/a
```

Giac [A]
time = 0.53, size = 142, normalized size = 0.88

$$\frac{\cos\left(\frac{2cf}{d}\right)\operatorname{Ci}\left(-\frac{2(dfx+cf)}{d}\right)+\cos(2e)\log(dx+c)+i\operatorname{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\sin\left(\frac{2cf}{d}\right)+i\log(dx+c)\sin(2e)-i\cos\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)+\sin\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)}{2\left(ad\cos(2e)+iad\sin(2e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(cos(2*c*f/d)*cos_integral(-2*(d*f*x + c*f)/d) + cos(2*e)*log(d*x + c)
+ I*cos_integral(-2*(d*f*x + c*f)/d)*sin(2*c*f/d) + I*log(d*x + c)*sin(2*e)
```

```
 - I*cos(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + sin(2*c*f/d)*sin_integr
al(2*(d*f*x + c*f)/d))/(a*d*cos(2*e) + I*a*d*sin(2*e))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+a\tan(e+f\,x)\ 1\mathrm{i})\ (c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c + d*x)),x)
[Out] int(1/((a + a*tan(e + f*x)*1i)*(c + d*x)), x)
```

3.22 $\int \frac{1}{(c+dx)^2(a+ia\tan(e+fx))}\,dx$

Optimal. Leaf size=168

$$-\frac{if\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{ad^2}-\frac{f\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^2}-\frac{f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2c}{d}\right.}{ad^2}$$

```
[Out] -I*f*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a/d^2-f*cos(-2*e+2*c*f/d)*Si(2*c*f
/d+2*f*x)/a/d^2+f*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d^2-I*f*Si(2*c*f/d+
2*f*x)*sin(-2*e+2*c*f/d)/a/d^2-1/d/(d*x+c)/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.17, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3805, 3384, 3380, 3383}

$$-\frac{f\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^2}-\frac{if\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{ad^2}+\frac{if\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{ad^2}-\frac{f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{ad^2}-\frac{1}{d(c+dx)(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] ((-I)*f*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) - (f*C
osIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a*d^2) - (f*Cos[2*e -
(2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) + (I*f*Sin[2*e - (2*c*f)
/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) - 1/(d*(c + d*x)*(a + I*a*Tan[e
 + f*x]))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3805

```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Dist[f/(a*d)
, Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*
x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+dx)^2(a+ia\tan(e+fx))}\,dx &= -\frac{1}{d(c+dx)(a+ia\tan(e+fx))} - \frac{(if)\int \frac{\cos(2e+2fx)}{c+dx}\,dx}{ad} - \frac{f\int \frac{\sin(2e+}{c+d}}{ad} \\ &= -\frac{1}{d(c+dx)(a+ia\tan(e+fx))} - \frac{\left(if\cos\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cos\left(\frac{2cf}{d}+2fx\right)}{c+dx}}{ad} \\ &= -\frac{if\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{ad^2} - \frac{f\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^2}\end{aligned}$$

Mathematica [A]

time = 0.88, size = 224, normalized size = 1.33

$$\frac{\sec(e+fx)\left(\cos\left(\frac{cf}{d}\right)+i\sin\left(\frac{cf}{d}\right)\right)\left(d\left(i\cos\left(e+f\left(-\frac{c}{d}+x\right)\right)+i\cos\left(e+f\left(\frac{c}{d}+x\right)\right)-\sin\left(e+f\left(-\frac{c}{d}+x\right)\right)+\sin\left(e+f\left(\frac{c}{d}+x\right)\right)\right)-2f(c+dx)\mathrm{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\left(\cos\left(e-\frac{f(c+dx)}{d}\right)-i\sin\left(e-\frac{f(c+dx)}{d}\right)\right)+2f(c+dx)\left(i\cos\left(e-\frac{f(c+dx)}{d}\right)+\sin\left(e-\frac{f(c+dx)}{d}\right)\right)\mathrm{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{2ad^2(c+dx)(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (Sec[e + f*x]*(Cos[(c*f)/d] + I*Sin[(c*f)/d])*(d*(I*Cos[e + f*(-(c/d) + x)]
 + I*Cos[e + f*(c/d + x)] - Sin[e + f*(-(c/d) + x)] + Sin[e + f*(c/d + x)])
 - 2*f*(c + d*x)*CosIntegral[(2*f*(c + d*x))/d]*(Cos[e - (f*(c + d*x))/d] -
 I*Sin[e - (f*(c + d*x))/d]) + 2*f*(c + d*x)*(I*Cos[e - (f*(c + d*x))/d] +
Sin[e - (f*(c + d*x))/d])*SinIntegral[(2*f*(c + d*x))/d]))/(2*a*d^2*(c + d*
x)*(-I + Tan[e + f*x]))
```

Maple [A]

time = 0.37, size = 285, normalized size = 1.70

method	result
risch	$-\frac{1}{2da(dx+c)}-\frac{f\,\mathrm{e}^{-2i(fx+e)}}{2a(dxf+cf)d}+\frac{if\,\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,2ifx+2ie+\frac{2i(cf-de)}{d}\right)}{a\,d^2}$
default	$-\frac{if^2\left(-\frac{2\sin(2fx+2e)}{(cf-de+d(fx+e))d}+\frac{4\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{4\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}+f^2\Big(-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/f*(-1/4*I*f^2*(-2*sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d+2*(2*Si(2*f*x+2*
e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(
c*f-d*e)/d)/d)/d)+1/4*f^2*(-2*cos(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d-2*(2*Si(
2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)
*sin(2*(c*f-d*e)/d)/d)/d)-1/2*f^2/(c*f-d*e+d*(f*x+e))/d)
```

Maxima [A]
time = 0.38, size = 127, normalized size = 0.76

$$-\frac{f^2\cos\left(\frac{2\,(cf-de)}{d}\right)E_2\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)+i\,f^2E_2\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{2\,(cf-de)}{d}\right)+f^2}{2\,((fx+e)ad^2+acdf-ad^2e)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(f^2*cos(2*(c*f - d*e)/d)*exp_integral_e(2, -2*(-I*(f*x + e)*d - I*c*f
 + I*d*e)/d) + I*f^2*exp_integral_e(2, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/
d)*sin(2*(c*f - d*e)/d) + f^2)/(((f*x + e)*a*d^2 + a*c*d*f - a*d^2*e)*f)
```

Fricas [A]
time = 0.37, size = 86, normalized size = 0.51

$$-\frac{\left(\left(2\,(i\,dfx+i\,cf)\mathrm{Ei}\left(-\frac{2\,(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{2\,(-i\,cf+i\,de)}{d}\right)}+d\right)e^{(2i\,fx+2i\,e)}+d\right)e^{(-2i\,fx-2i\,e)}}{2\,(ad^3x+acd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((2*(I*d*f*x + I*c*f)*Ei(-2*(I*d*f*x + I*c*f)/d)*e^(-2*(-I*c*f + I*d*e
)/d) + d)*e^(2*I*f*x + 2*I*e) + d)*e^(-2*I*f*x - 2*I*e)/(a*d^3*x + a*c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{c^2\tan{(e+fx)}-ic^2+2cdx\tan{(e+fx)}-2icdx+d^2x^2\tan{(e+fx)}-id^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(1/(c**2*tan(e + f*x) - I*c**2 + 2*c*d*x*tan(e + f*x) - 2*I*c*d*
x + d**2*x**2*tan(e + f*x) - I*d**2*x**2), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(166) = 332$.
time = 2.77, size = 1099, normalized size = 6.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(2*I*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos(2*(c*f - d*
e)/d)*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f
+ d*e)/d) - 2*I*c*f^3*cos(2*(c*f - d*e)/d)*cos_integral(-2*((d*x + c)*(c*f/
(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 2*I*d*f^2*cos(2*(c*f - d*e
)/d)*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f +
 d*e)/d)*e - 2*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos_integr
al(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*sin(2*
(c*f - d*e)/d) + 2*c*f^3*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*
e/(d*x + c)) - c*f + d*e)/d)*sin(2*(c*f - d*e)/d) - 2*d*f^2*cos_integral(-2
*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e*sin(2*(c*
f - d*e)/d) + 2*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos(2*(c*
f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))
- c*f + d*e)/d) - 2*c*f^3*cos(2*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(
c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 2*d*f^2*cos(2*(c*f - d
*e)/d)*e*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c
*f + d*e)/d) + 2*I*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*sin(2*
(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c
)) - c*f + d*e)/d) - 2*I*c*f^3*sin(2*(c*f - d*e)/d)*sin_integral(-2*((d*x +
 c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 2*I*d*f^2*e*sin(2
*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x +
c)) - c*f + d*e)/d) - d*f^2*cos(2*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x +
 c))/d) - I*d*f^2*sin(2*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d) -
d*f^2)*d^2/(((d*x + c)*a*d^4*(c*f/(d*x + c) - f - d*e/(d*x + c)) - a*c*d^4*
f + a*d^5*e)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*tan(e + f*x)*1i)*(c + d*x)^2), x)
```

3.23 $\int \frac{1}{(c+dx)^3(a+ia\tan(e+fx))}\,dx$

Optimal. Leaf size=227

$$-\frac{if}{2ad^2(c+dx)}-\frac{f^2\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{ad^3}+\frac{if^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^3}+\frac{if^2\text{ c}}{}$$

```
[Out] -1/2*I*f/a/d^2/(d*x+c)-f^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a/d^3+I*f^2*
cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a/d^3-I*f^2*Ci(2*c*f/d+2*f*x)*sin(-2*e+
2*c*f/d)/a/d^3-f^2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d^3-1/2/d/(d*x+c)^
2/(a+I*a*tan(f*x+e))+I*f/d^2/(d*x+c)/(a+I*a*tan(f*x+e))
```

Rubi [A]

time = 0.23, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3806, 3805, 3384, 3380, 3383}

$$\frac{if^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^3}-\frac{f^2\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{ad^3}+\frac{f^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{ad^3}+\frac{if^2\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{ad^3}+\frac{if}{d^2(c+dx)(a+ia\tan(e+fx))}-\frac{if}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^3*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] ((-1/2*I)*f)/(a*d^2*(c + d*x)) - (f^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c
*f)/d + 2*f*x])/(a*d^3) + (I*f^2*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (
2*c*f)/d])/(a*d^3) + (I*f^2*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*
f*x])/(a*d^3) + (f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(
a*d^3) - 1/(2*d*(c + d*x)^2*(a + I*a*Tan[e + f*x])) + (I*f)/(d^2*(c + d*x)*
(a + I*a*Tan[e + f*x]))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3805

```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Dist[f/(a*d)
, Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*
x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3806

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[f*((c + d*x)^(m + 2)/(b*d^2*(m + 1)*(m + 2))), x] + (Dist[2*b*
(f/(a*d*(m + 1))), Int[(c + d*x)^(m + 1)/(a + b*Tan[e + f*x]), x], x] + Sim
p[(c + d*x)^(m + 1)/(d*(m + 1)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(c+dx)^3(a+ia\tan(e+fx))}\,dx &= -\frac{if}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a+ia\tan(e+fx))}-\frac{(if)\int \frac{}{(c+dx)^2(a}}{} \\
&= -\frac{if}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a+ia\tan(e+fx))}+\frac{}{d^2(c+dx)(a+} \\
&= -\frac{if}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a+ia\tan(e+fx))}+\frac{}{d^2(c+dx)(a+} \\
&= -\frac{if}{2ad^2(c+dx)}-\frac{f^2\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{ad^3}+\frac{if^2\operatorname{Ci}\left(\frac{2cf}{d}+2f\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 1.24, size = 285, normalized size = 1.26

$$
\frac{\sec(e+fx)\left(\cos\left(\frac{cf}{d}\right)+i\sin\left(\frac{cf}{d}\right)\right)\left(d\left(id\cos\left(e+f\left(-\frac{c}{d}+x\right)\right)+(id+2cf+2dfx)\cos\left(e+f\left(\frac{c}{d}+x\right)\right)-d\sin\left(e+f\left(-\frac{c}{d}+x\right)\right)+d\sin\left(e+f\left(\frac{c}{d}+x\right)\right)-2icf\sin\left(e+f\left(\frac{c}{d}+x\right)\right)-2idfx\sin\left(e+f\left(\frac{c}{d}+x\right)\right)\right)+4f^2(c+dx)^2\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\left(i\cos\left(e-\frac{f(c+dx)}{d}\right)+\sin\left(e-\frac{f(c+dx)}{d}\right)\right)+4f^2(c+dx)^2\left(\cos\left(e-\frac{f(c+dx)}{d}\right)-i\sin\left(e-\frac{f(c+dx)}{d}\right)\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{4ad^3(c+dx)^2(-i+\tan(e+fx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^3*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (Sec[e + f*x]*(Cos[(c*f)/d] + I*Sin[(c*f)/d])*(d*(I*d*Cos[e + f*(-(c/d) + x
)] + (I*d + 2*c*f + 2*d*f*x)*Cos[e + f*(c/d + x)] - d*Sin[e + f*(-(c/d) + x
)] + d*Sin[e + f*(c/d + x)] - (2*I)*c*f*Sin[e + f*(c/d + x)] - (2*I)*d*f*x*
Sin[e + f*(c/d + x)]) + 4*f^2*(c + d*x)^2*CosIntegral[(2*f*(c + d*x))/d]*(I
*Cos[e - (f*(c + d*x))/d] + Sin[e - (f*(c + d*x))/d]) + 4*f^2*(c + d*x)^2*(
Cos[e - (f*(c + d*x))/d] - I*Sin[e - (f*(c + d*x))/d])*SinIntegral[(2*f*(c
+ d*x))/d]))/(4*a*d^3*(c + d*x)^2*(-I + Tan[e + f*x]))
```

Maple [A]
time = 0.45, size = 358, normalized size = 1.58

method	result
risch	$-\frac{1}{4da(dx+c)^2}+\frac{if^3\mathrm{e}^{-2i(fx+e)}x}{2ad(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2\mathrm{e}^{-2i(fx+e)}}{4ad(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{if^3\mathrm{e}^{-2i(fx+e)}c}{2a\,d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{f^2\mathrm{e}^{\frac{2i(cf-de}{d}}}{}$
default	$\frac{-\frac{if^3\left(-\frac{\sin(2fx+2e)}{(cf-de+d(fx+e))^2d}+\frac{-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)}{d}\right.}{d}}{d}\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/f*(-1/4*I*f^3*(-sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))^2/d+(-2*cos(2*f*x+2*
e)/(c*f-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d
)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d)/d)+1/4*f^3*(-cos
(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))^2/d-(-2*sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/
d+2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c
*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d)/d)/d)-1/4*f^3/(c*f-d*e+d*(f*x+e))^2/d)
```

Maxima [A]
time = 0.42, size = 167, normalized size = 0.74

$$-\frac{2\,f^3\cos\left(\frac{2\,(cf-de)}{d}\right)E_3\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)+2i\,f^3E_3\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{2\,(cf-de)}{d}\right)+f^3}{4\left((fx+e)^2ad^3+ac^2df^2-2\,acd^2fe+ad^3e^2+2\,(acd^2f-ad^3e)(fx+e)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*f^3*cos(2*(c*f - d*e)/d)*exp_integral_e(3, -2*(-I*(f*x + e)*d - I*c
*f + I*d*e)/d) + 2*I*f^3*exp_integral_e(3, -2*(-I*(f*x + e)*d - I*c*f + I*d
*e)/d)*sin(2*(c*f - d*e)/d) + f^3)/(((f*x + e)^2*a*d^3 + a*c^2*d*f^2 - 2*a*
c*d^2*f*e + a*d^3*e^2 + 2*(a*c*d^2*f - a*d^3*e)*(f*x + e))*f)
```

Fricas [A]
time = 0.37, size = 133, normalized size = 0.59

$$\frac{\left(2i\,d^2fx+2i\,cdf-d^2-\left(4\,(d^2f^2x^2+2\,cdf^2x+c^2f^2)\mathrm{Ei}\left(-\frac{2\,(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{2\,(-i\,cf+i\,de)}{d}\right)}+d^2\right)e^{(2i\,fx+2i\,e)}\right)e^{(-2i\,fx-2i\,e)}}{4\,(ad^5x^2+2\,acd^4x+ac^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*d^2*f*x + 2*I*c*d*f - d^2 - (4*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^
2)*Ei(-2*(I*d*f*x + I*c*f)/d)*e^(-2*(-I*c*f + I*d*e)/d) + d^2)*e^(2*I*f*x +
 2*I*e))*e^(-2*I*f*x - 2*I*e)/(a*d^5*x^2 + 2*a*c*d^4*x + a*c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{c^3\tan(e+fx)-ic^3+3c^2dx\tan(e+fx)-3ic^2dx+3cd^2x^2\tan(e+fx)-3icd^2x^2+d^3x^3\tan(e+fx)-id^3x^3}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(1/(c**3*tan(e + f*x) - I*c**3 + 3*c**2*d*x*tan(e + f*x) - 3*I*c
**2*d*x + 3*c*d**2*x**2*tan(e + f*x) - 3*I*c*d**2*x**2 + d**3*x**3*tan(e +
f*x) - I*d**3*x**3), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(218) = 436$.
time = 0.52, size = 540, normalized size = 2.38

$$-\frac{4\,d^2f^2x^2\cos\left(\frac{2cf}{d}\right)\operatorname{Ci}\left(-\frac{2(dfx+cf)}{d}\right)+4i\,d^2f^2x^2\operatorname{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\sin\left(\frac{2cf}{d}\right)-4i\,d^2f^2x^2\cos\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)+4\,d^2f^2x^2\sin\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)+8\,cdf^2x\cos\left(\frac{2cf}{d}\right)\operatorname{Ci}\left(-\frac{2(dfx+cf)}{d}\right)+8i\,cdf^2x\operatorname{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\sin\left(\frac{2cf}{d}\right)-8i\,cdf^2x\cos\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)+8\,cdf^2x\sin\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)+4\,c^2f^2\cos\left(\frac{2cf}{d}\right)\operatorname{Ci}\left(-\frac{2(dfx+cf)}{d}\right)+4i\,c^2f^2\operatorname{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\sin\left(\frac{2cf}{d}\right)-4i\,c^2f^2\cos\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)+4\,c^2f^2\sin\left(\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2(dfx+cf)}{d}\right)-2i\,d^2fx\cos(2fx)-2\,d^2fx\sin(2fx)-2i\,cdf\cos(2fx)-2\,cdf\sin(2fx)+d^2\cos(2fx)+d^2\cos(2e)-i\,d^2\sin(2fx)+i\,d^2\sin(2e)}{4\left(ad^5x^2\cos(2e)+i\,ad^5x^2\sin(2e)+2\,acd^4x\cos(2e)+2i\,acd^4x\sin(2e)+ac^2d^3\cos(2e)+i\,ac^2d^3\sin(2e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/4*(4*d^2*f^2*x^2*cos(2*c*f/d)*cos_integral(-2*(d*f*x + c*f)/d) + 4*I*d^2
*f^2*x^2*cos_integral(-2*(d*f*x + c*f)/d)*sin(2*c*f/d) - 4*I*d^2*f^2*x^2*co
s(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 4*d^2*f^2*x^2*sin(2*c*f/d)*sin
_integral(2*(d*f*x + c*f)/d) + 8*c*d*f^2*x*cos(2*c*f/d)*cos_integral(-2*(d*
f*x + c*f)/d) + 8*I*c*d*f^2*x*cos_integral(-2*(d*f*x + c*f)/d)*sin(2*c*f/d)
 - 8*I*c*d*f^2*x*cos(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*c*d*f^2*x
*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 4*c^2*f^2*cos(2*c*f/d)*cos_
integral(-2*(d*f*x + c*f)/d) + 4*I*c^2*f^2*cos_integral(-2*(d*f*x + c*f)/d)
*sin(2*c*f/d) - 4*I*c^2*f^2*cos(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) +
4*c^2*f^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - 2*I*d^2*f*x*cos(2*
f*x) - 2*d^2*f*x*sin(2*f*x) - 2*I*c*d*f*cos(2*f*x) - 2*c*d*f*sin(2*f*x) + d
^2*cos(2*f*x) + d^2*cos(2*e) - I*d^2*sin(2*f*x) + I*d^2*sin(2*e))/(a*d^5*x^
2*cos(2*e) + I*a*d^5*x^2*sin(2*e) + 2*a*c*d^4*x*cos(2*e) + 2*I*a*c*d^4*x*si
n(2*e) + a*c^2*d^3*cos(2*e) + I*a*c^2*d^3*sin(2*e))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)\,{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)*(c + d*x)^3),x)
[Out] int(1/((a + a*tan(e + f*x)*1i)*(c + d*x)^3), x)
```

3.24 $\int \frac{(c+dx)^3}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=270

$$-\frac{3d^3e^{-2ie-2ifx}}{16a^2f^4}-\frac{3d^3e^{-4ie-4ifx}}{512a^2f^4}-\frac{3id^2e^{-2ie-2ifx}(c+dx)}{8a^2f^3}-\frac{3id^2e^{-4ie-4ifx}(c+dx)}{128a^2f^3}+\frac{3de^{-2ie-2ifx}(c+dx)^2}{8a^2f^2}+3de$$

```
[Out] -3/16*d^3*exp(-2*I*e-2*I*f*x)/a^2/f^4-3/512*d^3*exp(-4*I*e-4*I*f*x)/a^2/f^4
-3/8*I*d^2*exp(-2*I*e-2*I*f*x)*(d*x+c)/a^2/f^3-3/128*I*d^2*exp(-4*I*e-4*I*f
*x)*(d*x+c)/a^2/f^3+3/8*d*exp(-2*I*e-2*I*f*x)*(d*x+c)^2/a^2/f^2+3/64*d*exp(
-4*I*e-4*I*f*x)*(d*x+c)^2/a^2/f^2+1/4*I*exp(-2*I*e-2*I*f*x)*(d*x+c)^3/a^2/f
+1/16*I*exp(-4*I*e-4*I*f*x)*(d*x+c)^3/a^2/f+1/16*(d*x+c)^4/a^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3810, 2207, 2225}

$$-\frac{3id^2(c+dx)e^{-2ie-2ifx}}{8a^2f^3}-\frac{3id^2(c+dx)e^{-4ie-4ifx}}{128a^2f^3}+\frac{3d(c+dx)^2e^{-2ie-2ifx}}{8a^2f^2}+\frac{3d(c+dx)^2e^{-4ie-4ifx}}{64a^2f^2}+\frac{i(c+dx)^3e^{-2ie-2ifx}}{4a^2f}+\frac{i(c+dx)^3e^{-4ie-4ifx}}{16a^2f}+\frac{(c+dx)^4}{16a^2d}-\frac{3d^3e^{-2ie-2ifx}}{16a^2f^4}-\frac{3d^3e^{-4ie-4ifx}}{512a^2f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (-3*d^3*E^((-2*I)*e - (2*I)*f*x))/(16*a^2*f^4) - (3*d^3*E^((-4*I)*e - (4*I)
*f*x))/(512*a^2*f^4) - (((3*I)/8)*d^2*E^((-2*I)*e - (2*I)*f*x)*(c + d*x))/(
a^2*f^3) - (((3*I)/128)*d^2*E^((-4*I)*e - (4*I)*f*x)*(c + d*x))/(a^2*f^3) +
 (3*d*E^((-2*I)*e - (2*I)*f*x)*(c + d*x)^2)/(8*a^2*f^2) + (3*d*E^((-4*I)*e
- (4*I)*f*x)*(c + d*x)^2)/(64*a^2*f^2) + ((I/4)*E^((-2*I)*e - (2*I)*f*x)*(c
 + d*x)^3)/(a^2*f) + ((I/16)*E^((-4*I)*e - (4*I)*f*x)*(c + d*x)^3)/(a^2*f)
+ (c + d*x)^4/(16*a^2*d)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+ia\tan(e+fx))^2}\,dx &= \int \left(\frac{(c+dx)^3}{4a^2}+\frac{e^{-2ie-2ifx}(c+dx)^3}{2a^2}+\frac{e^{-4ie-4ifx}(c+dx)^3}{4a^2}\right)dx\\ &= \frac{(c+dx)^4}{16a^2d}+\frac{\int e^{-4ie-4ifx}(c+dx)^3\,dx}{4a^2}+\frac{\int e^{-2ie-2ifx}(c+dx)^3\,dx}{2a^2}\\ &= \frac{ie^{-2ie-2ifx}(c+dx)^3}{4a^2f}+\frac{ie^{-4ie-4ifx}(c+dx)^3}{16a^2f}+\frac{(c+dx)^4}{16a^2d}-\frac{(3id)\int e^{-4ie-4ifx}(c}{16a^2f}\\ &= \frac{3de^{-2ie-2ifx}(c+dx)^2}{8a^2f^2}+\frac{3de^{-4ie-4ifx}(c+dx)^2}{64a^2f^2}+\frac{ie^{-2ie-2ifx}(c+dx)^3}{4a^2f}+\frac{ie^{-4ie}}{}\\ &= -\frac{3id^2e^{-2ie-2ifx}(c+dx)}{8a^2f^3}-\frac{3id^2e^{-4ie-4ifx}(c+dx)}{128a^2f^3}+\frac{3de^{-2ie-2ifx}(c+dx)^2}{8a^2f^2}+\\ &= -\frac{3d^3e^{-2ie-2ifx}}{16a^2f^4}-\frac{3d^3e^{-4ie-4ifx}}{512a^2f^4}-\frac{3id^2e^{-2ie-2ifx}(c+dx)}{8a^2f^3}-\frac{3id^2e^{-4ie-4ifx}(c+}{128a^2f^3}\end{aligned}$$

Mathematica [A]
time = 1.02, size = 473, normalized size = 1.75

$$\frac{\sec^2(e+fx)(\cos(fx)+i\sin(fx))^2\left((4ic^3f^3+6c^2df^2(1+2ifx)+6cd^2f(-i+2fx+2if^2x^2)+d^3(-3-6ifx+6f^2x^2+4if^3x^3))\cos(2fx)+\frac{1}{32}(32ic^3f^3+24c^2df^2(1+4ifx)+12cd^2f(-i+4fx+8if^2x^2)+d^3(-3-12ifx+24f^2x^2+32if^3x^3))\cos(4fx)(\cos(2e)-i\sin(2e))+f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)(\cos(2e)+i\sin(2e))+(4c^3f^3+6c^2df^2(-i+2fx)+6cd^2f(-1-2ifx+2f^2x^2)+d^3(3i-6fx-6if^2x^2+4f^3x^3))\sin(2fx)+\frac{1}{32}(32c^3f^3+24c^2df^2(-i+4fx)+12cd^2f(-1-4ifx+8f^2x^2)+d^3(3i-12fx-24if^2x^2+32f^3x^3))(\cos(2e)-i\sin(2e))\sin(4fx)\right)}{16f^4(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*(((4*I)*c^3*f^3 + 6*c^2*d*f^2*(1
+ (2*I)*f*x) + 6*c*d^2*f*(-I + 2*f*x + (2*I)*f^2*x^2) + d^3*(-3 - (6*I)*f*x
 + 6*f^2*x^2 + (4*I)*f^3*x^3))*Cos[2*f*x] + (((32*I)*c^3*f^3 + 24*c^2*d*f^2
*(1 + (4*I)*f*x) + 12*c*d^2*f*(-I + 4*f*x + (8*I)*f^2*x^2) + d^3*(-3 - (12*
I)*f*x + 24*f^2*x^2 + (32*I)*f^3*x^3))*Cos[4*f*x]*(Cos[2*e] - I*Sin[2*e]))/
32 + f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(Cos[2*e] + I*Sin[2*
e]) + (4*c^3*f^3 + 6*c^2*d*f^2*(-I + 2*f*x) + 6*c*d^2*f*(-1 - (2*I)*f*x + 2
*f^2*x^2) + d^3*(3*I - 6*f*x - (6*I)*f^2*x^2 + 4*f^3*x^3))*Sin[2*f*x] + ((3
2*c^3*f^3 + 24*c^2*d*f^2*(-I + 4*f*x) + 12*c*d^2*f*(-1 - (4*I)*f*x + 8*f^2*
x^2) + d^3*(3*I - 12*f*x - (24*I)*f^2*x^2 + 32*f^3*x^3))*(Cos[2*e] - I*Sin[
2*e])*Sin[4*f*x])/32))/(16*f^4*(a + I*a*Tan[e + f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2045 vs. 2(224) = 448.
time = 0.53, size = 2046, normalized size = 7.58

method	result
risch	$\frac{d^3x^4}{16a^2} + \frac{d^2c\,x^3}{4a^2} + \frac{3d\,c^2x^2}{8a^2} + \frac{c^3x}{4a^2} + \frac{c^4}{16a^2d} + \frac{i(4d^3x^3f^3+12c\,d^2f^3x^2-6id^3f^2x^2+12c^2d\,f^3x-12ic\,d^2f^2x+4c^3f^3-6ic^2d\,f^2-6c}{16f^4a^2}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/f*(1/2*I*c^3*cos(f*x+e)^4+2/f^3*d^3*((f*x+e)^3*(1/4*(cos(f*x+e)^3+3/2
*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+3/16*(f*x+e)^2*cos(f*x+e)^4-3/8*(f*x
+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+45/128*(f*
x+e)^2-3/512*(2*cos(f*x+e)^2+3)^2+9/16*(f*x+e)^2*cos(f*x+e)^2-9/8*(f*x+e)*(
1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+9/32*sin(f*x+e)^2-9/32*(f*x+e)^4)+
3/f*c^2*d*e*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-3/f^2*c*d^2*e^2*(1/2*
cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+6/f^2*c*d^2*e*((f*x+e)*(1/2*cos(f*x+e)
*sin(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)+3/2*I/f^2*c*d^2*
e^2*cos(f*x+e)^4-3/2*I/f*c^2*d*e*cos(f*x+e)^4+12*I/f^2*c*d^2*e*(-1/4*(f*x+e
)*cos(f*x+e)^4+1/16*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/32*f*x+3/32*
e)-2/f^3*d^3*e^3*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*
e)+6/f^3*d^3*e^2*((f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8
*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*cos(f*x+e)^2+3)^2)-6/f^3*d^3*e*((f*x+e)^
2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+1/8*(f*x+e)*
cos(f*x+e)^4-1/32*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)-15/64*f*x-15/64*
e+3/8*(f*x+e)*cos(f*x+e)^2-3/16*cos(f*x+e)*sin(f*x+e)-1/4*(f*x+e)^3)-2*I/f^
3*d^3*(-1/4*(f*x+e)^3*cos(f*x+e)^4+3/4*(f*x+e)^2*(1/4*(cos(f*x+e)^3+3/2*cos
(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+3/32*(f*x+e)*cos(f*x+e)^4-3/128*(cos(f*x
+e)^3+3/2*cos(f*x+e))*sin(f*x+e)-45/256*f*x-45/256*e+9/32*(f*x+e)*cos(f*x+e
)^2-9/64*cos(f*x+e)*sin(f*x+e)-3/16*(f*x+e)^3)+6/f*c^2*d*((f*x+e)*(1/4*(cos
(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*
cos(f*x+e)^2+3)^2)+6/f^2*c*d^2*((f*x+e)^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e)
)*sin(f*x+e)+3/8*f*x+3/8*e)+1/8*(f*x+e)*cos(f*x+e)^4-1/32*(cos(f*x+e)^3+3/2
*cos(f*x+e))*sin(f*x+e)-15/64*f*x-15/64*e+3/8*(f*x+e)*cos(f*x+e)^2-3/16*cos
(f*x+e)*sin(f*x+e)-1/4*(f*x+e)^3)-3/f*c^2*d*((f*x+e)*(1/2*cos(f*x+e)*sin(f*
x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)-3/f^2*c*d^2*((f*x+e)^2*
(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+1/2*(f*x+e)*cos(f*x+e)^2-1/4*cos(
f*x+e)*sin(f*x+e)-1/4*f*x-1/4*e-1/3*(f*x+e)^3)+1/f^3*d^3*e^3*(1/2*cos(f*x+e
)*sin(f*x+e)+1/2*f*x+1/2*e)-3/f^3*d^3*e^2*((f*x+e)*(1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)+3/f^3*d^3*e*((f*x+e)^2*(1
/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+1/2*(f*x+e)*cos(f*x+e)^2-1/4*cos(f*
x+e)*sin(f*x+e)-1/4*f*x-1/4*e-1/3*(f*x+e)^3)-c^3*(1/2*cos(f*x+e)*sin(f*x+e)
+1/2*f*x+1/2*e)-1/f^3*d^3*((f*x+e)^3*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2
*e)+3/4*(f*x+e)^2*cos(f*x+e)^2-3/2*(f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f
*x+1/2*e)+3/8*(f*x+e)^2+3/8*sin(f*x+e)^2-3/8*(f*x+e)^4)-1/2*I/f^3*d^3*e^3*c
os(f*x+e)^4-6/f*c^2*d*e*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f
```

```
*x+3/8*e)+6/f^2*c*d^2*e^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8
*f*x+3/8*e)-12/f^2*c*d^2*e*((f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(
f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*cos(f*x+e)^2+3)^2)-6*I/f^2*c*d
^2*(-1/4*(f*x+e)^2*cos(f*x+e)^4+1/2*(f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+
e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/32*(f*x+e)^2+1/128*(2*cos(f*x+e)^2+3)^2)-6*
I/f^3*d^3*e^2*(-1/4*(f*x+e)*cos(f*x+e)^4+1/16*(cos(f*x+e)^3+3/2*cos(f*x+e))
*sin(f*x+e)+3/32*f*x+3/32*e)+6*I/f^3*d^3*e*(-1/4*(f*x+e)^2*cos(f*x+e)^4+1/2
*(f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/32*
(f*x+e)^2+1/128*(2*cos(f*x+e)^2+3)^2)-6*I/f*c^2*d*(-1/4*(f*x+e)*cos(f*x+e)^
4+1/16*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/32*f*x+3/32*e)+2*c^3*(1/4
*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 270, normalized size = 1.00

$$\frac{\left(32 i\,d^3 f^3 x^3 + 32 i\,c^3 f^3 + 24\,c^2 d f^2 - 12 i\,c d^2 f - 3\,d^3 - 24\,(-4 i\,c d^2 f^3 - d^3 f^2) x^2 - 12\,(-8 i\,c^2 d f^3 - 4\,c d^2 f^2 + i\,d^3 f) x + 32\,(d^3 f^4 x^4 + 4\,c d^2 f^4 x^3 + 6\,c^2 d f^4 x^2 + 4\,c^3 f^4 x) e^{(4 i\,f x + 4 i\,e)} - 32\,(-4 i\,d^3 f^3 x^3 - 4 i\,c^3 f^3 - 6\,c^2 d f^2 + 6 i\,c d^2 f + 3\,d^3 + 6\,(-2 i\,c d^2 f^3 - d^3 f^2) x^2 + 6\,(-2 i\,c^2 d f^3 - 2\,c d^2 f^2 + i\,d^3 f) x) e^{(2 i\,f x + 2 i\,e)}\right) e^{(-4 i\,f x - 4 i\,e)}}{512\,a^2 f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/512*(32*I*d^3*f^3*x^3 + 32*I*c^3*f^3 + 24*c^2*d*f^2 - 12*I*c*d^2*f - 3*d^
3 - 24*(-4*I*c*d^2*f^3 - d^3*f^2)*x^2 - 12*(-8*I*c^2*d*f^3 - 4*c*d^2*f^2 +
I*d^3*f)*x + 32*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2 + 4*c^3*f^
4*x)*e^(4*I*f*x + 4*I*e) - 32*(-4*I*d^3*f^3*x^3 - 4*I*c^3*f^3 - 6*c^2*d*f^2
 + 6*I*c*d^2*f + 3*d^3 + 6*(-2*I*c*d^2*f^3 - d^3*f^2)*x^2 + 6*(-2*I*c^2*d*f
^3 - 2*c*d^2*f^2 + I*d^3*f)*x)*e^(2*I*f*x + 2*I*e))*e^(-4*I*f*x - 4*I*e)/(a
^2*f^4)
```

Sympy [A]
time = 0.36, size = 665, normalized size = 2.46

$$\begin{cases} \frac{\left(\left(512ia^2c^3f^7e^{2ie} + 1536ia^2c^2df^7xe^{2ie} + 384a^2c^2df^6e^{2ie} + 1536ia^2cd^2f^7x^2e^{2ie} + 768a^2cd^2f^6xe^{2ie} - 192ia^2cd^2f^5e^{2ie} + 512ia^2d^3f^7x^3e^{2ie} + 384a^2d^3f^6x^2e^{2ie} - 192ia^2d^3f^5xe^{2ie} - 48a^2d^3f^4e^{2ie}\right)e^{-4ifx} + \left(2048ia^2c^3f^7e^{4ie} + 6144ia^2c^2df^7xe^{4ie} + 3072a^2c^2df^6e^{4ie} + 6144ia^2cd^2f^7x^2e^{4ie} + 6144a^2cd^2f^6xe^{4ie} - 3072ia^2cd^2f^5e^{4ie} + 2048ia^2d^3f^7x^3e^{4ie} + 3072a^2d^3f^6x^2e^{4ie} - 3072ia^2d^3f^5xe^{4ie} - 1536a^2d^3f^4e^{4ie}\right)e^{-2ifx}\right)e^{-6ie}}{8192a^4f^8} & \text{for}\ a^4f^8e^{6ie} \neq 0 \\ \frac{x^4 \cdot \left(2d^3e^{2ie} + d^3\right)e^{-4ie}}{16a^2} + \frac{x^3 \cdot \left(2cd^2e^{2ie} + cd^2\right)e^{-4ie}}{4a^2} + \frac{x^2 \cdot \left(6c^2de^{2ie} + 3c^2d\right)e^{-4ie}}{8a^2} + \frac{x\left(2c^3e^{2ie} + c^3\right)e^{-4ie}}{4a^2} & \text{otherwise} \end{cases} + \frac{c^3x}{4a^2} + \frac{3c^2dx^2}{8a^2} + \frac{cd^2x^3}{4a^2} + \frac{d^3x^4}{16a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*tan(f*x+e))**2,x)
```

[Out] Piecewise((((512*I*a**2*c**3*f**7*exp(2*I*e) + 1536*I*a**2*c**2*d*f**7*x*ex
p(2*I*e) + 384*a**2*c**2*d*f**6*exp(2*I*e) + 1536*I*a**2*c*d**2*f**7*x**2*e
xp(2*I*e) + 768*a**2*c*d**2*f**6*x*exp(2*I*e) - 192*I*a**2*c*d**2*f**5*exp(
2*I*e) + 512*I*a**2*d**3*f**7*x**3*exp(2*I*e) + 384*a**2*d**3*f**6*x**2*exp
(2*I*e) - 192*I*a**2*d**3*f**5*x*exp(2*I*e) - 48*a**2*d**3*f**4*exp(2*I*e))
*exp(-4*I*f*x) + (2048*I*a**2*c**3*f**7*exp(4*I*e) + 6144*I*a**2*c**2*d*f**
7*x*exp(4*I*e) + 3072*a**2*c**2*d*f**6*exp(4*I*e) + 6144*I*a**2*c*d**2*f**7
*x**2*exp(4*I*e) + 6144*a**2*c*d**2*f**6*x*exp(4*I*e) - 3072*I*a**2*c*d**2*
f**5*exp(4*I*e) + 2048*I*a**2*d**3*f**7*x**3*exp(4*I*e) + 3072*a**2*d**3*f*
*6*x**2*exp(4*I*e) - 3072*I*a**2*d**3*f**5*x*exp(4*I*e) - 1536*a**2*d**3*f*
*4*exp(4*I*e))*exp(-2*I*f*x))*exp(-6*I*e)/(8192*a**4*f**8), Ne(a**4*f**8*ex
p(6*I*e), 0)), (x**4*(2*d**3*exp(2*I*e) + d**3)*exp(-4*I*e)/(16*a**2) + x**
3*(2*c*d**2*exp(2*I*e) + c*d**2)*exp(-4*I*e)/(4*a**2) + x**2*(6*c**2*d*exp(
2*I*e) + 3*c**2*d)*exp(-4*I*e)/(8*a**2) + x*(2*c**3*exp(2*I*e) + c**3)*exp(
-4*I*e)/(4*a**2), True)) + c**3*x/(4*a**2) + 3*c**2*d*x**2/(8*a**2) + c*d**
2*x**3/(4*a**2) + d**3*x**4/(16*a**2)

Giac [A]
time = 0.59, size = 383, normalized size = 1.42

$$\frac{\left(32\,d^{3}f^{4}x^{4}e^{\left(4i\,fx+4i\,e\right)}+128\,cd^{2}f^{4}x^{3}e^{\left(4i\,fx+4i\,e\right)}+192\,c^{2}df^{4}x^{2}e^{\left(4i\,fx+4i\,e\right)}+128i\,d^{3}f^{3}x^{3}e^{\left(2i\,fx+2i\,e\right)}+32i\,d^{3}f^{3}x^{3}+128\,c^{3}f^{4}xe^{\left(4i\,fx+4i\,e\right)}+384i\,cd^{2}f^{3}x^{2}e^{\left(2i\,fx+2i\,e\right)}+96i\,cd^{2}f^{3}x^{2}+384i\,c^{2}df^{3}xe^{\left(2i\,fx+2i\,e\right)}+192\,d^{3}f^{2}x^{2}e^{\left(2i\,fx+2i\,e\right)}+96i\,c^{2}df^{3}x+24\,d^{3}f^{2}x^{2}+128i\,c^{3}f^{3}e^{\left(2i\,fx+2i\,e\right)}+384\,cd^{2}f^{2}xe^{\left(2i\,fx+2i\,e\right)}+32i\,c^{3}f^{3}+48\,cd^{2}f^{2}x+192\,c^{2}df^{2}e^{\left(2i\,fx+2i\,e\right)}-192i\,d^{3}fxe^{\left(2i\,fx+2i\,e\right)}+24\,c^{2}df^{2}-12i\,d^{3}fx-192i\,cd^{2}fe^{\left(2i\,fx+2i\,e\right)}-12i\,cd^{2}f-96\,d^{3}e^{\left(2i\,fx+2i\,e\right)}-3\,d^{3}\right)e^{\left(-4i\,fx-4i\,e\right)}}{512\,a^{2}f^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")

[Out] 1/512*(32*d^3*f^4*x^4*e^(4*I*f*x + 4*I*e) + 128*c*d^2*f^4*x^3*e^(4*I*f*x +
4*I*e) + 192*c^2*d*f^4*x^2*e^(4*I*f*x + 4*I*e) + 128*I*d^3*f^3*x^3*e^(2*I*f
*x + 2*I*e) + 32*I*d^3*f^3*x^3 + 128*c^3*f^4*x*e^(4*I*f*x + 4*I*e) + 384*I*
c*d^2*f^3*x^2*e^(2*I*f*x + 2*I*e) + 96*I*c*d^2*f^3*x^2 + 384*I*c^2*d*f^3*x*
e^(2*I*f*x + 2*I*e) + 192*d^3*f^2*x^2*e^(2*I*f*x + 2*I*e) + 96*I*c^2*d*f^3*
x + 24*d^3*f^2*x^2 + 128*I*c^3*f^3*e^(2*I*f*x + 2*I*e) + 384*c*d^2*f^2*x*e^
(2*I*f*x + 2*I*e) + 32*I*c^3*f^3 + 48*c*d^2*f^2*x + 192*c^2*d*f^2*e^(2*I*f*
x + 2*I*e) - 192*I*d^3*f*x*e^(2*I*f*x + 2*I*e) + 24*c^2*d*f^2 - 12*I*d^3*f*
x - 192*I*c*d^2*f*e^(2*I*f*x + 2*I*e) - 12*I*c*d^2*f - 96*d^3*e^(2*I*f*x +
2*I*e) - 3*d^3)*e^(-4*I*f*x - 4*I*e)/(a^2*f^4)

Mupad [B]
time = 3.23, size = 289, normalized size = 1.07

$$\mathrm{e}^{-e\,2\mathrm{i}-f\,x\,2\mathrm{i}}\,\left(\frac{\left(4\,c^3\,f^3-c^2\,d\,f^2\,6\mathrm{i}-6\,c\,d^2\,f+d^3\,3\mathrm{i}\right)\,1\mathrm{i}}{16\,a^2\,f^4}+\frac{d^3\,x^3\,1\mathrm{i}}{4\,a^2\,f}-\frac{d\,x\,\left(-2\,c^2\,f^2+c\,d\,f\,2\mathrm{i}+d^2\right)\,3\mathrm{i}}{8\,a^2\,f^3}-\frac{d^2\,x^2\,\left(-2\,c\,f+d\,1\mathrm{i}\right)\,3\mathrm{i}}{8\,a^2\,f^2}\right)+\mathrm{e}^{-e\,4\mathrm{i}-f\,x\,4\mathrm{i}}\,\left(\frac{\left(32\,c^3\,f^3-c^2\,d\,f^2\,24\mathrm{i}-12\,c\,d^2\,f+d^3\,3\mathrm{i}\right)\,1\mathrm{i}}{512\,a^2\,f^4}+\frac{d^3\,x^3\,1\mathrm{i}}{16\,a^2\,f}-\frac{d\,x\,\left(-8\,c^2\,f^2+c\,d\,f\,4\mathrm{i}+d^2\right)\,3\mathrm{i}}{128\,a^2\,f^3}-\frac{d^2\,x^2\,\left(-4\,c\,f+d\,1\mathrm{i}\right)\,3\mathrm{i}}{64\,a^2\,f^2}\right)+\frac{c^3\,x}{4\,a^2}+\frac{d^3\,x^4}{16\,a^2}+\frac{3\,c^2\,d\,x^2}{8\,a^2}+\frac{c\,d^2\,x^3}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^3/(a + a*tan(e + f*x)*1i)^2,x)

[Out] exp(- e*2i - f*x*2i)*(((d^3*3i + 4*c^3*f^3 - c^2*d*f^2*6i - 6*c*d^2*f)*1i)/
(16*a^2*f^4) + (d^3*x^3*1i)/(4*a^2*f) - (d*x*(d^2 - 2*c^2*f^2 + c*d*f*2i)*3

```
i)/(8*a^2*f^3) - (d^2*x^2*(d*1i - 2*c*f)*3i)/(8*a^2*f^2)) + exp(- e*4i - f*
x*4i)*(((d^3*3i + 32*c^3*f^3 - c^2*d*f^2*24i - 12*c*d^2*f)*1i)/(512*a^2*f^4
) + (d^3*x^3*1i)/(16*a^2*f) - (d*x*(d^2 - 8*c^2*f^2 + c*d*f*4i)*3i)/(128*a^
2*f^3) - (d^2*x^2*(d*1i - 4*c*f)*3i)/(64*a^2*f^2)) + (c^3*x)/(4*a^2) + (d^3
*x^4)/(16*a^2) + (3*c^2*d*x^2)/(8*a^2) + (c*d^2*x^3)/(4*a^2)
```

3.25 $\int \frac{(c+dx)^2}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=202

$$-\frac{id^2e^{-2ie-2ifx}}{8a^2f^3}-\frac{id^2e^{-4ie-4ifx}}{128a^2f^3}+\frac{de^{-2ie-2ifx}(c+dx)}{4a^2f^2}+\frac{de^{-4ie-4ifx}(c+dx)}{32a^2f^2}+\frac{ie^{-2ie-2ifx}(c+dx)^2}{4a^2f}+\frac{ie^{-4ie-4ifx}(}{16a^2}$$

```
[Out] -1/8*I*d^2*exp(-2*I*e-2*I*f*x)/a^2/f^3-1/128*I*d^2*exp(-4*I*e-4*I*f*x)/a^2/
f^3+1/4*d*exp(-2*I*e-2*I*f*x)*(d*x+c)/a^2/f^2+1/32*d*exp(-4*I*e-4*I*f*x)*(d
*x+c)/a^2/f^2+1/4*I*exp(-2*I*e-2*I*f*x)*(d*x+c)^2/a^2/f+1/16*I*exp(-4*I*e-4
*I*f*x)*(d*x+c)^2/a^2/f+1/12*(d*x+c)^3/a^2/d
```

Rubi [A]

time = 0.14, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3810, 2207, 2225}

$$\frac{d(c+dx)e^{-2ie-2ifx}}{4a^2f^2}+\frac{d(c+dx)e^{-4ie-4ifx}}{32a^2f^2}+\frac{i(c+dx)^2e^{-2ie-2ifx}}{4a^2f}+\frac{i(c+dx)^2e^{-4ie-4ifx}}{16a^2f}+\frac{(c+dx)^3}{12a^2d}-\frac{id^2e^{-2ie-2ifx}}{8a^2f^3}-\frac{id^2e^{-4ie-4ifx}}{128a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((-1/8*I)*d^2*E^((-2*I)*e - (2*I)*f*x))/(a^2*f^3) - ((I/128)*d^2*E^((-4*I)*
e - (4*I)*f*x))/(a^2*f^3) + (d*E^((-2*I)*e - (2*I)*f*x)*(c + d*x))/(4*a^2*f
^2) + (d*E^((-4*I)*e - (4*I)*f*x)*(c + d*x))/(32*a^2*f^2) + ((I/4)*E^((-2*I
)*e - (2*I)*f*x)*(c + d*x)^2)/(a^2*f) + ((I/16)*E^((-4*I)*e - (4*I)*f*x)*(c
 + d*x)^2)/(a^2*f) + (c + d*x)^3/(12*a^2*d)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
```

```
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+ia\tan(e+fx))^2}\,dx &= \int \left(\frac{(c+dx)^2}{4a^2}+\frac{e^{-2ie-2ifx}(c+dx)^2}{2a^2}+\frac{e^{-4ie-4ifx}(c+dx)^2}{4a^2}\right)dx\\ &= \frac{(c+dx)^3}{12a^2d}+\frac{\int e^{-4ie-4ifx}(c+dx)^2\,dx}{4a^2}+\frac{\int e^{-2ie-2ifx}(c+dx)^2\,dx}{2a^2}\\ &= \frac{ie^{-2ie-2ifx}(c+dx)^2}{4a^2f}+\frac{ie^{-4ie-4ifx}(c+dx)^2}{16a^2f}+\frac{(c+dx)^3}{12a^2d}-\frac{(id)\int e^{-4ie-4ifx}(c+}{8a^2f}\\ &= \frac{de^{-2ie-2ifx}(c+dx)}{4a^2f^2}+\frac{de^{-4ie-4ifx}(c+dx)}{32a^2f^2}+\frac{ie^{-2ie-2ifx}(c+dx)^2}{4a^2f}+\frac{ie^{-4ie-4ifx}}{16a}\\ &= -\frac{id^2e^{-2ie-2ifx}}{8a^2f^3}-\frac{id^2e^{-4ie-4ifx}}{128a^2f^3}+\frac{de^{-2ie-2ifx}(c+dx)}{4a^2f^2}+\frac{de^{-4ie-4ifx}(c+dx)}{32a^2f^2}+\end{aligned}$$

Mathematica [A]
time = 0.69, size = 282, normalized size = 1.40

$$\frac{\sec^2(e+fx)(\cos(fx)+i\sin(fx))^2\left((d+(1+i)cf+(1+i)dfx)((1+i)cf+d(-i+(1+i)fx))\cos(2fx)+\frac{1}{16}(d+(2+2i)cf+(2+2i)dfx)((2+2i)cf+d(-i+(2+2i)fx))\cos(4fx)(\cos(2e)-i\sin(2e))+\frac{2}{3}f^3x(3c^2+3cdx+d^2x^2)(\cos(2e)+i\sin(2e))-i(d+(1+i)cf+(1+i)dfx)((1+i)cf+d(-i+(1+i)fx))\sin(2fx)-\frac{1}{16}i(d+(2+2i)cf+(2+2i)dfx)((2+2i)cf+d(-i+(2+2i)fx))(\cos(2e)-i\sin(2e))\sin(4fx)\right)}{8f^3(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2*((d + (1 + I)*c*f + (1 + I)*d*f*x
)*((1 + I)*c*f + d*(-I + (1 + I)*f*x))*Cos[2*f*x] + ((d + (2 + 2*I)*c*f + (
2 + 2*I)*d*f*x)*((2 + 2*I)*c*f + d*(-I + (2 + 2*I)*f*x))*Cos[4*f*x]*(Cos[2*
e] - I*Sin[2*e]))/16 + (2*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2)*(Cos[2*e] + I*S
in[2*e]))/3 - I*(d + (1 + I)*c*f + (1 + I)*d*f*x)*((1 + I)*c*f + d*(-I + (1
 + I)*f*x))*Sin[2*f*x] - (I/16)*(d + (2 + 2*I)*c*f + (2 + 2*I)*d*f*x)*((2 +
 2*I)*c*f + d*(-I + (2 + 2*I)*f*x))*(Cos[2*e] - I*Sin[2*e])*Sin[4*f*x]))/(8
*f^3*(a + I*a*Tan[e + f*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(166) = 332$.
time = 0.49, size = 956, normalized size = 4.73

method	result
risch	$\frac{d^2x^3}{12a^2}+\frac{dc\,x^2}{4a^2}+\frac{c^2x}{4a^2}+\frac{c^3}{12a^2d}+\frac{i(2d^2x^2f^2+4cd\,f^2x-2id^2fx+2c^2f^2-2icdf-d^2)\mathrm{e}^{-2i(fx+e)}}{8f^3a^2}+\frac{i(8d^2x^2f^2+16cd\,f^2x-4id^2fx+}{128f}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/f*(1/2*I/f^2*d^2*e^2*cos(f*x+e)^4-2*I/f^2*d^2*(-1/4*(f*x+e)^2*cos(f*x
+e)^4+1/2*(f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8
*e)-3/32*(f*x+e)^2+1/128*(2*cos(f*x+e)^2+3)^2)-4*I/f*c*d*(-1/4*(f*x+e)*cos(
f*x+e)^4+1/16*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/32*f*x+3/32*e)+4*I
/f^2*d^2*e*(-1/4*(f*x+e)*cos(f*x+e)^4+1/16*(cos(f*x+e)^3+3/2*cos(f*x+e))*si
n(f*x+e)+3/32*f*x+3/32*e)-I/f*c*d*e*cos(f*x+e)^4+1/2*I*c^2*cos(f*x+e)^4+2*c
^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-4/f*c*d*e*(
1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+4/f*c*d*((f*x+e
)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)
^2+1/64*(2*cos(f*x+e)^2+3)^2)+2/f^2*d^2*e^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+
e))*sin(f*x+e)+3/8*f*x+3/8*e)-4/f^2*d^2*e*((f*x+e)*(1/4*(cos(f*x+e)^3+3/2*c
os(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*cos(f*x+e)^2+3)
^2)+2/f^2*d^2*((f*x+e)^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*
f*x+3/8*e)+1/8*(f*x+e)*cos(f*x+e)^4-1/32*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(
f*x+e)-15/64*f*x-15/64*e+3/8*(f*x+e)*cos(f*x+e)^2-3/16*cos(f*x+e)*sin(f*x+e
)-1/4*(f*x+e)^3)-c^2*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+2/f*c*d*e*(1
/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-2/f*c*d*((f*x+e)*(1/2*cos(f*x+e)*si
n(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)-1/f^2*d^2*e^2*(1/2*
cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+2/f^2*d^2*e*((f*x+e)*(1/2*cos(f*x+e)*s
in(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2-1/4*sin(f*x+e)^2)-1/f^2*d^2*((f*x+e)
^2*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+1/2*(f*x+e)*cos(f*x+e)^2-1/4*c
os(f*x+e)*sin(f*x+e)-1/4*f*x-1/4*e-1/3*(f*x+e)^3))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 166, normalized size = 0.82

$$\frac{\left(24i\,d^2f^2x^2+24i\,c^2f^2+12\,cdf-3i\,d^2-12\,(-4i\,cdf^2-d^2f)x+32\,(d^2f^3x^3+3\,cdf^3x^2+3\,c^2f^3x)e^{(4i\,fx+4i\,e)}-48\,(-2i\,d^2f^2x^2-2i\,c^2f^2-2\,cdf+i\,d^2+2\,(-2i\,cdf^2-d^2f)x)e^{(2i\,fx+2i\,e)}\right)e^{(-4i\,fx-4i\,e)}}{384\,a^2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(24*I*d^2*f^2*x^2 + 24*I*c^2*f^2 + 12*c*d*f - 3*I*d^2 - 12*(-4*I*c*d*
f^2 - d^2*f)*x + 32*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x)*e^(4*I*f*x
+ 4*I*e) - 48*(-2*I*d^2*f^2*x^2 - 2*I*c^2*f^2 - 2*c*d*f + I*d^2 + 2*(-2*I*c
*d*f^2 - d^2*f)*x)*e^(2*I*f*x + 2*I*e))*e^(-4*I*f*x - 4*I*e)/(a^2*f^3)
```

Sympy [A]
time = 0.28, size = 418, normalized size = 2.07

$$\begin{cases} \frac{\left(\left(64ia^{2}c^{2}f^{5}e^{2ie}+128ia^{2}cdf^{5}xe^{2ie}+32a^{2}cdf^{4}e^{2ie}+64ia^{2}d^{2}f^{5}x^{2}e^{2ie}+32a^{2}d^{2}f^{4}xe^{2ie}-8ia^{2}d^{2}f^{3}e^{2ie}\right)e^{-4ifx}+\left(256ia^{2}c^{2}f^{5}e^{4ie}+512ia^{2}cdf^{5}xe^{4ie}+256a^{2}cdf^{4}e^{4ie}+256ia^{2}d^{2}f^{5}x^{2}e^{4ie}+256a^{2}d^{2}f^{4}xe^{4ie}-128ia^{2}d^{2}f^{3}e^{4ie}\right)e^{-2ifx}\right)e^{-6ie}}{1024a^{4}f^{6}} & \text{for}\: a^{4}f^{6}e^{6ie}\neq 0 \\ \frac{x^{3}\cdot\left(2d^{2}e^{2ie}+d^{2}\right)e^{-4ie}}{12a^{2}}+\frac{x^{2}\cdot\left(2cde^{2ie}+cd\right)e^{-4ie}}{4a^{2}}+\frac{x\left(2c^{2}e^{2ie}+c^{2}\right)e^{-4ie}}{4a^{2}} & \text{otherwise} \end{cases}+\frac{c^{2}x}{4a^{2}}+\frac{cdx^{2}}{4a^{2}}+\frac{d^{2}x^{3}}{12a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((((64*I*a**2*c**2*f**5*exp(2*I*e) + 128*I*a**2*c*d*f**5*x*exp(2*I
*e) + 32*a**2*c*d*f**4*exp(2*I*e) + 64*I*a**2*d**2*f**5*x**2*exp(2*I*e) + 3
2*a**2*d**2*f**4*x*exp(2*I*e) - 8*I*a**2*d**2*f**3*exp(2*I*e))*exp(-4*I*f*x
) + (256*I*a**2*c**2*f**5*exp(4*I*e) + 512*I*a**2*c*d*f**5*x*exp(4*I*e) + 2
56*a**2*c*d*f**4*exp(4*I*e) + 256*I*a**2*d**2*f**5*x**2*exp(4*I*e) + 256*a*
*2*d**2*f**4*x*exp(4*I*e) - 128*I*a**2*d**2*f**3*exp(4*I*e))*exp(-2*I*f*x))
*exp(-6*I*e)/(1024*a**4*f**6), Ne(a**4*f**6*exp(6*I*e), 0)), (x**3*(2*d**2*
exp(2*I*e) + d**2)*exp(-4*I*e)/(12*a**2) + x**2*(2*c*d*exp(2*I*e) + c*d)*ex
p(-4*I*e)/(4*a**2) + x*(2*c**2*exp(2*I*e) + c**2)*exp(-4*I*e)/(4*a**2), Tru
e)) + c**2*x/(4*a**2) + c*d*x**2/(4*a**2) + d**2*x**3/(12*a**2)
```

Giac [A]
time = 0.59, size = 227, normalized size = 1.12

$$\frac{\left(32\,d^{2}f^{3}x^{3}e^{\left(4i\,fx+4i\,e\right)}+96\,cdf^{3}x^{2}e^{\left(4i\,fx+4i\,e\right)}+96\,c^{2}f^{3}xe^{\left(4i\,fx+4i\,e\right)}+96i\,d^{2}f^{2}x^{2}e^{\left(2i\,fx+2i\,e\right)}+24i\,d^{2}f^{2}x^{2}+192i\,cdf^{2}xe^{\left(2i\,fx+2i\,e\right)}+48i\,cdf^{2}x+96i\,c^{2}f^{2}e^{\left(2i\,fx+2i\,e\right)}+96\,d^{2}fxe^{\left(2i\,fx+2i\,e\right)}+24i\,c^{2}f^{2}+12\,d^{2}fx+96\,cdfe^{\left(2i\,fx+2i\,e\right)}+12\,cdf-48i\,d^{2}e^{\left(2i\,fx+2i\,e\right)}-3i\,d^{2}\right)e^{\left(-4i\,fx-4i\,e\right)}}{384\,a^{2}f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/384*(32*d^2*f^3*x^3*e^(4*I*f*x + 4*I*e) + 96*c*d*f^3*x^2*e^(4*I*f*x + 4*I
*e) + 96*c^2*f^3*x*e^(4*I*f*x + 4*I*e) + 96*I*d^2*f^2*x^2*e^(2*I*f*x + 2*I*
e) + 24*I*d^2*f^2*x^2 + 192*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) + 48*I*c*d*f^2*
x + 96*I*c^2*f^2*e^(2*I*f*x + 2*I*e) + 96*d^2*f*x*e^(2*I*f*x + 2*I*e) + 24*
I*c^2*f^2 + 12*d^2*f*x + 96*c*d*f*e^(2*I*f*x + 2*I*e) + 12*c*d*f - 48*I*d^2
*e^(2*I*f*x + 2*I*e) - 3*I*d^2)*e^(-4*I*f*x - 4*I*e)/(a^2*f^3)
```

Mupad [B]
time = 2.99, size = 183, normalized size = 0.91

$$\frac{c^2\,x}{4\,a^2}-{\mathrm{e}}^{-e\,4{}\mathrm{i}-f\,x\,4{}\mathrm{i}}\,\left(\frac{\left(-8\,c^2\,f^2+c\,d\,f\,4{}\mathrm{i}+d^2\right)\,1{}\mathrm{i}}{128\,a^2\,f^3}-\frac{d^2\,x^2\,1{}\mathrm{i}}{16\,a^2\,f}+\frac{d\,x\,\left(-4\,c\,f+d\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{32\,a^2\,f^2}\right)-{\mathrm{e}}^{-e\,2{}\mathrm{i}-f\,x\,2{}\mathrm{i}}\,\left(\frac{\left(-2\,c^2\,f^2+c\,d\,f\,2{}\mathrm{i}+d^2\right)\,1{}\mathrm{i}}{8\,a^2\,f^3}-\frac{d^2\,x^2\,1{}\mathrm{i}}{4\,a^2\,f}+\frac{d\,x\,\left(-2\,c\,f+d\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4\,a^2\,f^2}\right)+\frac{d^2\,x^3}{12\,a^2}+\frac{c\,d\,x^2}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] (c^2*x)/(4*a^2) - exp(- e*4i - f*x*4i)*(((d^2 - 8*c^2*f^2 + c*d*f*4i)*1i)/(
128*a^2*f^3) - (d^2*x^2*1i)/(16*a^2*f) + (d*x*(d*1i - 4*c*f)*1i)/(32*a^2*f^
2)) - exp(- e*2i - f*x*2i)*(((d^2 - 2*c^2*f^2 + c*d*f*2i)*1i)/(8*a^2*f^3) -
 (d^2*x^2*1i)/(4*a^2*f) + (d*x*(d*1i - 2*c*f)*1i)/(4*a^2*f^2)) + (d^2*x^3)/
(12*a^2) + (c*d*x^2)/(4*a^2)
```

3.26 $\int \frac{c+dx}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=151

$$-\frac{3idx}{16a^2f}-\frac{dx^2}{8a^2}+\frac{x(c+dx)}{4a^2}+\frac{d}{16f^2(a+ia\tan(e+fx))^2}+\frac{i(c+dx)}{4f(a+ia\tan(e+fx))^2}+\frac{3d}{16f^2\left(a^2+ia^2\tan(e+fx)\right)}$$

```
[Out] -3/16*I*d*x/a^2/f-1/8*d*x^2/a^2+1/4*x*(d*x+c)/a^2+1/16*d/f^2/(a+I*a*tan(f*x
+e))^2+1/4*I*(d*x+c)/f/(a+I*a*tan(f*x+e))^2+3/16*d/f^2/(a^2+I*a^2*tan(f*x+e
))+1/4*I*(d*x+c)/f/(a^2+I*a^2*tan(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3560, 8, 3811}

$$\frac{i(c+dx)}{4f\left(a^2+ia^2\tan(e+fx)\right)}+\frac{x(c+dx)}{4a^2}+\frac{3d}{16f^2\left(a^2+ia^2\tan(e+fx)\right)}-\frac{3idx}{16a^2f}-\frac{dx^2}{8a^2}+\frac{i(c+dx)}{4f(a+ia\tan(e+fx))^2}+\frac{d}{16f^2(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((-3*I)/16)*d*x)/(a^2*f) - (d*x^2)/(8*a^2) + (x*(c + d*x))/(4*a^2) + d/(16
*f^2*(a + I*a*Tan[e + f*x])^2) + ((I/4)*(c + d*x))/(f*(a + I*a*Tan[e + f*x]
)^2) + (3*d)/(16*f^2*(a^2 + I*a^2*Tan[e + f*x])) + ((I/4)*(c + d*x))/(f*(a^
2 + I*a^2*Tan[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3811

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)
^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x]] /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{(a+ia\tan(e+fx))^2}\,dx &= \frac{x(c+dx)}{4a^2}+\frac{i(c+dx)}{4f(a+ia\tan(e+fx))^2}+\frac{i(c+dx)}{4f\left(a^2+ia^2\tan(e+fx)\right)}-d\int \Bigg(\\
&= -\frac{dx^2}{8a^2}+\frac{x(c+dx)}{4a^2}+\frac{i(c+dx)}{4f(a+ia\tan(e+fx))^2}+\frac{i(c+dx)}{4f\left(a^2+ia^2\tan(e+fx)\right)} - \\
&= -\frac{dx^2}{8a^2}+\frac{x(c+dx)}{4a^2}+\frac{d}{16f^2(a+ia\tan(e+fx))^2}+\frac{i(c+dx)}{4f(a+ia\tan(e+fx))^2} \\
&= -\frac{idx}{8a^2f}-\frac{dx^2}{8a^2}+\frac{x(c+dx)}{4a^2}+\frac{d}{16f^2(a+ia\tan(e+fx))^2}+\frac{i(c+dx)}{4f(a+ia\tan(e} \\
&= -\frac{3idx}{16a^2f}-\frac{dx^2}{8a^2}+\frac{x(c+dx)}{4a^2}+\frac{d}{16f^2(a+ia\tan(e+fx))^2}+\frac{i(c+dx}{4f(a+ia\tan(e}
\end{aligned}
$$

Mathematica [A]
time = 0.54, size = 130, normalized size = 0.86

$$
-\frac{\sec^2(e+fx)\left(8(d+2icf+2idfx)+(4cf(i+4fx)+d(1+4ifx+8f^2x^2))\cos(2(e+fx))+(4cf(1+4ifx)+d(-i+4fx+8if^2x^2))\sin(2(e+fx))\right)}{64a^2f^2(-i+\tan(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] -1/64*(Sec[e + f*x]^2*(8*(d + (2*I)*c*f + (2*I)*d*f*x) + (4*c*f*(I + 4*f*x)
 + d*(1 + (4*I)*f*x + 8*f^2*x^2))*Cos[2*(e + f*x)] + (4*c*f*(1 + (4*I)*f*x)
 + d*(-I + 4*f*x + (8*I)*f^2*x^2))*Sin[2*(e + f*x)]))/(a^2*f^2*(-I + Tan[e
+ f*x])^2)
```

Maple [A]
time = 0.57, size = 82, normalized size = 0.54

method	result	size
risch	$\frac{d\,x^2}{8a^2}+\frac{cx}{4a^2}+\frac{i(2dxf+2cf-id)\mathrm{e}^{-2i(fx+e)}}{8f^2a^2}+\frac{i(4dxf+4cf-id)\mathrm{e}^{-4i(fx+e)}}{64f^2a^2}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d*x^2/a^2+1/4/a^2*c*x+1/8*I*(2*d*x*f+2*c*f-I*d)/f^2/a^2*exp(-2*I*(f*x+e
))+1/64*I*(4*d*x*f-I*d+4*c*f)/f^2/a^2*exp(-4*I*(f*x+e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.36, size = 82, normalized size = 0.54

$$\frac{\left(4i\,dfx+4i\,cf+8\left(df^2x^2+2\,cf^2x\right)e^{(4i\,fx+4i\,e)}-8\left(-2i\,dfx-2i\,cf-d\right)e^{(2i\,fx+2i\,e)}+d\right)e^{(-4i\,fx-4i\,e)}}{64\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/64*(4*I*d*f*x + 4*I*c*f + 8*(d*f^2*x^2 + 2*c*f^2*x)*e^(4*I*f*x + 4*I*e) -
 8*(-2*I*d*f*x - 2*I*c*f - d)*e^(2*I*f*x + 2*I*e) + d)*e^(-4*I*f*x - 4*I*e)
/(a^2*f^2)
```

Sympy [**A**]
time = 0.21, size = 226, normalized size = 1.50

$$\begin{cases}\frac{\left(\left(32ia^2cf^3e^{2ie}+32ia^2df^3xe^{2ie}+8a^2df^2e^{2ie}\right)e^{-4ifx}+\left(128ia^2cf^3e^{4ie}+128ia^2df^3xe^{4ie}+64a^2df^2e^{4ie}\right)e^{-2ifx}\right)e^{-6ie}}{512a^4f^4} & \text{for}\ a^4f^4e^{6ie}\neq 0\\ \frac{x^2\cdot\left(2de^{2ie}+d\right)e^{-4ie}}{8a^2}+\frac{x\left(2ce^{2ie}+c\right)e^{-4ie}}{4a^2} & \text{otherwise}\end{cases}+\frac{cx}{4a^2}+\frac{dx^2}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((((32*I*a**2*c*f**3*exp(2*I*e) + 32*I*a**2*d*f**3*x*exp(2*I*e) +
8*a**2*d*f**2*exp(2*I*e))*exp(-4*I*f*x) + (128*I*a**2*c*f**3*exp(4*I*e) + 1
28*I*a**2*d*f**3*x*exp(4*I*e) + 64*a**2*d*f**2*exp(4*I*e))*exp(-2*I*f*x))*e
xp(-6*I*e)/(512*a**4*f**4), Ne(a**4*f**4*exp(6*I*e), 0)), (x**2*(2*d*exp(2*
I*e) + d)*exp(-4*I*e)/(8*a**2) + x*(2*c*exp(2*I*e) + c)*exp(-4*I*e)/(4*a**2
), True)) + c*x/(4*a**2) + d*x**2/(8*a**2)
```

Giac [**A**]
time = 0.54, size = 107, normalized size = 0.71

$$\frac{\left(8\,df^2x^2e^{(4i\,fx+4i\,e)}+16\,cf^2xe^{(4i\,fx+4i\,e)}+16i\,dfxe^{(2i\,fx+2i\,e)}+4i\,dfx+16i\,cfe^{(2i\,fx+2i\,e)}+4i\,cf+8\,de^{(2i\,fx+2i\,e)}+d\right)e^{(-4i\,fx-4i\,e)}}{64\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*d*f^2*x^2*e^(4*I*f*x + 4*I*e) + 16*c*f^2*x*e^(4*I*f*x + 4*I*e) + 16
*I*d*f*x*e^(2*I*f*x + 2*I*e) + 4*I*d*f*x + 16*I*c*f*e^(2*I*f*x + 2*I*e) + 4
*I*c*f + 8*d*e^(2*I*f*x + 2*I*e) + d)*e^(-4*I*f*x - 4*I*e)/(a^2*f^2)
```

Mupad [B]
time = 2.77, size = 103, normalized size = 0.68

$$\frac{d\,x^2}{8\,a^2}-\mathrm{e}^{-e\,4\mathrm{i}-f\,x\,4\mathrm{i}}\left(\frac{(-4\,c\,f+d\,1\mathrm{i})\ 1\mathrm{i}}{64\,a^2\,f^2}-\frac{d\,x\,1\mathrm{i}}{16\,a^2\,f}\right)-\mathrm{e}^{-e\,2\mathrm{i}-f\,x\,2\mathrm{i}}\left(\frac{(-2\,c\,f+d\,1\mathrm{i})\ 1\mathrm{i}}{8\,a^2\,f^2}-\frac{d\,x\,1\mathrm{i}}{4\,a^2\,f}\right)+\frac{c\,x}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] (d*x^2)/(8*a^2) - exp(- e*4i - f*x*4i)*(((d*1i - 4*c*f)*1i)/(64*a^2*f^2) -
(d*x*1i)/(16*a^2*f)) - exp(- e*2i - f*x*2i)*(((d*1i - 2*c*f)*1i)/(8*a^2*f^2
) - (d*x*1i)/(4*a^2*f)) + (c*x)/(4*a^2)
```

3.27 $\int \frac{1}{(c+dx)(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=305

$$\frac{\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{2a^2d}+\frac{\cos\left(4e-\frac{4cf}{d}\right)\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)}{4a^2d}+\frac{\log(c+dx)}{4a^2d}-\frac{i\text{CosIntegral}\left(\right.}{}$$

```
[Out] 1/4*Ci(4*c*f/d+4*f*x)*cos(-4*e+4*c*f/d)/a^2/d+1/2*Ci(2*c*f/d+2*f*x)*cos(-2*
e+2*c*f/d)/a^2/d+1/4*ln(d*x+c)/a^2/d-1/2*I*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f
*x)/a^2/d-1/4*I*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a^2/d+1/4*I*Ci(4*c*f/d+
4*f*x)*sin(-4*e+4*c*f/d)/a^2/d+1/4*Si(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^2/
d+1/2*I*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^2/d+1/2*Si(2*c*f/d+2*f*x)*sin
(-2*e+2*c*f/d)/a^2/d
```

Rubi [A]
time = 0.55, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3809, 3384, 3380, 3383, 3393}

$$-\frac{i\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{2a^2d}-\frac{i\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4cf}{d}\right)}{4a^2d}+\frac{\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2a^2d}+\frac{\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\cos\left(4e-\frac{4cf}{d}\right)}{4a^2d}-\frac{\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2a^2d}-\frac{\sin\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{4a^2d}-\frac{i\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{2a^2d}-\frac{i\cos\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{4a^2d}+\frac{\log(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + (Cos[4*e
- (4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + Log[c + d*x]/(4*a^
2*d) - ((I/4)*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])/(a^2*d)
- ((I/2)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a^2*d) - ((I
/2)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^2*d) - (Sin[2*e
 - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) - ((I/4)*Cos[4*e -
(4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(a^2*d) - (Sin[4*e - (4*c*f)/d]*
SinIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(c+dx)(a+ia\tan(e+fx))^2}\,dx &= \int \left(\frac{1}{4a^2(c+dx)} + \frac{\cos(2e+2fx)}{2a^2(c+dx)} + \frac{\cos^2(2e+2fx)}{4a^2(c+dx)} - \frac{i\sin(2e+2f}{2a^2(c+dx}\right.\\
&= \frac{\log(c+dx)}{4a^2d} - \frac{i\int \frac{\sin(4e+4fx)}{c+dx}\,dx}{4a^2} - \frac{i\int \frac{\sin(2e+2fx)}{c+dx}\,dx}{2a^2} + \frac{\int \frac{\cos^2(2e+2fx)}{c+dx}\,d}{4a^2}\\
&= \frac{\log(c+dx)}{4a^2d} - \frac{\int \left(\frac{1}{2(c+dx)} - \frac{\cos(4e+4fx)}{2(c+dx)}\right)dx}{4a^2} + \frac{\int \left(\frac{1}{2(c+dx)} + \frac{\cos(4e+4fx)}{2(c+dx)}\right.}{4a^2}\\
&= \frac{\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} - \frac{i\mathrm{Ci}\left(\frac{4cf}{d}+4fx\right)\sin\left(4\right.}{4a^2d}\\
&= \frac{\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} - \frac{i\mathrm{Ci}\left(\frac{4cf}{d}+4fx\right)\sin\left(4\right.}{4a^2d}\\
&= \frac{\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} - \frac{i\mathrm{Ci}\left(\frac{4cf}{d}+4fx\right)\sin\left(4\right.}{4a^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.57, size = 211, normalized size = 0.69

$$
\frac{\left(\cos\left(2e-\frac{2cf}{d}\right)-i\sin\left(2e-\frac{2cf}{d}\right)\right)\left(2\mathrm{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)+\cos\left(2e-\frac{2cf}{d}\right)\log(f(c+dx))+\mathrm{CosIntegral}\left(\frac{4f(c+dx)}{d}\right)\left(\cos\left(2e-\frac{2cf}{d}\right)-i\sin\left(2e-\frac{2cf}{d}\right)\right)+i\log(f(c+dx))\sin\left(2e-\frac{2cf}{d}\right)-2i\mathrm{Si}\left(\frac{2f(c+dx)}{d}\right)-i\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Si}\left(\frac{4f(c+dx)}{d}\right)-\sin\left(2e-\frac{2cf}{d}\right)\mathrm{Si}\left(\frac{4f(c+dx)}{d}\right)\right)}{4a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] ((Cos[2*e - (2*c*f)/d] - I*Sin[2*e - (2*c*f)/d])*(2*CosIntegral[(2*f*(c + d
*x))/d] + Cos[2*e - (2*c*f)/d]*Log[f*(c + d*x)] + CosIntegral[(4*f*(c + d*x
))/d]*(Cos[2*e - (2*c*f)/d] - I*Sin[2*e - (2*c*f)/d]) + I*Log[f*(c + d*x)]*
Sin[2*e - (2*c*f)/d] - (2*I)*SinIntegral[(2*f*(c + d*x))/d] - I*Cos[2*e - (
2*c*f)/d]*SinIntegral[(4*f*(c + d*x))/d] - Sin[2*e - (2*c*f)/d]*SinIntegral
[(4*f*(c + d*x))/d]))/(4*a^2*d)
```

Maple [A]
time = 0.44, size = 377, normalized size = 1.24

method	result
risch	$\frac{\ln(dx+c)}{4a^2d} - \frac{e^{\frac{4i(cf-de)}{d}} \operatorname{expIntegral}\left(1,4ifx+4ie+\frac{4i(cf-de)}{d}\right)}{4a^2d} - \frac{e^{\frac{2i(cf-de)}{d}} \operatorname{expIntegral}\left(1,2ifx+2ie+\frac{2i(cf-de)}{d}\right)}{2a^2d}$
default	$-\frac{-\frac{if\left(\frac{2\operatorname{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\operatorname{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}-\frac{if\left(\frac{4\operatorname{sinIntegral}\left(4fx+4e+\frac{4cf-4}{d}\right.}{d}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/f*(-1/4*I*f*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci(
2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)-1/16*I*f*(4*Si(4*f*x+4*e+4*(
c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d-4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d
*e)/d)/d)+1/16*f*(4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d+4*Ci(4
*f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d)+1/4*f*(2*Si(2*f*x+2*e+2*(c*f-
d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/
d)/d)+1/4*f*ln(c*f-d*e+d*(f*x+e))/d)
```

Maxima [A]
time = 0.36, size = 208, normalized size = 0.68

$$-\frac{2\,f\cos\left(\frac{2\,(cf-de)}{d}\right)E_1\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)+f\cos\left(\frac{4\,(cf-de)}{d}\right)E_1\left(-\frac{4\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)+i\,fE_1\left(-\frac{4\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{4\,(cf-de)}{d}\right)+2i\,fE_1\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{2\,(cf-de)}{d}\right)-f\log\left((fx+e)d+cf-de\right)}{4\,a^2df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*f*cos(2*(c*f - d*e)/d)*exp_integral_e(1, -2*(-I*(f*x + e)*d - I*c*f
 + I*d*e)/d) + f*cos(4*(c*f - d*e)/d)*exp_integral_e(1, -4*(-I*(f*x + e)*d
- I*c*f + I*d*e)/d) + I*f*exp_integral_e(1, -4*(-I*(f*x + e)*d - I*c*f + I*
d*e)/d)*sin(4*(c*f - d*e)/d) + 2*I*f*exp_integral_e(1, -2*(-I*(f*x + e)*d -
 I*c*f + I*d*e)/d)*sin(2*(c*f - d*e)/d) - f*log((f*x + e)*d + c*f - d*e))/(
a^2*d*f)
```

Fricas [A]
time = 0.37, size = 86, normalized size = 0.28

$$\frac{2\,\mathrm{Ei}\left(-\frac{2\,(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{2\,(-i\,cf+i\,de)}{d}\right)}+\mathrm{Ei}\left(-\frac{4\,(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{4\,(-i\,cf+i\,de)}{d}\right)}+\log\left(\frac{dx+c}{d}\right)}{4\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*Ei(-2*(I*d*f*x + I*c*f)/d)*e^(-2*(-I*c*f + I*d*e)/d) + Ei(-4*(I*d*f*
x + I*c*f)/d)*e^(-4*(-I*c*f + I*d*e)/d) + log((d*x + c)/d))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{1}{c\tan^2(e+fx)-2ic\tan(e+fx)-c+dx\tan^2(e+fx)-2idx\tan(e+fx)-dx}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(1/(c*tan(e + f*x)**2 - 2*I*c*tan(e + f*x) - c + d*x*tan(e + f*x)*
*2 - 2*I*d*x*tan(e + f*x) - d*x), x)/a**2
```

Giac [A]
time = 0.55, size = 420, normalized size = 1.38

$$\frac{2\cos\left(\frac{2cf}{d}\right)\cos(2e)\,\mathrm{Ci}\left(-\frac{2(dfx+cf)}{d}\right)+\cos(2e)^2\log(dx+c)+2i\cos(2e)\,\mathrm{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\sin\left(\frac{2cf}{d}\right)+2i\cos\left(\frac{2cf}{d}\right)\mathrm{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\sin(2e)+2i\cos(2e)\log(dx+c)\sin(2e)-2\,\mathrm{Ci}\left(-\frac{2(dfx+cf)}{d}\right)\sin\left(\frac{2cf}{d}\right)\sin(2e)-\log(dx+c)\sin(2e)^2-2i\cos\left(\frac{2cf}{d}\right)\cos(2e)\,\mathrm{Si}\left(\frac{2(dfx+cf)}{d}\right)+2\cos(2e)\sin\left(\frac{2cf}{d}\right)\mathrm{Si}\left(\frac{2(dfx+cf)}{d}\right)+2\cos\left(\frac{2cf}{d}\right)\sin(2e)\,\mathrm{Si}\left(\frac{2(dfx+cf)}{d}\right)+2i\sin\left(\frac{2cf}{d}\right)\sin(2e)\,\mathrm{Si}\left(\frac{2(dfx+cf)}{d}\right)+\cos\left(\frac{4cf}{d}\right)\mathrm{Ci}\left(-\frac{4(dfx+cf)}{d}\right)+i\,\mathrm{Ci}\left(-\frac{4(dfx+cf)}{d}\right)\sin\left(\frac{4cf}{d}\right)-i\cos\left(\frac{4cf}{d}\right)\mathrm{Si}\left(\frac{4(dfx+cf)}{d}\right)+\sin\left(\frac{4cf}{d}\right)\mathrm{Si}\left(\frac{4(dfx+cf)}{d}\right)}{4\left(a^2d\cos(2e)^2+2i\,a^2d\cos(2e)\sin(2e)-a^2d\sin(2e)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*cos(2*c*f/d)*cos(2*e)*cos_integral(-2*(d*f*x + c*f)/d) + cos(2*e)^2*
log(d*x + c) + 2*I*cos(2*e)*cos_integral(-2*(d*f*x + c*f)/d)*sin(2*c*f/d) +
 2*I*cos(2*c*f/d)*cos_integral(-2*(d*f*x + c*f)/d)*sin(2*e) + 2*I*cos(2*e)*
log(d*x + c)*sin(2*e) - 2*cos_integral(-2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin
(2*e) - log(d*x + c)*sin(2*e)^2 - 2*I*cos(2*c*f/d)*cos(2*e)*sin_integral(2*
(d*f*x + c*f)/d) + 2*cos(2*e)*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d)
+ 2*cos(2*c*f/d)*sin(2*e)*sin_integral(2*(d*f*x + c*f)/d) + 2*I*sin(2*c*f/d
)*sin(2*e)*sin_integral(2*(d*f*x + c*f)/d) + cos(4*c*f/d)*cos_integral(-4*(
d*f*x + c*f)/d) + I*cos_integral(-4*(d*f*x + c*f)/d)*sin(4*c*f/d) - I*cos(4
*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + sin(4*c*f/d)*sin_integral(4*(d*f*
x + c*f)/d))/(a^2*d*cos(2*e)^2 + 2*I*a^2*d*cos(2*e)*sin(2*e) - a^2*d*sin(2*
e)^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c + d*x)),x)
[Out] int(1/((a + a*tan(e + f*x)*1i)^2*(c + d*x)), x)
```

3.28 $\int \frac{1}{(c+dx)^2(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=436

$$-\frac{1}{4a^2d(c+dx)}-\frac{\cos(2e+2fx)}{2a^2d(c+dx)}-\frac{\cos^2(2e+2fx)}{4a^2d(c+dx)}-\frac{if\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2}-\frac{if\cos\left(4e-\right.}{}$$

```
[Out] -1/4/a^2/d/(d*x+c)-I*f*Ci(4*c*f/d+4*f*x)*cos(-4*e+4*c*f/d)/a^2/d^2-I*f*Ci(2
*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^2/d^2-1/2*cos(2*f*x+2*e)/a^2/d/(d*x+c)-1/
4*cos(2*f*x+2*e)^2/a^2/d/(d*x+c)-f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^2/
d^2-f*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a^2/d^2+f*Ci(4*c*f/d+4*f*x)*sin(-
4*e+4*c*f/d)/a^2/d^2-I*f*Si(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^2/d^2+f*Ci(2
*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^2/d^2-I*f*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*
f/d)/a^2/d^2+1/2*I*sin(2*f*x+2*e)/a^2/d/(d*x+c)+1/4*sin(2*f*x+2*e)^2/a^2/d/
(d*x+c)+1/4*I*sin(4*f*x+4*e)/a^2/d/(d*x+c)
```

Rubi [A]

time = 0.51, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3809, 3378, 3384, 3380, 3383, 3394, 12}

$$-\frac{f\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4cf}{d}\right)}{a^2d^2}-\frac{f\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{if\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{if\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\cos\left(4e-\frac{4cf}{d}\right)}{a^2d^2}+\frac{if\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{a^2d^2}+\frac{if\sin\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{a^2d^2}-\frac{f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\cos\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{a^2d^2}+\frac{\sin^2(2e+2fx)}{4a^2d(c+dx)}+\frac{i\sin(2e+2fx)}{2a^2d(c+dx)}+\frac{i\sin(4e+4fx)}{4a^2d(c+dx)}-\frac{\cos^2(2e+2fx)}{4a^2d(c+dx)}-\frac{\cos(2e+2fx)}{2a^2d(c+dx)}-\frac{1}{4a^2d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] -1/4*1/(a^2*d*(c + d*x)) - Cos[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Cos[2*e +
 2*f*x]^2/(4*a^2*d*(c + d*x)) - (I*f*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*
f)/d + 2*f*x])/(a^2*d^2) - (I*f*Cos[4*e - (4*c*f)/d]*CosIntegral[(4*c*f)/d
+ 4*f*x])/(a^2*d^2) - (f*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d
])/(a^2*d^2) - (f*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a^2
*d^2) + ((I/2)*Sin[2*e + 2*f*x])/(a^2*d*(c + d*x)) + Sin[2*e + 2*f*x]^2/(4*
a^2*d*(c + d*x)) + ((I/4)*Sin[4*e + 4*f*x])/(a^2*d*(c + d*x)) - (f*Cos[2*e
- (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (I*f*Sin[2*e - (2*
c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*Cos[4*e - (4*c*f)/d]
*SinIntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2) + (I*f*Sin[4*e - (4*c*f)/d]*SinI
ntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia\tan(e+fx))^2}\,dx = \int \left(\frac{1}{4a^2(c+dx)^2} + \frac{\cos(2e+2fx)}{2a^2(c+dx)^2} + \frac{\cos^2(2e+2fx)}{4a^2(c+dx)^2} - \frac{i\sin(2e+2}{2a^2(c+d}\right.$$

$$= -\frac{1}{4a^2d(c+dx)} - \frac{i\int \frac{\sin(4e+4fx)}{(c+dx)^2}\,dx}{4a^2} - \frac{i\int \frac{\sin(2e+2fx)}{(c+dx)^2}\,dx}{2a^2} + \frac{\int \frac{\cos^2(2e+2}{(c+dx)^2}}{4a^2}$$

$$= -\frac{1}{4a^2d(c+dx)} - \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} + \frac{i\sin(2e+2f}{2a^2d(c+dx}$$

$$= -\frac{1}{4a^2d(c+dx)} - \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} + \frac{i\sin(2e+2f}{2a^2d(c+dx}$$

$$= -\frac{1}{4a^2d(c+dx)} - \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} - \frac{if\cos\left(2e-\right.}{}$$

$$= -\frac{1}{4a^2d(c+dx)} - \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} - \frac{if\cos\left(2e-\right.}{}$$

Mathematica [A]
time = 1.73, size = 467, normalized size = 1.07

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] -1/4*((Cos[2*(e + f*(-(c/d) + x))] - I*Sin[2*(e + f*(-(c/d) + x))])*(2*d*Co
s[(2*c*f)/d] + d*Cos[2*(e + f*(-(c/d) + x))] + d*Cos[2*(e + f*(c/d + x))] -
 (2*I)*d*Sin[(2*c*f)/d] + (4*I)*f*(c + d*x)*CosIntegral[(2*f*(c + d*x))/d]*
(Cos[2*f*x] + I*Sin[2*f*x]) + I*d*Sin[2*(e + f*(-(c/d) + x))] - I*d*Sin[2*(
e + f*(c/d + x))] + 4*f*(c + d*x)*CosIntegral[(4*f*(c + d*x))/d]*(I*Cos[2*e
 - (2*f*(c + d*x))/d] + Sin[2*e - (2*f*(c + d*x))/d]) + 4*c*f*Cos[2*f*x]*Si
nIntegral[(2*f*(c + d*x))/d] + 4*d*f*x*Cos[2*f*x]*SinIntegral[(2*f*(c + d*x
))/d] + (4*I)*c*f*Sin[2*f*x]*SinIntegral[(2*f*(c + d*x))/d] + (4*I)*d*f*x*S
in[2*f*x]*SinIntegral[(2*f*(c + d*x))/d] + 4*c*f*Cos[2*e - (2*f*(c + d*x))/
d]*SinIntegral[(4*f*(c + d*x))/d] + 4*d*f*x*Cos[2*e - (2*f*(c + d*x))/d]*Si
nIntegral[(4*f*(c + d*x))/d] - (4*I)*c*f*Sin[2*e - (2*f*(c + d*x))/d]*SinIn
tegral[(4*f*(c + d*x))/d] - (4*I)*d*f*x*Sin[2*e - (2*f*(c + d*x))/d]*SinInt
egral[(4*f*(c + d*x))/d]))/(a^2*d^2*(c + d*x))
```

Maple [A]
time = 0.48, size = 536, normalized size = 1.23

method	result
risch	$-\frac{1}{4a^2d(dx+c)}-\frac{f\,\mathrm{e}^{-4i(fx+e)}}{4a^2(dxf+cf)d}+\frac{if\,\mathrm{e}^{\frac{4i(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,4ifx+4ie+\frac{4i(cf-de)}{d}\right)}{a^2d^2}-\frac{f\,\mathrm{e}^{-2i(fx+e)}}{2a^2(dxf+cf)d}+\frac{if\,\mathrm{e}^{\frac{2i(cf-de)}{d}}\,\mathrm{expIn}}{}$
default	$-\frac{if^2\left(-\frac{2\sin(2fx+2e)}{(cf-de+d(fx+e))d}+\frac{\frac{4\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{4\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}}{d}\right)}{4}-\frac{if^2\left(-\frac{}{(}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)^2/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 1/a^2/f*(-1/4*I*f^2*(-2*sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d+2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d)/d)-1/16*I*f^2*(-4*sin(4*f*x+4*e)/(c*f-d*e+d*(f*x+e))/d+4*(4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d+4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d)/d)+1/16*f^2*(-4*cos(4*f*x+4*e)/(c*f-d*e+d*(f*x+e))/d-4*(4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d-4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d)/d)+1/4*f^2*(-2*cos(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d)-1/4*f^2/(c*f-d*e+d*(f*x+e))/d)

Maxima [A]
time = 0.44, size = 225, normalized size = 0.52

$$-\frac{2\,f^2\cos\left(\frac{2\,(cf-de)}{d}\right)E_2\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)+f^2\cos\left(\frac{4\,(cf-de)}{d}\right)E_2\left(-\frac{4\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)+i\,f^2E_2\left(-\frac{4\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{4\,(cf-de)}{d}\right)+2i\,f^2E_2\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{2\,(cf-de)}{d}\right)+f^2}{4\,((fx+e)a^2d^2+a^2cdf-a^2d^2e)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")

[Out] -1/4*(2*f^2*cos(2*(c*f - d*e)/d)*exp_integral_e(2, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d) + f^2*cos(4*(c*f - d*e)/d)*exp_integral_e(2, -4*(-I*(f*x + e)*d - I*c*f + I*d*e)/d) + I*f^2*exp_integral_e(2, -4*(-I*(f*x + e)*d - I*c*f + I*d*e)/d)*sin(4*(c*f - d*e)/d) + 2*I*f^2*exp_integral_e(2, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d)*sin(2*(c*f - d*e)/d) + f^2)/(((f*x + e)*a^2*d^2 + a^2*c*d*f - a^2*d^2*e)*f)

Fricas [A]
time = 0.38, size = 147, normalized size = 0.34

$$-\frac{\left(\left(4\,(i\,dfx+i\,cf)\mathrm{Ei}\left(-\frac{2\,(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{2\,(-i\,cf+i\,de)}{d}\right)}+4\,(i\,dfx+i\,cf)\mathrm{Ei}\left(-\frac{4\,(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{4\,(-i\,cf+i\,de)}{d}\right)}+d\right)e^{(4i\,fx+4i\,e)}+2\,de^{(2i\,fx+2i\,e)}+d\right)e^{(-4i\,fx-4i\,e)}}{4\,(a^2d^3x+a^2cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/4*((4*(I*d*f*x + I*c*f)*Ei(-2*(I*d*f*x + I*c*f)/d)*e^(-2*(-I*c*f + I*d*e
)/d) + 4*(I*d*f*x + I*c*f)*Ei(-4*(I*d*f*x + I*c*f)/d)*e^(-4*(-I*c*f + I*d*e
)/d) + d)*e^(4*I*f*x + 4*I*e) + 2*d*e^(2*I*f*x + 2*I*e) + d)*e^(-4*I*f*x -
4*I*e)/(a^2*d^3*x + a^2*c*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{c^2\tan^2(e+fx)-2ic^2\tan(e+fx)-c^2+2cdx\tan^2(e+fx)-4icdx\tan(e+fx)-2cdx+d^2x^2\tan^2(e+fx)-2id^2x^2\tan(e+fx)-d^2x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(1/(c**2*tan(e + f*x)**2 - 2*I*c**2*tan(e + f*x) - c**2 + 2*c*d*x*
tan(e + f*x)**2 - 4*I*c*d*x*tan(e + f*x) - 2*c*d*x + d**2*x**2*tan(e + f*x)
**2 - 2*I*d**2*x**2*tan(e + f*x) - d**2*x**2), x)/a**2
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2135 vs. 2(423) = 846.
time = 13.09, size = 2135, normalized size = 4.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*I*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos(2*(c*f - d*
e)/d)*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f
+ d*e)/d) - 4*I*c*f^3*cos(2*(c*f - d*e)/d)*cos_integral(-2*((d*x + c)*(c*f/
(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 4*I*(d*x + c)*(c*f/(d*x +
c) - f - d*e/(d*x + c))*f^2*cos(4*(c*f - d*e)/d)*cos_integral(-4*((d*x + c)
*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - 4*I*c*f^3*cos(4*(c*f
 - d*e)/d)*cos_integral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) -
 c*f + d*e)/d) + 4*I*d*f^2*cos(2*(c*f - d*e)/d)*cos_integral(-2*((d*x + c)*
(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e + 4*I*d*f^2*cos(4*(c*
f - d*e)/d)*cos_integral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))
- c*f + d*e)/d)*e - 4*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos
_integral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)
*sin(4*(c*f - d*e)/d) + 4*c*f^3*cos_integral(-4*((d*x + c)*(c*f/(d*x + c) -
 f - d*e/(d*x + c)) - c*f + d*e)/d)*sin(4*(c*f - d*e)/d) - 4*d*f^2*cos_inte
gral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e*si
n(4*(c*f - d*e)/d) - 4*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*co
s_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d
```

```
)*sin(2*(c*f - d*e)/d) + 4*c*f^3*cos_integral(-2*((d*x + c)*(c*f/(d*x + c)
- f - d*e/(d*x + c)) - c*f + d*e)/d)*sin(2*(c*f - d*e)/d) - 4*d*f^2*cos_int
egral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e*s
in(2*(c*f - d*e)/d) + 4*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*c
os(2*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*
x + c)) - c*f + d*e)/d) - 4*c*f^3*cos(2*(c*f - d*e)/d)*sin_integral(-2*((d*
x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 4*d*f^2*cos(2*
(c*f - d*e)/d)*e*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x +
 c)) - c*f + d*e)/d) + 4*I*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^
2*sin(2*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/
(d*x + c)) - c*f + d*e)/d) - 4*I*c*f^3*sin(2*(c*f - d*e)/d)*sin_integral(-2
*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 4*I*d*f^2
*e*sin(2*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e
/(d*x + c)) - c*f + d*e)/d) + 4*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c
))*f^2*cos(4*(c*f - d*e)/d)*sin_integral(-4*((d*x + c)*(c*f/(d*x + c) - f -
 d*e/(d*x + c)) - c*f + d*e)/d) - 4*c*f^3*cos(4*(c*f - d*e)/d)*sin_integral
(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 4*d*f^
2*cos(4*(c*f - d*e)/d)*e*sin_integral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*
e/(d*x + c)) - c*f + d*e)/d) + 4*I*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x
+ c))*f^2*sin(4*(c*f - d*e)/d)*sin_integral(-4*((d*x + c)*(c*f/(d*x + c) -
f - d*e/(d*x + c)) - c*f + d*e)/d) - 4*I*c*f^3*sin(4*(c*f - d*e)/d)*sin_int
egral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 4
*I*d*f^2*e*sin(4*(c*f - d*e)/d)*sin_integral(-4*((d*x + c)*(c*f/(d*x + c) -
 f - d*e/(d*x + c)) - c*f + d*e)/d) - d*f^2*cos(4*(d*x + c)*(c*f/(d*x + c)
- f - d*e/(d*x + c))/d) - 2*d*f^2*cos(2*(d*x + c)*(c*f/(d*x + c) - f - d*e/
(d*x + c))/d) - I*d*f^2*sin(4*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))
/d) - 2*I*d*f^2*sin(2*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d) - d*
f^2)*d^2/((((d*x + c)*a^2*d^4*(c*f/(d*x + c) - f - d*e/(d*x + c)) - a^2*c*d^
4*f + a^2*d^5*e)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + a \tan(e + f x)\ 1\mathrm{i})^2 (c + d x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^2*(c + d*x)^2),x)
[Out] int(1/((a + a*tan(e + f*x)*1i)^2*(c + d*x)^2), x)
```

3.29 $\int \frac{(c+dx)^3}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=396

$$-\frac{9d^3e^{-2ie-2ifx}}{64a^3f^4}-\frac{9d^3e^{-4ie-4ifx}}{1024a^3f^4}-\frac{d^3e^{-6ie-6ifx}}{1728a^3f^4}-\frac{9id^2e^{-2ie-2ifx}(c+dx)}{32a^3f^3}-\frac{9id^2e^{-4ie-4ifx}(c+dx)}{256a^3f^3}-\frac{id^2e^{-6ie-6ifx}}{288a^3}$$

```
[Out] -9/64*d^3*exp(-2*I*e-2*I*f*x)/a^3/f^4-9/1024*d^3*exp(-4*I*e-4*I*f*x)/a^3/f^
4-1/1728*d^3*exp(-6*I*e-6*I*f*x)/a^3/f^4-9/32*I*d^2*exp(-2*I*e-2*I*f*x)*(d*
x+c)/a^3/f^3-9/256*I*d^2*exp(-4*I*e-4*I*f*x)*(d*x+c)/a^3/f^3-1/288*I*d^2*ex
p(-6*I*e-6*I*f*x)*(d*x+c)/a^3/f^3+9/32*d*exp(-2*I*e-2*I*f*x)*(d*x+c)^2/a^3/
f^2+9/128*d*exp(-4*I*e-4*I*f*x)*(d*x+c)^2/a^3/f^2+1/96*d*exp(-6*I*e-6*I*f*x
)*(d*x+c)^2/a^3/f^2+3/16*I*exp(-2*I*e-2*I*f*x)*(d*x+c)^3/a^3/f+3/32*I*exp(-
4*I*e-4*I*f*x)*(d*x+c)^3/a^3/f+1/48*I*exp(-6*I*e-6*I*f*x)*(d*x+c)^3/a^3/f+1
/32*(d*x+c)^4/a^3/d
```

Rubi [A]
time = 0.27, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3810, 2207, 2225}

$$-\frac{9id^2(c+dx)e^{-2ie-2ifx}}{32a^3f^3}-\frac{9id^2(c+dx)e^{-4ie-4ifx}}{256a^3f^3}-\frac{id^2(c+dx)e^{-6ie-6ifx}}{288a^3f^3}+\frac{9d(c+dx)^2e^{-2ie-2ifx}}{32a^3f^2}+\frac{9d(c+dx)^2e^{-4ie-4ifx}}{128a^3f^2}+\frac{d(c+dx)^2e^{-6ie-6ifx}}{96a^3f^2}+\frac{3i(c+dx)^3e^{-2ie-2ifx}}{16a^3f}+\frac{3i(c+dx)^3e^{-4ie-4ifx}}{32a^3f}+\frac{i(c+dx)^3e^{-6ie-6ifx}}{48a^3f}+\frac{(c+dx)^4}{32a^3d}-\frac{9d^3e^{-2ie-2ifx}}{64a^3f^4}-\frac{9d^3e^{-4ie-4ifx}}{1024a^3f^4}-\frac{d^3e^{-6ie-6ifx}}{1728a^3f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (-9*d^3*E^((-2*I)*e - (2*I)*f*x))/(64*a^3*f^4) - (9*d^3*E^((-4*I)*e - (4*I)
*f*x))/(1024*a^3*f^4) - (d^3*E^((-6*I)*e - (6*I)*f*x))/(1728*a^3*f^4) - (((
9*I)/32)*d^2*E^((-2*I)*e - (2*I)*f*x)*(c + d*x))/(a^3*f^3) - (((9*I)/256)*d
^2*E^((-4*I)*e - (4*I)*f*x)*(c + d*x))/(a^3*f^3) - ((I/288)*d^2*E^((-6*I)*e
 - (6*I)*f*x)*(c + d*x))/(a^3*f^3) + (9*d*E^((-2*I)*e - (2*I)*f*x)*(c + d*x
)^2)/(32*a^3*f^2) + (9*d*E^((-4*I)*e - (4*I)*f*x)*(c + d*x)^2)/(128*a^3*f^2
) + (d*E^((-6*I)*e - (6*I)*f*x)*(c + d*x)^2)/(96*a^3*f^2) + (((3*I)/16)*E^(
(-2*I)*e - (2*I)*f*x)*(c + d*x)^3)/(a^3*f) + (((3*I)/32)*E^((-4*I)*e - (4*I
)*f*x)*(c + d*x)^3)/(a^3*f) + ((I/48)*E^((-6*I)*e - (6*I)*f*x)*(c + d*x)^3)
/(a^3*f) + (c + d*x)^4/(32*a^3*d)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{(a+ia\tan(e+fx))^3}\,dx &= \int \left(\frac{(c+dx)^3}{8a^3}+\frac{3e^{-2ie-2ifx}(c+dx)^3}{8a^3}+\frac{3e^{-4ie-4ifx}(c+dx)^3}{8a^3}+\frac{e^{-6ie-6ifx}(c+}{8a^3}\right.\\
&= \frac{(c+dx)^4}{32a^3d}+\frac{\int e^{-6ie-6ifx}(c+dx)^3\,dx}{8a^3}+\frac{3\int e^{-2ie-2ifx}(c+dx)^3\,dx}{8a^3}+\frac{3\int e^{-4ie-}}{}\\
&= \frac{3ie^{-2ie-2ifx}(c+dx)^3}{16a^3f}+\frac{3ie^{-4ie-4ifx}(c+dx)^3}{32a^3f}+\frac{ie^{-6ie-6ifx}(c+dx)^3}{48a^3f}+\frac{(c+d}{32a^3}\\
&= \frac{9de^{-2ie-2ifx}(c+dx)^2}{32a^3f^2}+\frac{9de^{-4ie-4ifx}(c+dx)^2}{128a^3f^2}+\frac{de^{-6ie-6ifx}(c+dx)^2}{96a^3f^2}+\frac{3ie^{-2}}{}\\
&= -\frac{9id^2e^{-2ie-2ifx}(c+dx)}{32a^3f^3}-\frac{9id^2e^{-4ie-4ifx}(c+dx)}{256a^3f^3}-\frac{id^2e^{-6ie-6ifx}(c+dx)}{288a^3f^3}+\frac{9}{}\\
&= -\frac{9d^3e^{-2ie-2ifx}}{64a^3f^4}-\frac{9d^3e^{-4ie-4ifx}}{1024a^3f^4}-\frac{d^3e^{-6ie-6ifx}}{1728a^3f^4}-\frac{9id^2e^{-2ie-2ifx}(c+dx)}{32a^3f^3}-\frac{9id}{}
\end{aligned}$$

Mathematica [A]
time = 2.43, size = 667, normalized size = 1.68

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I/27648)*Sec[e + f*x]^3*(243*((32*I)*c^3*f^3 + 8*c^2*d*f^2*(5 + (12*I)*f*
x) + 4*c*d^2*f*(-9*I + 20*f*x + (24*I)*f^2*x^2) + d^3*(-17 - (36*I)*f*x + 4
0*f^2*x^2 + (32*I)*f^3*x^3))*Cos[e + f*x] + 16*(36*c^3*f^3*(I + 6*f*x) + 18
*c^2*d*f^2*(1 + (6*I)*f*x + 18*f^2*x^2) + 6*c*d^2*f*(-I + 6*f*x + (18*I)*f^
2*x^2 + 36*f^3*x^3) + d^3*(-1 - (6*I)*f*x + 18*f^2*x^2 + (36*I)*f^3*x^3 + 5
4*f^4*x^4))*Cos[3*(e + f*x)] - (3645*I)*d^3*Sin[e + f*x] + 6804*c*d^2*f*Sin
```

```
[e + f*x] + (5832*I)*c^2*d*f^2*Sin[e + f*x] - 2592*c^3*f^3*Sin[e + f*x] + 6
804*d^3*f*x*Sin[e + f*x] + (11664*I)*c*d^2*f^2*x*Sin[e + f*x] - 7776*c^2*d*
f^3*x*Sin[e + f*x] + (5832*I)*d^3*f^2*x^2*Sin[e + f*x] - 7776*c*d^2*f^3*x^2
*Sin[e + f*x] - 2592*d^3*f^3*x^3*Sin[e + f*x] + (16*I)*d^3*Sin[3*(e + f*x)]
 - 96*c*d^2*f*Sin[3*(e + f*x)] - (288*I)*c^2*d*f^2*Sin[3*(e + f*x)] + 576*c
^3*f^3*Sin[3*(e + f*x)] - 96*d^3*f*x*Sin[3*(e + f*x)] - (576*I)*c*d^2*f^2*x
*Sin[3*(e + f*x)] + 1728*c^2*d*f^3*x*Sin[3*(e + f*x)] + (3456*I)*c^3*f^4*x*
Sin[3*(e + f*x)] - (288*I)*d^3*f^2*x^2*Sin[3*(e + f*x)] + 1728*c*d^2*f^3*x^
2*Sin[3*(e + f*x)] + (5184*I)*c^2*d*f^4*x^2*Sin[3*(e + f*x)] + 576*d^3*f^3*
x^3*Sin[3*(e + f*x)] + (3456*I)*c*d^2*f^4*x^3*Sin[3*(e + f*x)] + (864*I)*d^
3*f^4*x^4*Sin[3*(e + f*x)]))/(a^3*f^4*(-I + Tan[e + f*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3395 vs. $2(328) = 656$.
time = 0.62, size = 3396, normalized size = 8.58

method	result
risch	$\frac{d^3 x^4}{32a^3}+\frac{d^2 c x^3}{8a^3}+\frac{3d c^2 x^2}{16a^3}+\frac{c^3 x}{8a^3}+\frac{c^4}{32a^3 d}+\frac{3i(4d^3 x^3 f^3+12c d^2 f^3 x^2-6i d^3 f^2 x^2+12c^2 d f^3 x-12i c d^2 f^2 x+4c^3 f^3-6i c^2 d f^2-}{64a^3 f^4}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/f*(3*I/f*c^2*d*(-1/4*(f*x+e)*cos(f*x+e)^4+1/16*(cos(f*x+e)^3+3/2*cos(
f*x+e))*sin(f*x+e)+3/32*f*x+3/32*e)+3*I/f^2*c*d^2*(-1/4*(f*x+e)^2*cos(f*x+e
)^4+1/2*(f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e
)-3/32*(f*x+e)^2+1/128*(2*cos(f*x+e)^2+3)^2)-6*I/f^2*c*d^2*e*(-1/4*(f*x+e)*
cos(f*x+e)^4+1/16*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/32*f*x+3/32*e)
+24*I/f^2*c*d^2*e*(-1/6*(f*x+e)*cos(f*x+e)^6+1/36*(cos(f*x+e)^5+5/4*cos(f*x
+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/96*f*x+5/96*e)-3/4*I/f^2*c*d^2*e^2*cos(
f*x+e)^4+2*I/f^2*c*d^2*e^2*cos(f*x+e)^6-3/f^3*d^3*((f*x+e)^3*(1/4*(cos(f*x+
e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+3/16*(f*x+e)^2*cos(f*x+e)^4-
3/8*(f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+45
/128*(f*x+e)^2-3/512*(2*cos(f*x+e)^2+3)^2+9/16*(f*x+e)^2*cos(f*x+e)^2-9/8*(
f*x+e)*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+9/32*sin(f*x+e)^2-9/32*(f*
x+e)^4)-12/f^3*d^3*e*((f*x+e)^2*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*co
s(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)+1/18*(f*x+e)*cos(f*x+e)^6-1/108*(cos(
f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)-245/1152*f*x-245/1152
*e+5/48*(f*x+e)*cos(f*x+e)^4-5/192*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)
+5/16*(f*x+e)*cos(f*x+e)^2-5/32*cos(f*x+e)*sin(f*x+e)-5/24*(f*x+e)^3)+3/4*I
/f*c^2*d*e*cos(f*x+e)^4+3/f^3*d^3*e^3*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*si
n(f*x+e)+3/8*f*x+3/8*e)-9/f^3*d^3*e^2*((f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f
*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*cos(f*x+e)^2+3)^2)+
9/f^3*d^3*e*((f*x+e)^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*
```

```
x+3/8*e)+1/8*(f*x+e)*cos(f*x+e)^4-1/32*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*
x+e)-15/64*f*x-15/64*e+3/8*(f*x+e)*cos(f*x+e)^2-3/16*cos(f*x+e)*sin(f*x+e)-
1/4*(f*x+e)^3)-9/f*c^2*d*((f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*
x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*cos(f*x+e)^2+3)^2)-9/f^2*c*d^2*(
(f*x+e)^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+1/8*
(f*x+e)*cos(f*x+e)^4-1/32*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)-15/64*f*
x-15/64*e+3/8*(f*x+e)*cos(f*x+e)^2-3/16*cos(f*x+e)*sin(f*x+e)-1/4*(f*x+e)^3
)-2*I/f*c^2*d*e*cos(f*x+e)^6-4*I/f^3*d^3*(-1/6*(f*x+e)^3*cos(f*x+e)^6+1/2*(
f*x+e)^2*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/
16*f*x+5/16*e)+1/36*(f*x+e)*cos(f*x+e)^6-1/216*(cos(f*x+e)^5+5/4*cos(f*x+e)
^3+15/8*cos(f*x+e))*sin(f*x+e)-245/2304*f*x-245/2304*e+5/96*(f*x+e)*cos(f*x
+e)^4-5/384*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+5/32*(f*x+e)*cos(f*x+e
)^2-5/64*cos(f*x+e)*sin(f*x+e)-5/48*(f*x+e)^3)+I/f^3*d^3*(-1/4*(f*x+e)^3*co
s(f*x+e)^4+3/4*(f*x+e)^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*
f*x+3/8*e)+3/32*(f*x+e)*cos(f*x+e)^4-3/128*(cos(f*x+e)^3+3/2*cos(f*x+e))*si
n(f*x+e)-45/256*f*x-45/256*e+9/32*(f*x+e)*cos(f*x+e)^2-9/64*cos(f*x+e)*sin(
f*x+e)-3/16*(f*x+e)^3)+12/f^3*d^3*e^2*((f*x+e)*(1/6*(cos(f*x+e)^5+5/4*cos(f
*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)-5/32*(f*x+e)^2+1/36*co
s(f*x+e)^6+5/96*cos(f*x+e)^4+5/32*cos(f*x+e)^2)+12/f*c^2*d*((f*x+e)*(1/6*(c
os(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)-5
/32*(f*x+e)^2+1/36*cos(f*x+e)^6+5/96*cos(f*x+e)^4+5/32*cos(f*x+e)^2)+12/f^2
*c*d^2*((f*x+e)^2*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(
f*x+e)+5/16*f*x+5/16*e)+1/18*(f*x+e)*cos(f*x+e)^6-1/108*(cos(f*x+e)^5+5/4*c
os(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)-245/1152*f*x-245/1152*e+5/48*(f*x+e
)*cos(f*x+e)^4-5/192*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+5/16*(f*x+e)*
cos(f*x+e)^2-5/32*cos(f*x+e)*sin(f*x+e)-5/24*(f*x+e)^3)-4/f^3*d^3*e^3*(1/6*
(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)
+2/3*I*c^3*cos(f*x+e)^6-1/4*I*c^3*cos(f*x+e)^4+4/f^3*d^3*((f*x+e)^3*(1/6*(c
os(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)+1
/12*(f*x+e)^2*cos(f*x+e)^6-1/6*(f*x+e)*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+
15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)+245/768*(f*x+e)^2-1/216*cos(f*
x+e)^6-5/576*cos(f*x+e)^4-5/192*cos(f*x+e)^2+5/32*(f*x+e)^2*cos(f*x+e)^4-5/
16*(f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-5/1
024*(2*cos(f*x+e)^2+3)^2+15/32*(f*x+e)^2*cos(f*x+e)^2-15/16*(f*x+e)*(1/2*co
s(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+15/64*sin(f*x+e)^2-15/64*(f*x+e)^4)-24/f
^2*c*d^2*e*((f*x+e)*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*si
n(f*x+e)+5/16*f*x+5/16*e)-5/32*(f*x+e)^2+1/36*cos(f*x+e)^6+5/96*cos(f*x+e)^
4+5/32*cos(f*x+e)^2)+1/4*I/f^3*d^3*e^3*cos(f*x+e)^4-2/3*I/f^3*d^3*e^3*cos(f
*x+e)^6-12/f*c^2*d*e*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*s
in(f*x+e)+5/16*f*x+5/16*e)+12/f^2*c*d^2*e^2*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+
e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)-12*I/f*c^2*d*(-1/6*(f*x+e
)*cos(f*x+e)^6+1/36*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x
+e)+5/96*f*x+5/96*e)-12*I/f^2*c*d^2*(-1/6*(f*x+e)^2*cos(f*x+e)^6+1/3*(f*x+e
)*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+
5/16*e)-5/96*(f*x+e)^2+1/108*cos(f*x+e)^6+5/288*cos(f*x+e)^4+5/96*cos(f*x+e
```

```
)^2)+3*I/f^3*d^3*e^2*(-1/4*(f*x+e)*cos(f*x+e)^4...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 374, normalized size = 0.94

$$\frac{\left(576i\,d^{3}f^{3}x^{3}+576i\,c^{3}f^{3}+288\,c^{2}df^{2}-96i\,cd^{2}f-16\,d^{3}-288\,(-6i\,cd^{2}f^{3}-d^{3}f^{2})x^{2}-96\,(-18i\,c^{2}df^{3}-6\,cd^{2}f^{2}+i\,d^{3}f)x+864\,(d^{3}f^{4}x^{4}+4\,cd^{2}f^{4}x^{3}+6\,c^{2}df^{4}x^{2}+4\,c^{3}f^{4}x)e^{(6i\,fx+6i\,e)}-1296\,(-4i\,d^{3}f^{3}x^{3}-4i\,c^{3}f^{3}-6\,c^{2}df^{2}+6i\,cd^{2}f+3\,d^{3}+6\,(-2i\,cd^{2}f^{3}-d^{3}f^{2})x^{2}+6\,(-2i\,c^{2}df^{3}-2\,cd^{2}f^{2}+i\,d^{3}f)x)e^{(4i\,fx+4i\,e)}-81\,(-32i\,d^{3}f^{3}x^{3}-32i\,c^{3}f^{3}-24\,c^{2}df^{2}+12i\,cd^{2}f+3\,d^{3}+24\,(-4i\,cd^{2}f^{3}-d^{3}f^{2})x^{2}+12\,(-8i\,c^{2}df^{3}-4\,cd^{2}f^{2}+i\,d^{3}f)x)e^{(2i\,fx+2i\,e)}\right)e^{(-6i\,fx-6i\,e)}}{27648\,a^{3}f^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/27648*(576*I*d^3*f^3*x^3 + 576*I*c^3*f^3 + 288*c^2*d*f^2 - 96*I*c*d^2*f -
 16*d^3 - 288*(-6*I*c*d^2*f^3 - d^3*f^2)*x^2 - 96*(-18*I*c^2*d*f^3 - 6*c*d^
2*f^2 + I*d^3*f)*x + 864*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2 +
 4*c^3*f^4*x)*e^(6*I*f*x + 6*I*e) - 1296*(-4*I*d^3*f^3*x^3 - 4*I*c^3*f^3 -
6*c^2*d*f^2 + 6*I*c*d^2*f + 3*d^3 + 6*(-2*I*c*d^2*f^3 - d^3*f^2)*x^2 + 6*(-
2*I*c^2*d*f^3 - 2*c*d^2*f^2 + I*d^3*f)*x)*e^(4*I*f*x + 4*I*e) - 81*(-32*I*d
^3*f^3*x^3 - 32*I*c^3*f^3 - 24*c^2*d*f^2 + 12*I*c*d^2*f + 3*d^3 + 24*(-4*I*
c*d^2*f^3 - d^3*f^2)*x^2 + 12*(-8*I*c^2*d*f^3 - 4*c*d^2*f^2 + I*d^3*f)*x)*e
^(2*I*f*x + 2*I*e))*e^(-6*I*f*x - 6*I*e)/(a^3*f^4)
```

Sympy [A]
time = 0.51, size = 945, normalized size = 2.39

$$\begin{cases} \frac{[illegible]}{[illegible]} & \text{for}\: a^{9} f^{12} e^{12 i e} \neq 0 \\ [illegible] & \text{otherwise} \end{cases} + \frac{c^{3} x}{8 a^{3}} + \frac{3 c^{2} d x^{2}}{16 a^{3}} + \frac{c d^{2} x^{3}}{8 a^{3}} + \frac{d^{3} x^{4}}{32 a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((((2359296*I*a**6*c**3*f**11*exp(6*I*e) + 7077888*I*a**6*c**2*d*f
**11*x*exp(6*I*e) + 1179648*a**6*c**2*d*f**10*exp(6*I*e) + 7077888*I*a**6*c
*d**2*f**11*x**2*exp(6*I*e) + 2359296*a**6*c*d**2*f**10*x*exp(6*I*e) - 3932
16*I*a**6*c*d**2*f**9*exp(6*I*e) + 2359296*I*a**6*d**3*f**11*x**3*exp(6*I*e
) + 1179648*a**6*d**3*f**10*x**2*exp(6*I*e) - 393216*I*a**6*d**3*f**9*x*exp
(6*I*e) - 65536*a**6*d**3*f**8*exp(6*I*e))*exp(-6*I*f*x) + (10616832*I*a**6
*c**3*f**11*exp(8*I*e) + 31850496*I*a**6*c**2*d*f**11*x*exp(8*I*e) + 796262
```

```
4*a**6*c**2*d*f**10*exp(8*I*e) + 31850496*I*a**6*c*d**2*f**11*x**2*exp(8*I*
e) + 15925248*a**6*c*d**2*f**10*x*exp(8*I*e) - 3981312*I*a**6*c*d**2*f**9*e
xp(8*I*e) + 10616832*I*a**6*d**3*f**11*x**3*exp(8*I*e) + 7962624*a**6*d**3*
f**10*x**2*exp(8*I*e) - 3981312*I*a**6*d**3*f**9*x*exp(8*I*e) - 995328*a**6
*d**3*f**8*exp(8*I*e))*exp(-4*I*f*x) + (21233664*I*a**6*c**3*f**11*exp(10*I
*e) + 63700992*I*a**6*c**2*d*f**11*x*exp(10*I*e) + 31850496*a**6*c**2*d*f**
10*exp(10*I*e) + 63700992*I*a**6*c*d**2*f**11*x**2*exp(10*I*e) + 63700992*a
**6*c*d**2*f**10*x*exp(10*I*e) - 31850496*I*a**6*c*d**2*f**9*exp(10*I*e) +
21233664*I*a**6*d**3*f**11*x**3*exp(10*I*e) + 31850496*a**6*d**3*f**10*x**2
*exp(10*I*e) - 31850496*I*a**6*d**3*f**9*x*exp(10*I*e) - 15925248*a**6*d**3
*f**8*exp(10*I*e))*exp(-2*I*f*x))*exp(-12*I*e)/(113246208*a**9*f**12), Ne(a
**9*f**12*exp(12*I*e), 0)), (x**4*(3*d**3*exp(4*I*e) + 3*d**3*exp(2*I*e) +
d**3)*exp(-6*I*e)/(32*a**3) + x**3*(3*c*d**2*exp(4*I*e) + 3*c*d**2*exp(2*I*
e) + c*d**2)*exp(-6*I*e)/(8*a**3) + x**2*(9*c**2*d*exp(4*I*e) + 9*c**2*d*ex
p(2*I*e) + 3*c**2*d)*exp(-6*I*e)/(16*a**3) + x*(3*c**3*exp(4*I*e) + 3*c**3*
exp(2*I*e) + c**3)*exp(-6*I*e)/(8*a**3), True)) + c**3*x/(8*a**3) + 3*c**2*
d*x**2/(16*a**3) + c*d**2*x**3/(8*a**3) + d**3*x**4/(32*a**3)
```

Giac [A]
time = 0.75, size = 573, normalized size = 1.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/27648*(864*d^3*f^4*x^4*e^(6*I*f*x + 6*I*e) + 3456*c*d^2*f^4*x^3*e^(6*I*f*
x + 6*I*e) + 5184*c^2*d*f^4*x^2*e^(6*I*f*x + 6*I*e) + 5184*I*d^3*f^3*x^3*e^
(4*I*f*x + 4*I*e) + 2592*I*d^3*f^3*x^3*e^(2*I*f*x + 2*I*e) + 576*I*d^3*f^3*
x^3 + 3456*c^3*f^4*x*e^(6*I*f*x + 6*I*e) + 15552*I*c*d^2*f^3*x^2*e^(4*I*f*x
 + 4*I*e) + 7776*I*c*d^2*f^3*x^2*e^(2*I*f*x + 2*I*e) + 1728*I*c*d^2*f^3*x^2
 + 15552*I*c^2*d*f^3*x*e^(4*I*f*x + 4*I*e) + 7776*d^3*f^2*x^2*e^(4*I*f*x +
4*I*e) + 7776*I*c^2*d*f^3*x*e^(2*I*f*x + 2*I*e) + 1944*d^3*f^2*x^2*e^(2*I*f
*x + 2*I*e) + 1728*I*c^2*d*f^3*x + 288*d^3*f^2*x^2 + 5184*I*c^3*f^3*e^(4*I*
f*x + 4*I*e) + 15552*c*d^2*f^2*x*e^(4*I*f*x + 4*I*e) + 2592*I*c^3*f^3*e^(2*
I*f*x + 2*I*e) + 3888*c*d^2*f^2*x*e^(2*I*f*x + 2*I*e) + 576*I*c^3*f^3 + 576
*c*d^2*f^2*x + 7776*c^2*d*f^2*e^(4*I*f*x + 4*I*e) - 7776*I*d^3*f*x*e^(4*I*f
*x + 4*I*e) + 1944*c^2*d*f^2*e^(2*I*f*x + 2*I*e) - 972*I*d^3*f*x*e^(2*I*f*x
 + 2*I*e) + 288*c^2*d*f^2 - 96*I*d^3*f*x - 7776*I*c*d^2*f*e^(4*I*f*x + 4*I*
e) - 972*I*c*d^2*f*e^(2*I*f*x + 2*I*e) - 96*I*c*d^2*f - 3888*d^3*e^(4*I*f*x
 + 4*I*e) - 243*d^3*e^(2*I*f*x + 2*I*e) - 16*d^3)*e^(-6*I*f*x - 6*I*e)/(a^3
*f^4)
```

Mupad [B]
time = 3.85, size = 411, normalized size = 1.04

$$\mathrm{e}^{-e\,2\mathrm{i}-f\,x\,2\mathrm{i}}\,\left(\frac{\left(12\,c^3\,f^3-c^2\,d\,f^2\,18\mathrm{i}-18\,c\,d^2\,f+d^3\,9\mathrm{i}\right)\,1\mathrm{i}}{64\,a^3\,f^4}+\frac{d^3\,x^3\,3\mathrm{i}}{16\,a^3\,f}-\frac{d\,x\,\left(-2\,c^2\,f^2+c\,d\,f\,2\mathrm{i}+d^2\right)\,9\mathrm{i}}{32\,a^3\,f^3}-\frac{d^2\,x^2\,\left(-2\,c\,f+d\,1\mathrm{i}\right)\,9\mathrm{i}}{32\,a^3\,f^2}\right)+\mathrm{e}^{-e\,4\mathrm{i}-f\,x\,4\mathrm{i}}\,\left(\frac{\left(96\,c^3\,f^3-c^2\,d\,f^2\,72\mathrm{i}-36\,c\,d^2\,f+d^3\,9\mathrm{i}\right)\,1\mathrm{i}}{1024\,a^3\,f^4}+\frac{d^3\,x^3\,3\mathrm{i}}{32\,a^3\,f}-\frac{d\,x\,\left(-8\,c^2\,f^2+c\,d\,f\,4\mathrm{i}+d^2\right)\,9\mathrm{i}}{256\,a^3\,f^3}-\frac{d^2\,x^2\,\left(-4\,c\,f+d\,1\mathrm{i}\right)\,9\mathrm{i}}{128\,a^3\,f^2}\right)+\mathrm{e}^{-e\,6\mathrm{i}-f\,x\,6\mathrm{i}}\,\left(\frac{\left(36\,c^3\,f^3-c^2\,d\,f^2\,18\mathrm{i}-6\,c\,d^2\,f+d^3\,1\mathrm{i}\right)\,1\mathrm{i}}{1728\,a^3\,f^4}+\frac{d^3\,x^3\,1\mathrm{i}}{48\,a^3\,f}-\frac{d\,x\,\left(-18\,c^2\,f^2+c\,d\,f\,6\mathrm{i}+d^2\right)\,1\mathrm{i}}{288\,a^3\,f^3}-\frac{d^2\,x^2\,\left(-6\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{96\,a^3\,f^2}\right)+\frac{c^3\,x}{8\,a^3}+\frac{d^3\,x^4}{32\,a^3}+\frac{3\,c^2\,d\,x^2}{16\,a^3}+\frac{c\,d^2\,x^3}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] exp(- e*2i - f*x*2i)*(((d^3*9i + 12*c^3*f^3 - c^2*d*f^2*18i - 18*c*d^2*f)*1
i)/(64*a^3*f^4) + (d^3*x^3*3i)/(16*a^3*f) - (d*x*(d^2 - 2*c^2*f^2 + c*d*f*2
i)*9i)/(32*a^3*f^3) - (d^2*x^2*(d*1i - 2*c*f)*9i)/(32*a^3*f^2)) + exp(- e*4
i - f*x*4i)*(((d^3*9i + 96*c^3*f^3 - c^2*d*f^2*72i - 36*c*d^2*f)*1i)/(1024*
a^3*f^4) + (d^3*x^3*3i)/(32*a^3*f) - (d*x*(d^2 - 8*c^2*f^2 + c*d*f*4i)*9i)/
(256*a^3*f^3) - (d^2*x^2*(d*1i - 4*c*f)*9i)/(128*a^3*f^2)) + exp(- e*6i - f
*x*6i)*(((d^3*1i + 36*c^3*f^3 - c^2*d*f^2*18i - 6*c*d^2*f)*1i)/(1728*a^3*f^
4) + (d^3*x^3*1i)/(48*a^3*f) - (d*x*(d^2 - 18*c^2*f^2 + c*d*f*6i)*1i)/(288*
a^3*f^3) - (d^2*x^2*(d*1i - 6*c*f)*1i)/(96*a^3*f^2)) + (c^3*x)/(8*a^3) + (d
^3*x^4)/(32*a^3) + (3*c^2*d*x^2)/(16*a^3) + (c*d^2*x^3)/(8*a^3)
```

3.30 $\int \frac{(c+dx)^2}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=294

$$-\frac{3id^2e^{-2ie-2ifx}}{32a^3f^3}-\frac{3id^2e^{-4ie-4ifx}}{256a^3f^3}-\frac{id^2e^{-6ie-6ifx}}{864a^3f^3}+\frac{3de^{-2ie-2ifx}(c+dx)}{16a^3f^2}+\frac{3de^{-4ie-4ifx}(c+dx)}{64a^3f^2}+\frac{de^{-6ie-6ifx}(c+}{144a^3f^2}$$

```
[Out] -3/32*I*d^2*exp(-2*I*e-2*I*f*x)/a^3/f^3-3/256*I*d^2*exp(-4*I*e-4*I*f*x)/a^3
/f^3-1/864*I*d^2*exp(-6*I*e-6*I*f*x)/a^3/f^3+3/16*d*exp(-2*I*e-2*I*f*x)*(d*
x+c)/a^3/f^2+3/64*d*exp(-4*I*e-4*I*f*x)*(d*x+c)/a^3/f^2+1/144*d*exp(-6*I*e-
6*I*f*x)*(d*x+c)/a^3/f^2+3/16*I*exp(-2*I*e-2*I*f*x)*(d*x+c)^2/a^3/f+3/32*I*
exp(-4*I*e-4*I*f*x)*(d*x+c)^2/a^3/f+1/48*I*exp(-6*I*e-6*I*f*x)*(d*x+c)^2/a^
3/f+1/24*(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.19, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3810, 2207, 2225}

$$\frac{3d(c+dx)e^{-2ie-2ifx}}{16a^3f^2}+\frac{3d(c+dx)e^{-4ie-4ifx}}{64a^3f^2}+\frac{d(c+dx)e^{-6ie-6ifx}}{144a^3f^2}+\frac{3i(c+dx)^2e^{-2ie-2ifx}}{16a^3f}+\frac{3i(c+dx)^2e^{-4ie-4ifx}}{32a^3f}+\frac{i(c+dx)^2e^{-6ie-6ifx}}{48a^3f}+\frac{(c+dx)^3}{24a^3d}-\frac{3id^2e^{-2ie-2ifx}}{32a^3f^3}-\frac{3id^2e^{-4ie-4ifx}}{256a^3f^3}-\frac{id^2e^{-6ie-6ifx}}{864a^3f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (((-3*I)/32)*d^2*E^((-2*I)*e - (2*I)*f*x))/(a^3*f^3) - (((3*I)/256)*d^2*E^(
(-4*I)*e - (4*I)*f*x))/(a^3*f^3) - ((I/864)*d^2*E^((-6*I)*e - (6*I)*f*x))/(
a^3*f^3) + (3*d*E^((-2*I)*e - (2*I)*f*x)*(c + d*x))/(16*a^3*f^2) + (3*d*E^(
(-4*I)*e - (4*I)*f*x)*(c + d*x))/(64*a^3*f^2) + (d*E^((-6*I)*e - (6*I)*f*x)
*(c + d*x))/(144*a^3*f^2) + (((3*I)/16)*E^((-2*I)*e - (2*I)*f*x)*(c + d*x)^
2)/(a^3*f) + (((3*I)/32)*E^((-4*I)*e - (4*I)*f*x)*(c + d*x)^2)/(a^3*f) + ((
I/48)*E^((-6*I)*e - (6*I)*f*x)*(c + d*x)^2)/(a^3*f) + (c + d*x)^3/(24*a^3*d
)
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^2}{(a+ia\tan(e+fx))^3}\,dx &= \int \left(\frac{(c+dx)^2}{8a^3}+\frac{3e^{-2ie-2ifx}(c+dx)^2}{8a^3}+\frac{3e^{-4ie-4ifx}(c+dx)^2}{8a^3}+\frac{e^{-6ie-6ifx}(c \cdots}{8a^3} \cdots\right. \\
&= \frac{(c+dx)^3}{24a^3d}+\frac{\int e^{-6ie-6ifx}(c+dx)^2\,dx}{8a^3}+\frac{3\int e^{-2ie-2ifx}(c+dx)^2\,dx}{8a^3}+\frac{3\int e^{-4ie \cdots}{\cdots} \\
&= \frac{3ie^{-2ie-2ifx}(c+dx)^2}{16a^3f}+\frac{3ie^{-4ie-4ifx}(c+dx)^2}{32a^3f}+\frac{ie^{-6ie-6ifx}(c+dx)^2}{48a^3f}+\frac{(c+\cdots}{24a\cdots} \\
&= \frac{3de^{-2ie-2ifx}(c+dx)}{16a^3f^2}+\frac{3de^{-4ie-4ifx}(c+dx)}{64a^3f^2}+\frac{de^{-6ie-6ifx}(c+dx)}{144a^3f^2}+\frac{3ie^{-2ie \cdots}{\cdots} \\
&= -\frac{3id^2e^{-2ie-2ifx}}{32a^3f^3}-\frac{3id^2e^{-4ie-4ifx}}{256a^3f^3}-\frac{id^2e^{-6ie-6ifx}}{864a^3f^3}+\frac{3de^{-2ie-2ifx}(c+dx)}{16a^3f^2}+\frac{3\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 1.62, size = 405, normalized size = 1.38

$$
\frac{i\sec^3(e+fx)\left(81\left(24ic^2f^2+4cdf(5+12ifx)+d^2(-9i+20fx+24if^2x^2)\right)\cos(e+fx)+8\left(18c^2f^2(i+6fx)+6cdf(1+6ifx+18f^2x^2)+d^2(-i+6fx+18if^2x^2+36f^3x^3)\right)\cos(3(e+fx))+567d^2\sin(e+fx)+972icdf\sin(e+fx)-648c^2f^2\sin(e+fx)+972id^2fx\sin(e+fx)-1296cdf^2x\sin(e+fx)-648d^2f^2x^2\sin(e+fx)-8d^2\sin(3(e+fx))-48icdf\sin(3(e+fx))+144c^2f^2\sin(3(e+fx))-48id^2fx\sin(3(e+fx))+288cdf^2x\sin(3(e+fx))+864ic^2f^3x\sin(3(e+fx))+144d^2f^2x^2\sin(3(e+fx))+864icdf^3x^2\sin(3(e+fx))+288id^2f^3x^3\sin(3(e+fx))\right)}{6912a^3f^3(-i+\tan(e+fx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I/6912)*Sec[e + f*x]^3*(81*((24*I)*c^2*f^2 + 4*c*d*f*(5 + (12*I)*f*x) + d
^2*(-9*I + 20*f*x + (24*I)*f^2*x^2))*Cos[e + f*x] + 8*(18*c^2*f^2*(I + 6*f*
x) + 6*c*d*f*(1 + (6*I)*f*x + 18*f^2*x^2) + d^2*(-I + 6*f*x + (18*I)*f^2*x^
2 + 36*f^3*x^3))*Cos[3*(e + f*x)] + 567*d^2*Sin[e + f*x] + (972*I)*c*d*f*Si
n[e + f*x] - 648*c^2*f^2*Sin[e + f*x] + (972*I)*d^2*f*x*Sin[e + f*x] - 1296
*c*d*f^2*x*Sin[e + f*x] - 648*d^2*f^2*x^2*Sin[e + f*x] - 8*d^2*Sin[3*(e + f
*x)] - (48*I)*c*d*f*Sin[3*(e + f*x)] + 144*c^2*f^2*Sin[3*(e + f*x)] - (48*I
)*d^2*f*x*Sin[3*(e + f*x)] + 288*c*d*f^2*x*Sin[3*(e + f*x)] + (864*I)*c^2*f
^3*x*Sin[3*(e + f*x)] + 144*d^2*f^2*x^2*Sin[3*(e + f*x)] + (864*I)*c*d*f^3*
x^2*Sin[3*(e + f*x)] + (288*I)*d^2*f^3*x^3*Sin[3*(e + f*x)]))/(a^3*f^3*(-I
+ Tan[e + f*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1512 vs. 2(241) = 482.
time = 0.54, size = 1513, normalized size = 5.15

method	result
risch	$\frac{d^2x^3}{24a^3}+\frac{dc\,x^2}{8a^3}+\frac{c^2x}{8a^3}+\frac{c^3}{24a^3d}+\frac{3i(2d^2x^2f^2+4cd\,f^2x-2id^2fx+2c^2f^2-2icdf-d^2)\,\mathrm{e}^{-2i(fx+e)}}{32a^3f^3}+\frac{3i(8d^2x^2f^2+16cd\,f^2x-4id^2f}{256}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/f*(2/3*I/f^2*d^2*e^2*cos(f*x+e)^6-4*I/f^2*d^2*(-1/6*(f*x+e)^2*cos(f*x
+e)^6+1/3*(f*x+e)*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(
f*x+e)+5/16*f*x+5/16*e)-5/96*(f*x+e)^2+1/108*cos(f*x+e)^6+5/288*cos(f*x+e)^
4+5/96*cos(f*x+e)^2)+1/2*I/f*c*d*e*cos(f*x+e)^4-4/3*I/f*c*d*e*cos(f*x+e)^6-
1/4*I*c^2*cos(f*x+e)^4+2/3*I*c^2*cos(f*x+e)^6+4*c^2*(1/6*(cos(f*x+e)^5+5/4*
cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)-8/f*c*d*e*(1/6*(c
os(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)+8
/f*c*d*((f*x+e)*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*
x+e)+5/16*f*x+5/16*e)-5/32*(f*x+e)^2+1/36*cos(f*x+e)^6+5/96*cos(f*x+e)^4+5/
32*cos(f*x+e)^2)+4/f^2*d^2*e^2*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos
(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)-8/f^2*d^2*e*((f*x+e)*(1/6*(cos(f*x+e)^
5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)-5/32*(f*x+e
)^2+1/36*cos(f*x+e)^6+5/96*cos(f*x+e)^4+5/32*cos(f*x+e)^2)+4/f^2*d^2*((f*x+
e)^2*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f
*x+5/16*e)+1/18*(f*x+e)*cos(f*x+e)^6-1/108*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+1
5/8*cos(f*x+e))*sin(f*x+e)-245/1152*f*x-245/1152*e+5/48*(f*x+e)*cos(f*x+e)^
4-5/192*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+5/16*(f*x+e)*cos(f*x+e)^2-
5/32*cos(f*x+e)*sin(f*x+e)-5/24*(f*x+e)^3)-8*I/f*c*d*(-1/6*(f*x+e)*cos(f*x+
e)^6+1/36*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/96*f
*x+5/96*e)+2*I/f*c*d*(-1/4*(f*x+e)*cos(f*x+e)^4+1/16*(cos(f*x+e)^3+3/2*cos(
f*x+e))*sin(f*x+e)+3/32*f*x+3/32*e)+I/f^2*d^2*(-1/4*(f*x+e)^2*cos(f*x+e)^4+
1/2*(f*x+e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/
32*(f*x+e)^2+1/128*(2*cos(f*x+e)^2+3)^2)-1/4*I/f^2*d^2*e^2*cos(f*x+e)^4+8*I
/f^2*d^2*e*(-1/6*(f*x+e)*cos(f*x+e)^6+1/36*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+1
5/8*cos(f*x+e))*sin(f*x+e)+5/96*f*x+5/96*e)-2*I/f^2*d^2*e*(-1/4*(f*x+e)*cos
(f*x+e)^4+1/16*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/32*f*x+3/32*e)-3*
c^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+6/f*c*d*e*
(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-6/f*c*d*((f*x+
e)*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e
)^2+1/64*(2*cos(f*x+e)^2+3)^2)-3/f^2*d^2*e^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x
+e))*sin(f*x+e)+3/8*f*x+3/8*e)+6/f^2*d^2*e*((f*x+e)*(1/4*(cos(f*x+e)^3+3/2*
cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*cos(f*x+e)^2+3
)^2)-3/f^2*d^2*((f*x+e)^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8
*f*x+3/8*e)+1/8*(f*x+e)*cos(f*x+e)^4-1/32*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin
(f*x+e)-15/64*f*x-15/64*e+3/8*(f*x+e)*cos(f*x+e)^2-3/16*cos(f*x+e)*sin(f*x+
e)-1/4*(f*x+e)^3))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 225, normalized size = 0.77

$$\frac{\left(144i\,d^2f^2x^2+144i\,c^2f^2+48\,cdf-8i\,d^2-48\left(-6i\,cdf^2-d^2f\right)x+288\left(d^2f^3x^3+3\,cdf^3x^2+3\,c^2f^3x\right)e^{(6i\,fx+6i\,e)}-648\left(-2i\,d^2f^2x^2-2i\,c^2f^2-2\,cdf+i\,d^2+2\left(-2i\,cdf^2-d^2f\right)x\right)e^{(4i\,fx+4i\,e)}-81\left(-8i\,d^2f^2x^2-8i\,c^2f^2-4\,cdf+i\,d^2+4\left(-4i\,cdf^2-d^2f\right)x\right)e^{(2i\,fx+2i\,e)}\right)e^{(-6i\,fx-6i\,e)}}{6912\,a^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6912*(144*I*d^2*f^2*x^2 + 144*I*c^2*f^2 + 48*c*d*f - 8*I*d^2 - 48*(-6*I*c
*d*f^2 - d^2*f)*x + 288*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x)*e^(6*I*
f*x + 6*I*e) - 648*(-2*I*d^2*f^2*x^2 - 2*I*c^2*f^2 - 2*c*d*f + I*d^2 + 2*(-
2*I*c*d*f^2 - d^2*f)*x)*e^(4*I*f*x + 4*I*e) - 81*(-8*I*d^2*f^2*x^2 - 8*I*c^
2*f^2 - 4*c*d*f + I*d^2 + 4*(-4*I*c*d*f^2 - d^2*f)*x)*e^(2*I*f*x + 2*I*e))*
e^(-6*I*f*x - 6*I*e)/(a^3*f^3)
```

Sympy [A]
time = 0.39, size = 588, normalized size = 2.00

$$\begin{cases} \frac{\left(\left(147456ia^6c^2f^8e^{6ie}+294912ia^6cdf^8xe^{6ie}+49152a^6cdf^7e^{6ie}+147456ia^6d^2f^8x^2e^{6ie}+49152a^6d^2f^7xe^{6ie}-8192ia^6d^2f^6e^{6ie}\right)e^{-6ifx}+\left(663552ia^6c^2f^8e^{8ie}+1327104ia^6cdf^8xe^{8ie}+331776a^6cdf^7e^{8ie}+663552ia^6d^2f^8x^2e^{8ie}+331776a^6d^2f^7xe^{8ie}-82944ia^6d^2f^6e^{8ie}\right)e^{-4ifx}+\left(1327104ia^6c^2f^8e^{10ie}+2654208ia^6cdf^8xe^{10ie}+1327104a^6cdf^7e^{10ie}+1327104ia^6d^2f^8x^2e^{10ie}+1327104a^6d^2f^7xe^{10ie}-663552ia^6d^2f^6e^{10ie}\right)e^{-2ifx}\right)e^{-12ie}}{7077888a^9f^9} & \text{for}\: a^9f^9e^{12ie}\neq 0 \\ \frac{x^3\cdot\left(3d^2e^{4ie}+3d^2e^{2ie}+d^2\right)e^{-6ie}}{24a^3}+\frac{x^2\cdot\left(3cde^{4ie}+3cde^{2ie}+cd\right)e^{-6ie}}{8a^3}+\frac{x\left(3c^2e^{4ie}+3c^2e^{2ie}+c^2\right)e^{-6ie}}{8a^3} & \text{otherwise} \end{cases}+\frac{c^2x}{8a^3}+\frac{cdx^2}{8a^3}+\frac{d^2x^3}{24a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((((147456*I*a**6*c**2*f**8*exp(6*I*e) + 294912*I*a**6*c*d*f**8*x*
exp(6*I*e) + 49152*a**6*c*d*f**7*exp(6*I*e) + 147456*I*a**6*d**2*f**8*x**2*
exp(6*I*e) + 49152*a**6*d**2*f**7*x*exp(6*I*e) - 8192*I*a**6*d**2*f**6*exp(
6*I*e))*exp(-6*I*f*x) + (663552*I*a**6*c**2*f**8*exp(8*I*e) + 1327104*I*a**
6*c*d*f**8*x*exp(8*I*e) + 331776*a**6*c*d*f**7*exp(8*I*e) + 663552*I*a**6*d
**2*f**8*x**2*exp(8*I*e) + 331776*a**6*d**2*f**7*x*exp(8*I*e) - 82944*I*a**
6*d**2*f**6*exp(8*I*e))*exp(-4*I*f*x) + (1327104*I*a**6*c**2*f**8*exp(10*I*
e) + 2654208*I*a**6*c*d*f**8*x*exp(10*I*e) + 1327104*a**6*c*d*f**7*exp(10*I
*e) + 1327104*I*a**6*d**2*f**8*x**2*exp(10*I*e) + 1327104*a**6*d**2*f**7*x*
exp(10*I*e) - 663552*I*a**6*d**2*f**6*exp(10*I*e))*exp(-2*I*f*x))*exp(-12*I
*e)/(7077888*a**9*f**9), Ne(a**9*f**9*exp(12*I*e), 0)), (x**3*(3*d**2*exp(4
```

```
*I*e) + 3*d**2*exp(2*I*e) + d**2)*exp(-6*I*e)/(24*a**3) + x**2*(3*c*d*exp(4
*I*e) + 3*c*d*exp(2*I*e) + c*d)*exp(-6*I*e)/(8*a**3) + x*(3*c**2*exp(4*I*e)
 + 3*c**2*exp(2*I*e) + c**2)*exp(-6*I*e)/(8*a**3), True)) + c**2*x/(8*a**3)
 + c*d*x**2/(8*a**3) + d**2*x**3/(24*a**3)
```

Giac [A]
time = 0.73, size = 331, normalized size = 1.13

$$\frac{\left(288\,d^2f^3x^3e^{(6i\,fx+6i\,e)} + 864\,cdf^3x^2e^{(6i\,fx+6i\,e)} + 864\,c^2f^3xe^{(6i\,fx+6i\,e)} + 1296i\,d^2f^2x^2e^{(4i\,fx+4i\,e)} + 648i\,d^2f^2x^2e^{(2i\,fx+2i\,e)} + 144i\,d^2f^2x^2 + 2592i\,cdf^2xe^{(4i\,fx+4i\,e)} + 1296i\,cdf^2xe^{(2i\,fx+2i\,e)} + 288i\,cdf^2x + 1296i\,c^2f^2e^{(4i\,fx+4i\,e)} + 1296\,d^2fxe^{(4i\,fx+4i\,e)} + 648i\,c^2f^2e^{(2i\,fx+2i\,e)} + 324\,d^2fxe^{(2i\,fx+2i\,e)} + 144i\,c^2f^2 + 48\,d^2fx + 1296\,cdfe^{(4i\,fx+4i\,e)} + 324\,cdfe^{(2i\,fx+2i\,e)} + 48\,cdf - 648i\,d^2e^{(4i\,fx+4i\,e)} - 81i\,d^2e^{(2i\,fx+2i\,e)} - 8i\,d^2\right)e^{(-6i\,fx-6i\,e)}}{6912\,a^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/6912*(288*d^2*f^3*x^3*e^(6*I*f*x + 6*I*e) + 864*c*d*f^3*x^2*e^(6*I*f*x +
6*I*e) + 864*c^2*f^3*x*e^(6*I*f*x + 6*I*e) + 1296*I*d^2*f^2*x^2*e^(4*I*f*x
+ 4*I*e) + 648*I*d^2*f^2*x^2*e^(2*I*f*x + 2*I*e) + 144*I*d^2*f^2*x^2 + 2592
*I*c*d*f^2*x*e^(4*I*f*x + 4*I*e) + 1296*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) + 2
88*I*c*d*f^2*x + 1296*I*c^2*f^2*e^(4*I*f*x + 4*I*e) + 1296*d^2*f*x*e^(4*I*f
*x + 4*I*e) + 648*I*c^2*f^2*e^(2*I*f*x + 2*I*e) + 324*d^2*f*x*e^(2*I*f*x +
2*I*e) + 144*I*c^2*f^2 + 48*d^2*f*x + 1296*c*d*f*e^(4*I*f*x + 4*I*e) + 324*
c*d*f*e^(2*I*f*x + 2*I*e) + 48*c*d*f - 648*I*d^2*e^(4*I*f*x + 4*I*e) - 81*I
*d^2*e^(2*I*f*x + 2*I*e) - 8*I*d^2)*e^(-6*I*f*x - 6*I*e)/(a^3*f^3)
```

Mupad [B]
time = 3.37, size = 263, normalized size = 0.89

$$\frac{c^2\,x}{8\,a^3}-\mathrm{e}^{-e\,2\mathrm{i}-f\,x\,2\mathrm{i}}\,\left(\frac{\left(-6\,c^2\,f^2+c\,d\,f\,6\mathrm{i}+3\,d^2\right)\,1\mathrm{i}}{32\,a^3\,f^3}-\frac{d^2\,x^2\,3\mathrm{i}}{16\,a^3\,f}+\frac{d\,x\,\left(-2\,c\,f+d\,1\mathrm{i}\right)\,3\mathrm{i}}{16\,a^3\,f^2}\right)-\mathrm{e}^{-e\,4\mathrm{i}-f\,x\,4\mathrm{i}}\,\left(\frac{\left(-24\,c^2\,f^2+c\,d\,f\,12\mathrm{i}+3\,d^2\right)\,1\mathrm{i}}{256\,a^3\,f^3}-\frac{d^2\,x^2\,3\mathrm{i}}{32\,a^3\,f}+\frac{d\,x\,\left(-4\,c\,f+d\,1\mathrm{i}\right)\,3\mathrm{i}}{64\,a^3\,f^2}\right)-\mathrm{e}^{-e\,6\mathrm{i}-f\,x\,6\mathrm{i}}\,\left(\frac{\left(-18\,c^2\,f^2+c\,d\,f\,6\mathrm{i}+d^2\right)\,1\mathrm{i}}{864\,a^3\,f^3}-\frac{d^2\,x^2\,1\mathrm{i}}{48\,a^3\,f}+\frac{d\,x\,\left(-6\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{144\,a^3\,f^2}\right)+\frac{d^2\,x^3}{24\,a^3}+\frac{c\,d\,x^2}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] (c^2*x)/(8*a^3) - exp(- e*2i - f*x*2i)*(((3*d^2 - 6*c^2*f^2 + c*d*f*6i)*1i)
/(32*a^3*f^3) - (d^2*x^2*3i)/(16*a^3*f) + (d*x*(d*1i - 2*c*f)*3i)/(16*a^3*f
^2)) - exp(- e*4i - f*x*4i)*(((3*d^2 - 24*c^2*f^2 + c*d*f*12i)*1i)/(256*a^3
*f^3) - (d^2*x^2*3i)/(32*a^3*f) + (d*x*(d*1i - 4*c*f)*3i)/(64*a^3*f^2)) - e
xp(- e*6i - f*x*6i)*(((d^2 - 18*c^2*f^2 + c*d*f*6i)*1i)/(864*a^3*f^3) - (d^
2*x^2*1i)/(48*a^3*f) + (d*x*(d*1i - 6*c*f)*1i)/(144*a^3*f^2)) + (d^2*x^3)/(
24*a^3) + (c*d*x^2)/(8*a^3)
```

3.31 $\int \frac{c+dx}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=209

$$-\frac{11idx}{96a^3f}-\frac{dx^2}{16a^3}+\frac{x(c+dx)}{8a^3}+\frac{d}{36f^2(a+ia\tan(e+fx))^3}+\frac{i(c+dx)}{6f(a+ia\tan(e+fx))^3}+\frac{5d}{96af^2(a+ia\tan(e+f$$

```
[Out] -11/96*I*d*x/a^3/f-1/16*d*x^2/a^3+1/8*x*(d*x+c)/a^3+1/36*d/f^2/(a+I*a*tan(f
*x+e))^3+1/6*I*(d*x+c)/f/(a+I*a*tan(f*x+e))^3+5/96*d/a/f^2/(a+I*a*tan(f*x+e
))^2+1/8*I*(d*x+c)/a/f/(a+I*a*tan(f*x+e))^2+11/96*d/f^2/(a^3+I*a^3*tan(f*x+
e))+1/8*I*(d*x+c)/f/(a^3+I*a^3*tan(f*x+e))
```

Rubi [A]
time = 0.17, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3560, 8, 3811}

$$\frac{i(c+dx)}{8f\left(a^3+ia^3\tan(e+fx)\right)}+\frac{x(c+dx)}{8a^3}+\frac{11d}{96f^2\left(a^3+ia^3\tan(e+fx)\right)}-\frac{11idx}{96a^3f}-\frac{dx^2}{16a^3}+\frac{i(c+dx)}{8af(a+ia\tan(e+fx))^2}+\frac{i(c+dx)}{6f(a+ia\tan(e+fx))^3}+\frac{5d}{96af^2(a+ia\tan(e+fx))^2}+\frac{d}{36f^2(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (((-11*I)/96)*d*x)/(a^3*f) - (d*x^2)/(16*a^3) + (x*(c + d*x))/(8*a^3) + d/(
36*f^2*(a + I*a*Tan[e + f*x])^3) + ((I/6)*(c + d*x))/(f*(a + I*a*Tan[e + f*
x])^3) + (5*d)/(96*a*f^2*(a + I*a*Tan[e + f*x])^2) + ((I/8)*(c + d*x))/(a*f
*(a + I*a*Tan[e + f*x])^2) + (11*d)/(96*f^2*(a^3 + I*a^3*Tan[e + f*x])) + (
(I/8)*(c + d*x))/(f*(a^3 + I*a^3*Tan[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3811

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)
^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x]] /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+ia\tan(e+fx))^3}\,dx = \frac{x(c+dx)}{8a^3} + \frac{i(c+dx)}{6f(a+ia\tan(e+fx))^3} + \frac{i(c+dx)}{8af(a+ia\tan(e+fx))^2} + \frac{i}{8f\,(a^3+}$$

$$= -\frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{i(c+dx)}{6f(a+ia\tan(e+fx))^3} + \frac{i(c+dx)}{8af(a+ia\tan(e+fx))^2} +$$

$$= -\frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\tan(e+fx))^3} + \frac{i(c+dx)}{6f(a+ia\tan(e+fx))^3}$$

$$= -\frac{idx}{16a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\tan(e+fx))^3} + \frac{i(c+dx}{6f(a+ia\tan(e}$$

$$= -\frac{3idx}{32a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\tan(e+fx))^3} + \frac{i(c+dx}{6f(a+ia\tan(e}$$

$$= -\frac{11idx}{96a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\tan(e+fx))^3} + \frac{i(c+dx}{6f(a+ia\tan(e}$$

<u>Mathematica</u> [A]
time = 0.80, size = 205, normalized size = 0.98

$$\frac{i\sec^3(e+fx)\,(27(12icf+d(5+12ifx))\cos(e+fx)+4(6cf(i+6fx)+d(1+6ifx+18f^2x^2))\cos(3(e+fx))+81id\sin(e+fx)-108cf\sin(e+fx)-108dfx\sin(e+fx)-4id\sin(3(e+fx))+24cf\sin(3(e+fx))+24dfx\sin(3(e+fx))+144icf^2x\sin(3(e+fx))+72idf^2x^2\sin(3(e+fx)))}{1152a^3f^2(-i+\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] ((I/1152)*Sec[e + f*x]^3*(27*((12*I)*c*f + d*(5 + (12*I)*f*x))*Cos[e + f*x]
 + 4*(6*c*f*(I + 6*f*x) + d*(1 + (6*I)*f*x + 18*f^2*x^2))*Cos[3*(e + f*x)]
+ (81*I)*d*Sin[e + f*x] - 108*c*f*Sin[e + f*x] - 108*d*f*x*Sin[e + f*x] - (
4*I)*d*Sin[3*(e + f*x)] + 24*c*f*Sin[3*(e + f*x)] + 24*d*f*x*Sin[3*(e + f*x
)] + (144*I)*c*f^2*x*Sin[3*(e + f*x)] + (72*I)*d*f^2*x^2*Sin[3*(e + f*x)]))
/(a^3*f^2*(-I + Tan[e + f*x])^3)
```

<u>Maple</u> [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(181) = 362$.
time = 0.46, size = 523, normalized size = 2.50

method	result
risch	$\frac{d\,x^2}{16a^3} + \frac{cx}{8a^3} + \frac{3i(2dxf+2cf-id)\mathrm{e}^{-2i(fx+e)}}{32a^3f^2} + \frac{3i(4dxf+4cf-id)\mathrm{e}^{-4i(fx+e)}}{128a^3f^2} + \frac{i(6dxf+6cf-id)\mathrm{e}^{-6i(fx+e)}}{288a^3f^2}$

default	$\frac{\frac{2ic\left(\cos^6(fx+e)\right)}{3}-\frac{2ide\left(\cos^6(fx+e)\right)}{3f}-\frac{4id\left(-\frac{(fx+e)\left(\cos^6(fx+e)\right)}{6}+\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3(fx+e)\right)}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{36}+\frac{5fx}{96}+\frac{5e}{96}\right)}{f}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/f*(2/3*I*c*cos(f*x+e)^6-2/3*I/f*d*e*cos(f*x+e)^6-4*I/f*d*(-1/6*(f*x+e
)*cos(f*x+e)^6+1/36*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x
+e)+5/96*f*x+5/96*e)+4*c*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e
))*sin(f*x+e)+5/16*f*x+5/16*e)-4/f*d*e*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+
15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)+4/f*d*((f*x+e)*(1/6*(cos(f*x+e
)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e)-5/32*(f*x
+e)^2+1/36*cos(f*x+e)^6+5/96*cos(f*x+e)^4+5/32*cos(f*x+e)^2)-1/4*I*c*cos(f*
x+e)^4+1/4*I/f*d*e*cos(f*x+e)^4+I/f*d*(-1/4*(f*x+e)*cos(f*x+e)^4+1/16*(cos(
f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/32*f*x+3/32*e)-3*c*(1/4*(cos(f*x+e)^3
+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+3/f*d*e*(1/4*(cos(f*x+e)^3+3/2*c
os(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/f*d*((f*x+e)*(1/4*(cos(f*x+e)^3+3/2*
cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/64*(2*cos(f*x+e)^2+3
)^2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 109, normalized size = 0.52

$$\frac{\left(24i\,dfx+24i\,cf+72\left(df^2x^2+2\,cf^2x\right)e^{(6i\,fx+6i\,e)}-108\left(-2i\,dfx-2i\,cf-d\right)e^{(4i\,fx+4i\,e)}-27\left(-4i\,dfx-4i\,cf-d\right)e^{(2i\,fx+2i\,e)}+4\,d\right)e^{(-6i\,fx-6i\,e)}}{1152\,a^3f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/1152*(24*I*d*f*x + 24*I*c*f + 72*(d*f^2*x^2 + 2*c*f^2*x)*e^(6*I*f*x + 6*I
*e) - 108*(-2*I*d*f*x - 2*I*c*f - d)*e^(4*I*f*x + 4*I*e) - 27*(-4*I*d*f*x -
 4*I*c*f - d)*e^(2*I*f*x + 2*I*e) + 4*d)*e^(-6*I*f*x - 6*I*e)/(a^3*f^2)
```

Sympy [A]
time = 0.28, size = 311, normalized size = 1.49

$$\begin{cases} \frac{((24576ia^6cf^5e^{6ie}+24576ia^6df^5xe^{6ie}+4096a^6df^4e^{6ie})e^{-6ifx}+(110592ia^6cf^5e^{8ie}+110592ia^6df^5xe^{8ie}+27648a^6df^4e^{8ie})e^{-4ifx}+(221184ia^6cf^5e^{10ie}+221184ia^6df^5xe^{10ie}+110592a^6df^4e^{10ie})e^{-2ifx})e^{-12ie}}{1179648a^9f^6} & \text{for } a^9f^6e^{12ie}\neq 0 \\ \frac{x^2\cdot(3de^{4ie}+3de^{2ie}+d)e^{-6ie}}{16a^3}+\frac{x(3ce^{4ie}+3ce^{2ie}+c)e^{-6ie}}{8a^3} & \text{otherwise} \end{cases} + \frac{cx}{8a^3} + \frac{dx^2}{16a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((((24576*I*a**6*c*f**5*exp(6*I*e) + 24576*I*a**6*d*f**5*x*exp(6*I
*e) + 4096*a**6*d*f**4*exp(6*I*e))*exp(-6*I*f*x) + (110592*I*a**6*c*f**5*ex
p(8*I*e) + 110592*I*a**6*d*f**5*x*exp(8*I*e) + 27648*a**6*d*f**4*exp(8*I*e)
)*exp(-4*I*f*x) + (221184*I*a**6*c*f**5*exp(10*I*e) + 221184*I*a**6*d*f**5*
x*exp(10*I*e) + 110592*a**6*d*f**4*exp(10*I*e))*exp(-2*I*f*x))*exp(-12*I*e)
/(1179648*a**9*f**6), Ne(a**9*f**6*exp(12*I*e), 0)), (x**2*(3*d*exp(4*I*e)
+ 3*d*exp(2*I*e) + d)*exp(-6*I*e)/(16*a**3) + x*(3*c*exp(4*I*e) + 3*c*exp(2
*I*e) + c)*exp(-6*I*e)/(8*a**3), True)) + c*x/(8*a**3) + d*x**2/(16*a**3)
```

Giac [A]
time = 0.73, size = 151, normalized size = 0.72

$$\frac{\left(72\,df^2x^2e^{(6i\,fx+6i\,e)}+144\,cf^2xe^{(6i\,fx+6i\,e)}+216i\,dfxe^{(4i\,fx+4i\,e)}+108i\,dfxe^{(2i\,fx+2i\,e)}+24i\,dfx+216i\,cfe^{(4i\,fx+4i\,e)}+108i\,cfe^{(2i\,fx+2i\,e)}+24i\,cf+108\,de^{(4i\,fx+4i\,e)}+27\,de^{(2i\,fx+2i\,e)}+4\,d\right)e^{(-6i\,fx-6i\,e)}}{1152\,a^3f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/1152*(72*d*f^2*x^2*e^(6*I*f*x + 6*I*e) + 144*c*f^2*x*e^(6*I*f*x + 6*I*e)
+ 216*I*d*f*x*e^(4*I*f*x + 4*I*e) + 108*I*d*f*x*e^(2*I*f*x + 2*I*e) + 24*I*
d*f*x + 216*I*c*f*e^(4*I*f*x + 4*I*e) + 108*I*c*f*e^(2*I*f*x + 2*I*e) + 24*
I*c*f + 108*d*e^(4*I*f*x + 4*I*e) + 27*d*e^(2*I*f*x + 2*I*e) + 4*d)*e^(-6*I
*f*x - 6*I*e)/(a^3*f^2)
```

Mupad [B]
time = 3.03, size = 146, normalized size = 0.70

$$\frac{d\,x^2}{16\,a^3}-{\mathrm{e}}^{-e\,4\mathrm{i}-f\,x\,4\mathrm{i}}\,\left(\frac{\left(-12\,c\,f+d\,3\mathrm{i}\right)\,1\mathrm{i}}{128\,a^3\,f^2}-\frac{d\,x\,3\mathrm{i}}{32\,a^3\,f}\right)-{\mathrm{e}}^{-e\,6\mathrm{i}-f\,x\,6\mathrm{i}}\,\left(\frac{\left(-6\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{288\,a^3\,f^2}-\frac{d\,x\,1\mathrm{i}}{48\,a^3\,f}\right)-{\mathrm{e}}^{-e\,2\mathrm{i}-f\,x\,2\mathrm{i}}\,\left(\frac{\left(-6\,c\,f+d\,3\mathrm{i}\right)\,1\mathrm{i}}{32\,a^3\,f^2}-\frac{d\,x\,3\mathrm{i}}{16\,a^3\,f}\right)+\frac{c\,x}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*tan(e + f*x)*1i)^3,x)
```

```
[Out] (d*x^2)/(16*a^3) - exp(- e*4i - f*x*4i)*(((d*3i - 12*c*f)*1i)/(128*a^3*f^2)
 - (d*x*3i)/(32*a^3*f)) - exp(- e*6i - f*x*6i)*(((d*1i - 6*c*f)*1i)/(288*a^
3*f^2) - (d*x*1i)/(48*a^3*f)) - exp(- e*2i - f*x*2i)*(((d*3i - 6*c*f)*1i)/(
32*a^3*f^2) - (d*x*3i)/(16*a^3*f)) + (c*x)/(8*a^3)
```

3.32 $\int \frac{1}{(c+dx)(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=449

$$\frac{3\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{8a^3d}+\frac{3\cos\left(4e-\frac{4cf}{d}\right)\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)}{8a^3d}+\frac{\cos\left(6e-\frac{6cf}{d}\right)\text{CosInteg}}{8a^3d}$$

```
[Out] 1/8*Ci(6*c*f/d+6*f*x)*cos(-6*e+6*c*f/d)/a^3/d+3/8*Ci(4*c*f/d+4*f*x)*cos(-4*
e+4*c*f/d)/a^3/d+3/8*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^3/d+1/8*ln(d*x+c
)/a^3/d-3/8*I*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^3/d-3/8*I*cos(-4*e+4*c*
f/d)*Si(4*c*f/d+4*f*x)/a^3/d-1/8*I*cos(-6*e+6*c*f/d)*Si(6*c*f/d+6*f*x)/a^3/
d+1/8*I*Ci(6*c*f/d+6*f*x)*sin(-6*e+6*c*f/d)/a^3/d+1/8*Si(6*c*f/d+6*f*x)*sin
(-6*e+6*c*f/d)/a^3/d+3/8*I*Ci(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d+3/8*Si
(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d+3/8*I*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*
c*f/d)/a^3/d+3/8*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^3/d
```

Rubi [A]
time = 1.28, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3809, 3384, 3380, 3383, 3393, 4491, 4513}

$$-\frac{3i\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{8a^3d}-\frac{i\text{CosIntegral}\left(\frac{6cf}{d}+6fx\right)\sin\left(6e-\frac{6cf}{d}\right)}{8a^3d}-\frac{3i\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4cf}{d}\right)}{8a^3d}+\frac{3\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{8a^3d}+\frac{3\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\cos\left(4e-\frac{4cf}{d}\right)}{8a^3d}+\frac{\text{CosIntegral}\left(\frac{6cf}{d}+6fx\right)\cos\left(6e-\frac{6cf}{d}\right)}{8a^3d}-\frac{3\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{8a^3d}-\frac{3\sin\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{8a^3d}-\frac{\sin\left(6e-\frac{6cf}{d}\right)\text{Si}\left(6xf+\frac{6cf}{d}\right)}{8a^3d}-\frac{3i\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{8a^3d}-\frac{3i\cos\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{8a^3d}-\frac{i\cos\left(6e-\frac{6cf}{d}\right)\text{Si}\left(6xf+\frac{6cf}{d}\right)}{8a^3d}+\frac{\log(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + I*a*Tan[e + f*x])^3),x]
```

```
[Out] (3*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*Cos[
4*e - (4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (Cos[6*e - (6*
c*f)/d]*CosIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + Log[c + d*x]/(8*a^3*d)
- ((I/8)*CosIntegral[(6*c*f)/d + 6*f*x]*Sin[6*e - (6*c*f)/d])/(a^3*d) - (((
3*I)/8)*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])/(a^3*d) - (((3
*I)/8)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a^3*d) - (((3*
I)/8)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^3*d) - (3*Sin
[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) - (((3*I)/8)*Co
s[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(a^3*d) - (3*Sin[4*e - (
4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - ((I/8)*Cos[6*e - (6*c
*f)/d]*SinIntegral[(6*c*f)/d + 6*f*x])/(a^3*d) - (Sin[6*e - (6*c*f)/d]*SinI
ntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4513

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d
_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x
]^p*Sin[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] &&
 IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia\tan(e+fx))^3}\,dx = \int \left(\frac{1}{8a^3(c+dx)} + \frac{3\cos(2e+2fx)}{8a^3(c+dx)} + \frac{3\cos^2(2e+2fx)}{8a^3(c+dx)} + \frac{\cos^3(2e+\ldots}{8a^3(c+\ldots}\right.$$

$$= \frac{\log(c+dx)}{8a^3d} + \frac{i\int \frac{\sin^3(2e+2fx)}{c+dx}\,dx}{8a^3} - \frac{(3i)\int \frac{\sin(2e+2fx)}{c+dx}\,dx}{8a^3} - \frac{(3i)\int \frac{\cos^2(2\ldots}{}}{}$$

$$= \frac{\log(c+dx)}{8a^3d} + \frac{i\int \left(\frac{3\sin(2e+2fx)}{4(c+dx)} - \frac{\sin(6e+6fx)}{4(c+dx)}\right)dx}{8a^3} - \frac{(3i)\int \left(\frac{\sin(2e+2fx)}{4(c+dx)}\ldots\right.}{8\ldots}$$

$$= \frac{3\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} - \frac{3i\operatorname{Ci}\left(\frac{4cf}{d}+4fx\right)\sin\ldots}{8a^3d}$$

$$= \frac{3\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} - \frac{3i\operatorname{Ci}\left(\frac{4cf}{d}+4fx\right)\sin\ldots}{8a^3d}$$

$$= \frac{3\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Ci}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{\cos\left(6e-\frac{6cf}{d}\right)\operatorname{Ci}\left(\frac{6cf}{d}+6fx\right)}{8a^3d} + \frac{\log\ldots}{}$$

Mathematica [A]
time = 0.85, size = 336, normalized size = 0.75

$$\frac{\sec^3(e+fx)(\cos(fx)+i\sin(fx))^3\left(\cos(3e)\log(f(c+dx))+i\log(f(c+dx))\sin(3e)+\left(\cos\left(e-\frac{4cf}{d}\right)-i\sin\left(e-\frac{4cf}{d}\right)\right)\left(3\text{CosIntegral}\left(\frac{4f(c+dx)}{d}\right)+\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{6f(c+dx)}{d}\right)+3\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\left(\cos\left(2e-\frac{2cf}{d}\right)+i\sin\left(2e-\frac{2cf}{d}\right)\right)-i\text{CosIntegral}\left(\frac{6f(c+dx)}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)-3i\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)+3\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2f(c+dx)}{d}\right)-3i\text{Si}\left(\frac{4f(c+dx)}{d}\right)-i\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{6f(c+dx)}{d}\right)-\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{6f(c+dx)}{d}\right)\right)\right)}{8d(a+ia\tan(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Tan[e + f*x])^3),x]
```

```
[Out] (Sec[e + f*x]^3*(Cos[f*x] + I*Sin[f*x])^3*(Cos[3*e]*Log[f*(c + d*x)] + I*Lo
g[f*(c + d*x)]*Sin[3*e] + (Cos[e - (4*c*f)/d] - I*Sin[e - (4*c*f)/d])*(3*Co
sIntegral[(4*f*(c + d*x))/d] + Cos[2*e - (2*c*f)/d]*CosIntegral[(6*f*(c + d
*x))/d] + 3*CosIntegral[(2*f*(c + d*x))/d]*(Cos[2*e - (2*c*f)/d] + I*Sin[2*
e - (2*c*f)/d]) - I*CosIntegral[(6*f*(c + d*x))/d]*Sin[2*e - (2*c*f)/d] - (
3*I)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 3*Sin[2*e - (2*c
*f)/d]*SinIntegral[(2*f*(c + d*x))/d] - (3*I)*SinIntegral[(4*f*(c + d*x))/d
] - I*Cos[2*e - (2*c*f)/d]*SinIntegral[(6*f*(c + d*x))/d] - Sin[2*e - (2*c*
f)/d]*SinIntegral[(6*f*(c + d*x))/d])))/(8*d*(a + I*a*Tan[e + f*x])^3)
```

Maple [A]
time = 0.52, size = 550, normalized size = 1.22

method	result
risch	$\frac{\ln(dx+c)}{8a^3d} - \frac{e^{\frac{6i(cf-de)}{d}}\,\text{expIntegral}\left(1,6ifx+6ie+\frac{6i(cf-de)}{d}\right)}{8a^3d} - \frac{3\,e^{\frac{4i(cf-de)}{d}}\,\text{expIntegral}\left(1,4ifx+4ie+\frac{4i(cf-de)}{d}\right)}{8a^3d} - \frac{3\,e^{\frac{2i(cf-\ldots}{d}}}{}$

default	$-\frac{3if\left(\frac{2\,\mathrm{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}\right)}{16}-\frac{3if\left(\frac{4\,\mathrm{sinIntegral}\left(4fx+4e+\frac{4cf-}{}\right.}{d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/f*(-3/16*I*f*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci
(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)-3/32*I*f*(4*Si(4*f*x+4*e+4*
(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d-4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-
d*e)/d)/d)-1/48*I*f*(6*Si(6*f*x+6*e+6*(c*f-d*e)/d)*cos(6*(c*f-d*e)/d)/d-6*C
i(6*f*x+6*e+6*(c*f-d*e)/d)*sin(6*(c*f-d*e)/d)/d)+1/48*f*(6*Si(6*f*x+6*e+6*(
c*f-d*e)/d)*sin(6*(c*f-d*e)/d)/d+6*Ci(6*f*x+6*e+6*(c*f-d*e)/d)*cos(6*(c*f-d
*e)/d)/d)+3/32*f*(4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d+4*Ci(4
*f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d)+3/16*f*(2*Si(2*f*x+2*e+2*(c*f
-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)
/d)/d)+1/8*f*ln(c*f-d*e+d*(f*x+e))/d)
```

Maxima [A]
time = 0.39, size = 296, normalized size = 0.66

$$-\frac{3\,f\cos\left(\frac{2\,(cf-de)}{d}\right)E_1\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)+3\,f\cos\left(\frac{4\,(cf-de)}{d}\right)E_1\left(-\frac{4\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)+f\cos\left(\frac{6\,(cf-de)}{d}\right)E_1\left(-\frac{6\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)+i\,fE_1\left(-\frac{6\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{6\,(cf-de)}{d}\right)+3i\,fE_1\left(-\frac{4\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{4\,(cf-de)}{d}\right)+3i\,fE_1\left(-\frac{2\,(-i\,(fx+e)d-i\,cf+i\,de)}{d}\right)\sin\left(\frac{2\,(cf-de)}{d}\right)-f\log\left((fx+e)d+cf-de\right)}{8\,a^3df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*f*cos(2*(c*f - d*e)/d)*exp_integral_e(1, -2*(-I*(f*x + e)*d - I*c*f
 + I*d*e)/d) + 3*f*cos(4*(c*f - d*e)/d)*exp_integral_e(1, -4*(-I*(f*x + e)*
d - I*c*f + I*d*e)/d) + f*cos(6*(c*f - d*e)/d)*exp_integral_e(1, -6*(-I*(f*
x + e)*d - I*c*f + I*d*e)/d) + I*f*exp_integral_e(1, -6*(-I*(f*x + e)*d - I
*c*f + I*d*e)/d)*sin(6*(c*f - d*e)/d) + 3*I*f*exp_integral_e(1, -4*(-I*(f*x
 + e)*d - I*c*f + I*d*e)/d)*sin(4*(c*f - d*e)/d) + 3*I*f*exp_integral_e(1,
-2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d)*sin(2*(c*f - d*e)/d) - f*log((f*x +
e)*d + c*f - d*e))/(a^3*d*f)
```

Fricas [A]
time = 0.35, size = 120, normalized size = 0.27

$$\frac{3\,\mathrm{Ei}\left(-\frac{2\,(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{2\,(-i\,cf+i\,de)}{d}\right)}+3\,\mathrm{Ei}\left(-\frac{4\,(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{4\,(-i\,cf+i\,de)}{d}\right)}+\mathrm{Ei}\left(-\frac{6\,(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{6\,(-i\,cf+i\,de)}{d}\right)}+\log\left(\frac{dx+c}{d}\right)}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*Ei(-2*(I*d*f*x + I*c*f)/d)*e^(-2*(-I*c*f + I*d*e)/d) + 3*Ei(-4*(I*d*
f*x + I*c*f)/d)*e^(-4*(-I*c*f + I*d*e)/d) + Ei(-6*(I*d*f*x + I*c*f)/d)*e^(-
6*(-I*c*f + I*d*e)/d) + log((d*x + c)/d))/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{1}{c\tan^3(e+fx)-3ic\tan^2(e+fx)-3c\tan(e+fx)+ic+dx\tan^3(e+fx)-3idx\tan^2(e+fx)-3dx\tan(e+fx)+idx}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral(1/(c*tan(e + f*x)**3 - 3*I*c*tan(e + f*x)**2 - 3*c*tan(e + f*x)
+ I*c + d*x*tan(e + f*x)**3 - 3*I*d*x*tan(e + f*x)**2 - 3*d*x*tan(e + f*x)
+ I*d*x), x)/a**3
```

Giac [A]
time = 0.68, size = 846, normalized size = 1.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*cos(2*c*f/d)*cos(2*e)^2*cos_integral(-2*(d*f*x + c*f)/d) + cos(2*e)^
3*log(d*x + c) + 3*I*cos(2*e)^2*cos_integral(-2*(d*f*x + c*f)/d)*sin(2*c*f/
d) + 6*I*cos(2*c*f/d)*cos(2*e)*cos_integral(-2*(d*f*x + c*f)/d)*sin(2*e) +
3*I*cos(2*e)^2*log(d*x + c)*sin(2*e) - 6*cos(2*e)*cos_integral(-2*(d*f*x +
c*f)/d)*sin(2*c*f/d)*sin(2*e) - 3*cos(2*c*f/d)*cos_integral(-2*(d*f*x + c*f
)/d)*sin(2*e)^2 - 3*cos(2*e)*log(d*x + c)*sin(2*e)^2 - 3*I*cos_integral(-2*
(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(2*e)^2 - I*log(d*x + c)*sin(2*e)^3 - 3*I*
cos(2*c*f/d)*cos(2*e)^2*sin_integral(2*(d*f*x + c*f)/d) + 3*cos(2*e)^2*sin(
2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 6*cos(2*c*f/d)*cos(2*e)*sin(2*e)
*sin_integral(2*(d*f*x + c*f)/d) + 6*I*cos(2*e)*sin(2*c*f/d)*sin(2*e)*sin_i
ntegral(2*(d*f*x + c*f)/d) + 3*I*cos(2*c*f/d)*sin(2*e)^2*sin_integral(2*(d*
f*x + c*f)/d) - 3*sin(2*c*f/d)*sin(2*e)^2*sin_integral(2*(d*f*x + c*f)/d) +
 3*cos(4*c*f/d)*cos(2*e)*cos_integral(-4*(d*f*x + c*f)/d) + 3*I*cos(2*e)*co
s_integral(-4*(d*f*x + c*f)/d)*sin(4*c*f/d) + 3*I*cos(4*c*f/d)*cos_integral
(-4*(d*f*x + c*f)/d)*sin(2*e) - 3*cos_integral(-4*(d*f*x + c*f)/d)*sin(4*c*
f/d)*sin(2*e) - 3*I*cos(4*c*f/d)*cos(2*e)*sin_integral(4*(d*f*x + c*f)/d) +
 3*cos(2*e)*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 3*cos(4*c*f/d)*s
in(2*e)*sin_integral(4*(d*f*x + c*f)/d) + 3*I*sin(4*c*f/d)*sin(2*e)*sin_int
egral(4*(d*f*x + c*f)/d) + cos(6*c*f/d)*cos_integral(-6*(d*f*x + c*f)/d) +
I*cos_integral(-6*(d*f*x + c*f)/d)*sin(6*c*f/d) - I*cos(6*c*f/d)*sin_integr
al(6*(d*f*x + c*f)/d) + sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d))/(a^3*
d*cos(2*e)^3 + 3*I*a^3*d*cos(2*e)^2*sin(2*e) - 3*a^3*d*cos(2*e)*sin(2*e)^2
- I*a^3*d*sin(2*e)^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + a\tan(e + f\,x)\ 1\mathrm{i})^3\ (c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c + d*x)),x)
[Out] int(1/((a + a*tan(e + f*x)*1i)^3*(c + d*x)), x)
```

3.33 $\int \frac{1}{(c+dx)^2(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=712

$$-\frac{1}{8a^3d(c+dx)}-\frac{9\cos(2e+2fx)}{32a^3d(c+dx)}-\frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)}-\frac{\cos^3(2e+2fx)}{8a^3d(c+dx)}-\frac{3\cos(6e+6fx)}{32a^3d(c+dx)}-\frac{3if\cos\left(2e-\frac{2cf}{d}\right)}{}$$

```
[Out] -1/8/a^3/d/(d*x+c)-3/4*I*f*Si(6*c*f/d+6*f*x)*sin(-6*e+6*c*f/d)/a^3/d^2-3/4*
I*f*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^3/d^2-1/8*I*sin(2*f*x+2*e)^3/a^3/
d/(d*x+c)-9/32*cos(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*cos(2*f*x+2*e)^2/a^3/d/(d*x
+c)-1/8*cos(2*f*x+2*e)^3/a^3/d/(d*x+c)-3/32*cos(6*f*x+6*e)/a^3/d/(d*x+c)-3/
4*f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^3/d^2-3/2*f*cos(-4*e+4*c*f/d)*Si(
4*c*f/d+4*f*x)/a^3/d^2-3/4*f*cos(-6*e+6*c*f/d)*Si(6*c*f/d+6*f*x)/a^3/d^2+3/
4*f*Ci(6*c*f/d+6*f*x)*sin(-6*e+6*c*f/d)/a^3/d^2+3/32*I*sin(6*f*x+6*e)/a^3/d
/(d*x+c)+3/2*f*Ci(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d^2-3/4*I*f*Si(2*c*f
/d+2*f*x)*sin(-2*e+2*c*f/d)/a^3/d^2+3/4*f*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/
d)/a^3/d^2-3/2*I*f*Si(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d^2-3/2*I*f*Ci(4
*c*f/d+4*f*x)*cos(-4*e+4*c*f/d)/a^3/d^2+3/8*sin(2*f*x+2*e)^2/a^3/d/(d*x+c)+
15/32*I*sin(2*f*x+2*e)/a^3/d/(d*x+c)-3/4*I*f*Ci(6*c*f/d+6*f*x)*cos(-6*e+6*c
*f/d)/a^3/d^2+3/8*I*sin(4*f*x+4*e)/a^3/d/(d*x+c)
```

Rubi [A]
time = 1.25, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3809, 3378, 3384, 3380, 3383, 3394, 12, 4491, 4513}

$$-\frac{3f\,\text{CosIntegral}\left(\frac{6cf}{d}+6fx\right)\sin\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}-\frac{3f\,\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}-\frac{3f\,\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3if\,\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3if\,\text{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\cos\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}-\frac{3if\,\text{CosIntegral}\left(\frac{6cf}{d}+6fx\right)\cos\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}+\frac{3if\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{4a^3d^2}+\frac{3if\sin\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{2a^3d^2}+\frac{3if\sin\left(6e-\frac{6cf}{d}\right)\text{Si}\left(6xf+\frac{6cf}{d}\right)}{4a^3d^2}-\frac{3f\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf+\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3f\cos\left(4e-\frac{4cf}{d}\right)\text{Si}\left(4xf+\frac{4cf}{d}\right)}{2a^3d^2}-\frac{3f\cos\left(6e-\frac{6cf}{d}\right)\text{Si}\left(6xf+\frac{6cf}{d}\right)}{4a^3d^2}-\frac{i\sin^3(2e+2fx)}{8a^3d(c+dx)}+\frac{3\sin^2(2e+2fx)}{8a^3d(c+dx)}+\frac{15i\sin(2e+2fx)}{32a^3d(c+dx)}+\frac{3i\sin(4e+4fx)}{8a^3d(c+dx)}+\frac{3i\sin(6e+6fx)}{32a^3d(c+dx)}-\frac{\cos^3(2e+2fx)}{8a^3d(c+dx)}-\frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)}-\frac{9\cos(2e+2fx)}{32a^3d(c+dx)}-\frac{3\cos(6e+6fx)}{32a^3d(c+dx)}-\frac{1}{8a^3d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Tan[e + f*x])^3),x]
```

```
[Out] -1/8*1/(a^3*d*(c + d*x)) - (9*Cos[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3*C
os[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) - Cos[2*e + 2*f*x]^3/(8*a^3*d*(c + d
*x)) - (3*Cos[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) - (((3*I)/4)*f*Cos[2*e - (
2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a^3*d^2) - (((3*I)/2)*f*Cos[4*e
- (4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(a^3*d^2) - (((3*I)/4)*f*Cos[6
*e - (6*c*f)/d]*CosIntegral[(6*c*f)/d + 6*f*x])/(a^3*d^2) - (3*f*CosIntegra
l[(6*c*f)/d + 6*f*x]*Sin[6*e - (6*c*f)/d])/(4*a^3*d^2) - (3*f*CosIntegral[(
4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])/(2*a^3*d^2) - (3*f*CosIntegral[(2*c
*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(4*a^3*d^2) + (((15*I)/32)*Sin[2*e + 2
*f*x])/(a^3*d*(c + d*x)) + (3*Sin[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) - ((I
/8)*Sin[2*e + 2*f*x]^3)/(a^3*d*(c + d*x)) + (((3*I)/8)*Sin[4*e + 4*f*x])/(a
^3*d*(c + d*x)) + (((3*I)/32)*Sin[6*e + 6*f*x])/(a^3*d*(c + d*x)) - (3*f*Co
s[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) + (((3*I)/4)
*f*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^3*d^2) - (3*f*Co
```

```
s[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) + (((3*I)/2)
*f*Sin[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(a^3*d^2) - (3*f*Co
s[6*e - (6*c*f)/d]*SinIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) + (((3*I)/4)
*f*Sin[6*e - (6*c*f)/d]*SinIntegral[(6*c*f)/d + 6*f*x])/(a^3*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
```

```
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4513

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d
_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x
]^p*Sin[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] &&
 IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(c+dx)^2(a+ia\tan(e+fx))^3}\,dx &= \int \left(\frac{1}{8a^3(c+dx)^2} + \frac{3\cos(2e+2fx)}{8a^3(c+dx)^2} + \frac{3\cos^2(2e+2fx)}{8a^3(c+dx)^2} + \frac{\cos^3(2e\cdots}{8a^3(c\cdots}\right.\\
&= -\frac{1}{8a^3d(c+dx)} + \frac{i\int \frac{\sin^3(2e+2fx)}{(c+dx)^2}\,dx}{8a^3} - \frac{(3i)\int \frac{\sin(2e+2fx)}{(c+dx)^2}\,dx}{8a^3} - \frac{(3i)\int\cdots}{}\\
&= -\frac{1}{8a^3d(c+dx)} - \frac{3\cos(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} - \frac{\cos^3(2e+\cdots}{8a^3d(c+\cdots}\\
&= -\frac{1}{8a^3d(c+dx)} - \frac{3\cos(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} - \frac{\cos^3(2e+\cdots}{8a^3d(c+\cdots}\\
&= -\frac{1}{8a^3d(c+dx)} - \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} - \frac{\cos^3(2e+\cdots}{8a^3d(c+\cdots}\\
&= -\frac{1}{8a^3d(c+dx)} - \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} - \frac{\cos^3(2e+\cdots}{8a^3d(c+\cdots}\\
&= -\frac{1}{8a^3d(c+dx)} - \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} - \frac{\cos^3(2e+\cdots}{8a^3d(c+\cdots}
\end{aligned}$$

Mathematica [A]
time = 3.28, size = 833, normalized size = 1.17

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Tan[e + f*x])^3),x]
```

```
[Out] (Sec[e + f*x]^3*((-I)*Cos[(3*c*f)/d] + Sin[(3*c*f)/d])*(3*d*Cos[e + f*((-3*
c)/d + x)] + d*Cos[3*(e + f*(-(c/d) + x))] + d*Cos[3*(e + f*(c/d + x))] + 3
*d*Cos[e + f*((3*c)/d + x)] + (6*I)*c*f*Cos[3*e - (3*f*(c + d*x))/d]*CosInt
egral[(6*f*(c + d*x))/d] + (6*I)*d*f*x*Cos[3*e - (3*f*(c + d*x))/d]*CosInte
gral[(6*f*(c + d*x))/d] + (6*I)*f*(c + d*x)*CosIntegral[(2*f*(c + d*x))/d]*
(Cos[e - (c*f)/d + 3*f*x] + I*Sin[e - (c*f)/d + 3*f*x]) + (3*I)*d*Sin[e + f
*((-3*c)/d + x)] + I*d*Sin[3*(e + f*(-(c/d) + x))] - I*d*Sin[3*(e + f*(c/d
+ x))] - (3*I)*d*Sin[e + f*((3*c)/d + x)] + 6*c*f*CosIntegral[(6*f*(c + d*x
))/d]*Sin[3*e - (3*f*(c + d*x))/d] + 6*d*f*x*CosIntegral[(6*f*(c + d*x))/d]
*Sin[3*e - (3*f*(c + d*x))/d] + 12*f*(c + d*x)*CosIntegral[(4*f*(c + d*x))/
d]*(I*Cos[e - (f*(c + 3*d*x))/d] + Sin[e - (f*(c + 3*d*x))/d]) + 6*c*f*Cos[
e - (c*f)/d + 3*f*x]*SinIntegral[(2*f*(c + d*x))/d] + 6*d*f*x*Cos[e - (c*f)
/d + 3*f*x]*SinIntegral[(2*f*(c + d*x))/d] + (6*I)*c*f*Sin[e - (c*f)/d + 3*
f*x]*SinIntegral[(2*f*(c + d*x))/d] + (6*I)*d*f*x*Sin[e - (c*f)/d + 3*f*x]*
SinIntegral[(2*f*(c + d*x))/d] + 12*c*f*Cos[e - (f*(c + 3*d*x))/d]*SinInteg
ral[(4*f*(c + d*x))/d] + 12*d*f*x*Cos[e - (f*(c + 3*d*x))/d]*SinIntegral[(4
*f*(c + d*x))/d] - (12*I)*c*f*Sin[e - (f*(c + 3*d*x))/d]*SinIntegral[(4*f*(
c + d*x))/d] - (12*I)*d*f*x*Sin[e - (f*(c + 3*d*x))/d]*SinIntegral[(4*f*(c
+ d*x))/d] + 6*c*f*Cos[3*e - (3*f*(c + d*x))/d]*SinIntegral[(6*f*(c + d*x))
/d] + 6*d*f*x*Cos[3*e - (3*f*(c + d*x))/d]*SinIntegral[(6*f*(c + d*x))/d] -
 (6*I)*c*f*Sin[3*e - (3*f*(c + d*x))/d]*SinIntegral[(6*f*(c + d*x))/d] - (6
*I)*d*f*x*Sin[3*e - (3*f*(c + d*x))/d]*SinIntegral[(6*f*(c + d*x))/d]))/(8*
a^3*d^2*(c + d*x)*(-I + Tan[e + f*x])^3)
```

Maple [A]

time = 0.57, size = 787, normalized size = 1.11

method	result
risch	$-\frac{1}{8a^3d(dx+c)}-\frac{f\,e^{-6i(fx+e)}}{8a^3(dxf+cf)d}+\frac{3if\,e^{\frac{6i(cf-de)}{d}}\,\text{expIntegral}\left(1,6ifx+6ie+\frac{6i(cf-de)}{d}\right)}{4a^3d^2}-\frac{3f\,e^{-4i(fx+e)}}{8a^3(dxf+cf)d}+\frac{3if\,e^{\frac{4i(cf-de)}{d}}\,\text{exp}$
default	$\frac{-\frac{3if^2\left(-\frac{2\sin(2fx+2e)}{(cf-de+d(fx+e))d}+\frac{\frac{4\,\text{sinIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{4\,\text{cosineIntegral}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}}{d}\right)}{16}-\frac{3if^2\left(-\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/f*(-3/16*I*f^2*(-2*sin(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d+2*(2*Si(2*f*x
+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(
2*(c*f-d*e)/d)/d)/d)-3/32*I*f^2*(-4*sin(4*f*x+4*e)/(c*f-d*e+d*(f*x+e))/d+4*
```

```
(4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d+4*Ci(4*f*x+4*e+4*(c*f-d
*e)/d)*cos(4*(c*f-d*e)/d)/d)/d)-1/48*I*f^2*(-6*sin(6*f*x+6*e)/(c*f-d*e+d*(f
*x+e))/d+6*(6*Si(6*f*x+6*e+6*(c*f-d*e)/d)*sin(6*(c*f-d*e)/d)/d+6*Ci(6*f*x+6
*e+6*(c*f-d*e)/d)*cos(6*(c*f-d*e)/d)/d)/d)+1/48*f^2*(-6*cos(6*f*x+6*e)/(c*f
-d*e+d*(f*x+e))/d-6*(6*Si(6*f*x+6*e+6*(c*f-d*e)/d)*cos(6*(c*f-d*e)/d)/d-6*C
i(6*f*x+6*e+6*(c*f-d*e)/d)*sin(6*(c*f-d*e)/d)/d)/d)+3/32*f^2*(-4*cos(4*f*x+
4*e)/(c*f-d*e+d*(f*x+e))/d-4*(4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)
/d)/d-4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d)/d)+3/16*f^2*(-2*c
os(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*
(c*f-d*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d)-1/8*f
^2/(c*f-d*e+d*(f*x+e))/d)
```

Maxima [A]
time = 0.47, size = 317, normalized size = 0.45

$$-\frac{3 f^2 \cos\left(\frac{2(cf-de)}{d}\right) E_2\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right) + 3 f^2 \cos\left(\frac{4(cf-de)}{d}\right) E_2\left(-\frac{4(-i(fx+e)d-icf+ide)}{d}\right) + f^2 \cos\left(\frac{6(cf-de)}{d}\right) E_2\left(-\frac{6(-i(fx+e)d-icf+ide)}{d}\right) + i f^2 E_2\left(-\frac{6(-i(fx+e)d-icf+ide)}{d}\right) \sin\left(\frac{6(cf-de)}{d}\right) + 3i f^2 E_2\left(-\frac{4(-i(fx+e)d-icf+ide)}{d}\right) \sin\left(\frac{4(cf-de)}{d}\right) + 3i f^2 E_2\left(-\frac{2(-i(fx+e)d-icf+ide)}{d}\right) \sin\left(\frac{2(cf-de)}{d}\right) + f^2}{8\left((fx+e)a^3d^2 + a^3cdf - a^3d^2e\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*f^2*cos(2*(c*f - d*e)/d)*exp_integral_e(2, -2*(-I*(f*x + e)*d - I*c
*f + I*d*e)/d) + 3*f^2*cos(4*(c*f - d*e)/d)*exp_integral_e(2, -4*(-I*(f*x +
 e)*d - I*c*f + I*d*e)/d) + f^2*cos(6*(c*f - d*e)/d)*exp_integral_e(2, -6*(
-I*(f*x + e)*d - I*c*f + I*d*e)/d) + I*f^2*exp_integral_e(2, -6*(-I*(f*x +
e)*d - I*c*f + I*d*e)/d)*sin(6*(c*f - d*e)/d) + 3*I*f^2*exp_integral_e(2, -
4*(-I*(f*x + e)*d - I*c*f + I*d*e)/d)*sin(4*(c*f - d*e)/d) + 3*I*f^2*exp_in
tegral_e(2, -2*(-I*(f*x + e)*d - I*c*f + I*d*e)/d)*sin(2*(c*f - d*e)/d) + f
^2)/(((f*x + e)*a^3*d^2 + a^3*c*d*f - a^3*d^2*e)*f)
```

Fricas [A]
time = 0.37, size = 204, normalized size = 0.29

$$-\frac{\left(\left(6\,(i\,dfx+i\,cf)\mathrm{Ei}\left(-\frac{2(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{2(-i\,cf+i\,de)}{d}\right)} + 12\,(i\,dfx+i\,cf)\mathrm{Ei}\left(-\frac{4(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{4(-i\,cf+i\,de)}{d}\right)} + 6\,(i\,dfx+i\,cf)\mathrm{Ei}\left(-\frac{6(i\,dfx+i\,cf)}{d}\right)e^{\left(-\frac{6(-i\,cf+i\,de)}{d}\right)} + d\right)e^{(6i\,fx+6i\,e)} + 3\,de^{(4i\,fx+4i\,e)} + 3\,de^{(2i\,fx+2i\,e)} + d\right)e^{(-6i\,fx-6i\,e)}}{8\,(a^3d^3x + a^3cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/8*((6*(I*d*f*x + I*c*f)*Ei(-2*(I*d*f*x + I*c*f)/d)*e^(-2*(-I*c*f + I*d*e
)/d) + 12*(I*d*f*x + I*c*f)*Ei(-4*(I*d*f*x + I*c*f)/d)*e^(-4*(-I*c*f + I*d*
e)/d) + 6*(I*d*f*x + I*c*f)*Ei(-6*(I*d*f*x + I*c*f)/d)*e^(-6*(-I*c*f + I*d*
e)/d) + d)*e^(6*I*f*x + 6*I*e) + 3*d*e^(4*I*f*x + 4*I*e) + 3*d*e^(2*I*f*x +
 2*I*e) + d)*e^(-6*I*f*x - 6*I*e)/(a^3*d^3*x + a^3*c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{1}{c^2\tan^3(e+fx) - 3ic^2\tan^2(e+fx) - 3c^2\tan(e+fx) + ic^2 + 2cdx\tan^3(e+fx) - 6icdx\tan^2(e+fx) - 6cdx\tan(e+fx) + 2icdx + d^2x^2\tan^3(e+fx) - 3id^2x^2\tan^2(e+fx) - 3d^2x^2\tan(e+fx) + id^2x^2}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral(1/(c**2*tan(e + f*x)**3 - 3*I*c**2*tan(e + f*x)**2 - 3*c**2*tan(
e + f*x) + I*c**2 + 2*c*d*x*tan(e + f*x)**3 - 6*I*c*d*x*tan(e + f*x)**2 - 6
*c*d*x*tan(e + f*x) + 2*I*c*d*x + d**2*x**2*tan(e + f*x)**3 - 3*I*d**2*x**2
*tan(e + f*x)**2 - 3*d**2*x**2*tan(e + f*x) + I*d**2*x**2), x)/a**3
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3165 vs. $2(669) = 1338$.
time = 33.04, size = 3165, normalized size = 4.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*(6*I*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*cos(2*(c*f - d*
e)/d)*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f
+ d*e)/d) - 6*I*c*f^3*cos(2*(c*f - d*e)/d)*cos_integral(-2*((d*x + c)*(c*f/
(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 12*I*(d*x + c)*(c*f/(d*x +
 c) - f - d*e/(d*x + c))*f^2*cos(4*(c*f - d*e)/d)*cos_integral(-4*((d*x + c
)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - 12*I*c*f^3*cos(4*(c
*f - d*e)/d)*cos_integral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))
 - c*f + d*e)/d) + 6*I*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*co
s(6*(c*f - d*e)/d)*cos_integral(-6*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x
 + c)) - c*f + d*e)/d) - 6*I*c*f^3*cos(6*(c*f - d*e)/d)*cos_integral(-6*((d
*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 6*I*d*f^2*cos
(2*(c*f - d*e)/d)*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x
+ c)) - c*f + d*e)/d)*e + 12*I*d*f^2*cos(4*(c*f - d*e)/d)*cos_integral(-4*(
(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e + 6*I*d*f^2
*cos(6*(c*f - d*e)/d)*cos_integral(-6*((d*x + c)*(c*f/(d*x + c) - f - d*e/(
d*x + c)) - c*f + d*e)/d)*e - 6*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c
))*f^2*cos_integral(-6*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f
 + d*e)/d)*sin(6*(c*f - d*e)/d) + 6*c*f^3*cos_integral(-6*((d*x + c)*(c*f/(
d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*sin(6*(c*f - d*e)/d) - 6*d*f^
2*cos_integral(-6*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*
e)/d)*e*sin(6*(c*f - d*e)/d) - 12*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x +
 c))*f^2*cos_integral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c
*f + d*e)/d)*sin(4*(c*f - d*e)/d) + 12*c*f^3*cos_integral(-4*((d*x + c)*(c*
f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*sin(4*(c*f - d*e)/d) - 12*
d*f^2*cos_integral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f
+ d*e)/d)*e*sin(4*(c*f - d*e)/d) - 6*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*
x + c))*f^2*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))
- c*f + d*e)/d)*sin(2*(c*f - d*e)/d) + 6*c*f^3*cos_integral(-2*((d*x + c)*(
```

```
c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*sin(2*(c*f - d*e)/d) - 6
*d*f^2*cos_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f
 + d*e)/d)*e*sin(2*(c*f - d*e)/d) + 6*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d
*x + c))*f^2*cos(2*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x + c)
 - f - d*e/(d*x + c)) - c*f + d*e)/d) - 6*c*f^3*cos(2*(c*f - d*e)/d)*sin_in
tegral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) +
6*d*f^2*cos(2*(c*f - d*e)/d)*e*sin_integral(-2*((d*x + c)*(c*f/(d*x + c) -
f - d*e/(d*x + c)) - c*f + d*e)/d) + 6*I*(d*x + c)*(c*f/(d*x + c) - f - d*e
/(d*x + c))*f^2*sin(2*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x +
 c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - 6*I*c*f^3*sin(2*(c*f - d*e)/d)*s
in_integral(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/
d) + 6*I*d*f^2*e*sin(2*(c*f - d*e)/d)*sin_integral(-2*((d*x + c)*(c*f/(d*x
+ c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 12*(d*x + c)*(c*f/(d*x + c) - f
 - d*e/(d*x + c))*f^2*cos(4*(c*f - d*e)/d)*sin_integral(-4*((d*x + c)*(c*f/
(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - 12*c*f^3*cos(4*(c*f - d*e)
/d)*sin_integral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f +
d*e)/d) + 12*d*f^2*cos(4*(c*f - d*e)/d)*e*sin_integral(-4*((d*x + c)*(c*f/(
d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 12*I*(d*x + c)*(c*f/(d*x +
c) - f - d*e/(d*x + c))*f^2*sin(4*(c*f - d*e)/d)*sin_integral(-4*((d*x + c)
*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - 12*I*c*f^3*sin(4*(c*
f - d*e)/d)*sin_integral(-4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))
- c*f + d*e)/d) + 12*I*d*f^2*e*sin(4*(c*f - d*e)/d)*sin_integral(-4*((d*x +
 c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 6*(d*x + c)*(c*f/
(d*x + c) - f - d*e/(d*x + c))*f^2*cos(6*(c*f - d*e)/d)*sin_integral(-6*((d
*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - 6*c*f^3*cos(6
*(c*f - d*e)/d)*sin_integral(-6*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x +
c)) - c*f + d*e)/d) + 6*d*f^2*cos(6*(c*f - d*e)/d)*e*sin_integral(-6*((d*x
+ c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) + 6*I*(d*x + c)*(c
*f/(d*x + c) - f - d*e/(d*x + c))*f^2*sin(6*(c*f - d*e)/d)*sin_integral(-6*
((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - 6*I*c*f^3*
sin(6*(c*f - d*e)/d)*sin_integral(-6*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d
*x + c)) - c*f + d*e)/d) + 6*I*d*f^2*e*sin(6*(c*f - d*e)/d)*sin_integral(-6
*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d) - d*f^2*cos
(6*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d) - 3*d*f^2*cos(4*(d*x +
c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d) - 3*d*f^2*cos(2*(d*x + c)*(c*f/(d
*x + c) - f - d*e/(d*x + c))/d) - I*d*f^2*sin(6*(d*x + c)*(c*f/(d*x + c) -
f - d*e/(d*x + c))/d) - 3*I*d*f^2*sin(4*(d*x + c)*(c*f/(d*x + c) - f - d*e/
(d*x + c))/d) - 3*I*d*f^2*sin(2*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c
))/d) - d*f^2)*d^2/(((d*x + c)*a^3*d^4*(c*f/(d*...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^3\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tan(e + f*x)*1i)^3*(c + d*x)^2),x)
[Out] int(1/((a + a*tan(e + f*x)*1i)^3*(c + d*x)^2), x)
```

3.34 $\int (c+dx)^m(a+ia\tan(e+fx))^2\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left((c+dx)^m(a+ia\tan(e+fx))^2,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*(a+I*a*tan(f*x+e))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m(a+ia\tan(e+fx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + I*a*Tan[e + f*x])^2, x]
```

Rubi steps

$$\int (c+dx)^m(a+ia\tan(e+fx))^2\,dx = \int (c+dx)^m(a+ia\tan(e+fx))^2\,dx$$

Mathematica [A]
time = 30.54, size = 0, normalized size = 0.00

$$\int (c+dx)^m(a+ia\tan(e+fx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m*(a + I*a*Tan[e + f*x])^2, x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int (dx+c)^m\,(a+ia\tan(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+I*a*tan(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(m + 1)*a^2/(d*(m + 1)) + integrate((3*(d*x + c)^m*a^2*cos(4*f*x
+ 4*e)^2 - 4*(d*x + c)^m*a^2*cos(2*f*x + 2*e)^2 + 3*(d*x + c)^m*a^2*sin(4*f
*x + 4*e)^2 + 4*(d*x + c)^m*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - 4*(d*x
+ c)^m*a^2*sin(2*f*x + 2*e)^2 - 4*(d*x + c)^m*a^2*cos(2*f*x + 2*e) - (d*x +
 c)^m*a^2 + 2*(2*(d*x + c)^m*a^2*cos(2*f*x + 2*e) + (d*x + c)^m*a^2)*cos(4*
f*x + 4*e))/(2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)
^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1), x) + I*integrat
e(-4*(2*(d*x + c)^m*a^2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e) - (2*(d*x + c)^m*
a^2*cos(2*f*x + 2*e) + (d*x + c)^m*a^2)*sin(4*f*x + 4*e))/(2*(2*cos(2*f*x +
 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + s
in(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e
)^2 + 4*cos(2*f*x + 2*e) + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*(d*x + c)^m*a^2 + (f*e^(2*I*f*x + 2*I*e) + f)*integral(-2*(-I*a^2*d*m
 - 2*(a^2*d*f*x + a^2*c*f)*e^(2*I*f*x + 2*I*e))*(d*x + c)^m/(d*f*x + c*f +
(d*f*x + c*f)*e^(2*I*f*x + 2*I*e)), x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int (c+dx)^m\tan^2(e+fx)\,dx+\int \left(-2i(c+dx)^m\tan(e+fx)\right)\,dx+\int \left(-(c+dx)^m\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral((c + d*x)**m*tan(e + f*x)**2, x) + Integral(-2*I*(c + d*x)*
*m*tan(e + f*x), x) + Integral(-(c + d*x)**m, x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2*(c + d*x)^m,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^2*(c + d*x)^m, x)
```

3.35 $\int (c+dx)^m(a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}((c+dx)^m(a+ia\tan(e+fx)),x)$$

```
[Out] Unintegrable((d*x+c)^m*(a+I*a*tan(f*x+e)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m(a+ia\tan(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + I*a*Tan[e + f*x]),x]
[Out] Defer[Int][(c + d*x)^m*(a + I*a*Tan[e + f*x]), x]
```

Rubi steps

$$\int (c+dx)^m(a+ia\tan(e+fx))\,dx = \int (c+dx)^m(a+ia\tan(e+fx))\,dx$$

Mathematica [A]
time = 14.60, size = 0, normalized size = 0.00

$$\int (c+dx)^m(a+ia\tan(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Tan[e + f*x]),x]
[Out] Integrate[(c + d*x)^m*(a + I*a*Tan[e + f*x]), x]
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int (dx+c)^m\,(a+ia\tan(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+I*a*tan(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*I*a*integrate((d*x + c)^m*sin(2*f*x + 2*e)/(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(2*(d*x + c)^m*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(c+dx)^{m}\right)\,dx+\int(c+dx)^{m}\tan{(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*tan(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*(c + d*x)**m, x) + Integral((c + d*x)**m*tan(e + f*x), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + a\tan(e + f\,x)\ 1\mathrm{i})\ (c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)*(c + d*x)^m,x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)*(c + d*x)^m, x)
```

3.36 $\int \frac{(c+dx)^m}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=98

$$\frac{(c+dx)^{1+m}}{2ad(1+m)}+\frac{i2^{-2-m}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,\frac{2if(c+dx)}{d}\right)}{af}$$

```
[Out] 1/2*(d*x+c)^(1+m)/a/d/(1+m)+I*2^(-2-m)*(d*x+c)^m*GAMMA(1+m,2*I*f*(d*x+c)/d)
/a/exp(2*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3808, 2212}

$$\frac{(c+dx)^{m+1}}{2ad(m+1)}+\frac{i2^{-m-2}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2if(c+dx)}{d}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (c + d*x)^(1 + m)/(2*a*d*(1 + m)) + (I*2^(-2 - m)*(c + d*x)^m*Gamma[1 + m,
((2*I)*f*(c + d*x))/d])/(a*E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3808

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + Dist[1/(2*a), Int[(c +
 d*x)^m*E^(2*(a/b)*(e + f*x)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[a^2 + b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+ia\tan(e+fx)}\,dx = \frac{(c+dx)^{1+m}}{2ad(1+m)} + \frac{\int e^{-2i(e+fx)}(c+dx)^m\,dx}{2a}$$
$$= \frac{(c+dx)^{1+m}}{2ad(1+m)} + \frac{i2^{-2-m}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{af}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 205 vs. 2(98) = 196.
time = 1.40, size = 205, normalized size = 2.09

$$\frac{2^{-2-m}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^m\left(\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\sec(e+fx)\left(2^{1+m}f(c+dx)\left(\frac{if(c+dx)}{d}\right)^m\left(\cos\left(e-\frac{cf}{d}\right)+i\sin\left(e-\frac{cf}{d}\right)\right)+d(1+m)\mathrm{Gamma}\left(1+m,\frac{2if(c+dx)}{d}\right)\left(i\cos\left(e-\frac{cf}{d}\right)+\sin\left(e-\frac{cf}{d}\right)\right)\right)\left(-i\cos\left(f\left(\frac{c}{d}+x\right)\right)+\sin\left(f\left(\frac{c}{d}+x\right)\right)\right)}{adf(1+m)(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (2^(-2 - m)*(c + d*x)^m*(((-I)*f*(c + d*x))/d)^m*Sec[e + f*x]*(2^(1 + m)*f*
(c + d*x)*((I*f*(c + d*x))/d)^m*(Cos[e - (c*f)/d] + I*Sin[e - (c*f)/d]) + d
*(1 + m)*Gamma[1 + m, ((2*I)*f*(c + d*x))/d]*(I*Cos[e - (c*f)/d] + Sin[e -
(c*f)/d]))*((-I)*Cos[f*(c/d + x)] + Sin[f*(c/d + x)]))/(a*d*f*(1 + m)*((f^2
*(c + d*x)^2)/d^2)^m*(-I + Tan[e + f*x]))
```

Maple [**F**]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a+ia\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+I*a*tan(f*x+e)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) - (I*d*m + I*d)*i
ntegrate((d*x + c)^m*sin(2*f*x + 2*e), x) + e^(m*log(d*x + c) + log(d*x + c
)))/(a*d*m + a*d)
```

Fricas [A]
time = 0.12, size = 86, normalized size = 0.88

$$\frac{(i\,dm + i\,d)e^{\left(-\frac{dm\log\left(\frac{2i\,f}{d}\right) - 2i\,cf + 2i\,de}{d}\right)}\Gamma\left(m+1, -\frac{2\,(-i\,dfx - i\,cf)}{d}\right) + 2\,(dfx + cf)(dx + c)^m}{4\,(adfm + adf)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*((I*d*m + I*d)*e^(-(d*m*log(2*I*f/d) - 2*I*c*f + 2*I*d*e)/d)*gamma(m +
1, -2*(-I*d*f*x - I*c*f)/d) + 2*(d*f*x + c*f)*(d*x + c)^m)/(a*d*f*m + a*d*f
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{(c+dx)^m}{\tan(e+fx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral((c + d*x)**m/(tan(e + f*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^m}{a + a\tan(e + f\,x)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*tan(e + f*x)*1i),x)
```

```
[Out] int((c + d*x)^m/(a + a*tan(e + f*x)*1i), x)
```

3.37 $\int \frac{(c+dx)^m}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=171

$$\frac{(c+dx)^{1+m}}{4a^2d(1+m)}+\frac{i2^{-2-m}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,\frac{2if(c+dx)}{d}\right)}{a^2f}+\frac{i4^{-2-m}e^{-4i\left(e-\frac{cf}{d}\right)}(c+dx}{}$$

```
[Out] 1/4*(d*x+c)^(1+m)/a^2/d/(1+m)+I*2^(-2-m)*(d*x+c)^m*GAMMA(1+m,2*I*f*(d*x+c)/
d)/a^2/exp(2*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+I*4^(-2-m)*(d*x+c)^m*GAMMA(
1+m,4*I*f*(d*x+c)/d)/a^2/exp(4*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3810, 2212}

$$\frac{i2^{-m-2}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2if(c+dx)}{d}\right)}{a^2f}+\frac{i4^{-m-2}e^{-4i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{4if(c+dx)}{d}\right)}{a^2f}+\frac{(c+dx)^{m+1}}{4a^2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(4*a^2*d*(1 + m)) + (I*2^(-2 - m)*(c + d*x)^m*Gamma[1 + m
, ((2*I)*f*(c + d*x))/d])/(a^2*E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d
)^m) + (I*4^(-2 - m)*(c + d*x)^m*Gamma[1 + m, ((4*I)*f*(c + d*x))/d])/(a^2*
E^((4*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^m}{(a+ia\tan(e+fx))^2}\,dx &= \int \left(\frac{(c+dx)^m}{4a^2}+\frac{e^{-2ie-2ifx}(c+dx)^m}{2a^2}+\frac{e^{-4ie-4ifx}(c+dx)^m}{4a^2}\right)dx\\ &= \frac{(c+dx)^{1+m}}{4a^2d(1+m)}+\frac{\int e^{-4ie-4ifx}(c+dx)^m\,dx}{4a^2}+\frac{\int e^{-2ie-2ifx}(c+dx)^m\,dx}{2a^2}\\ &= \frac{(c+dx)^{1+m}}{4a^2d(1+m)}+\frac{i2^{-2-m}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{a^2f}\end{aligned}$$

Mathematica [A]
time = 151.50, size = 205, normalized size = 1.20

$$\frac{4^{-2-m}e^{-2ie}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\left(4^{1+m}e^{4ie}f(c+dx)\left(\frac{if(c+dx)}{d}\right)^m+i2^{2+m}de^{\frac{2i(de+cf)}{d}}(1+m)\mathrm{Gamma}\left(1+m,\frac{2if(c+dx)}{d}\right)+ide^{\frac{4icf}{d}}(1+m)\mathrm{Gamma}\left(1+m,\frac{4if(c+dx)}{d}\right)\right)\sec^2(e+fx)(\cos(fx)+i\sin(fx))^2}{df(1+m)(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (4^(-2 - m)*(c + d*x)^m*(4^(1 + m)*E^((4*I)*e)*f*(c + d*x)*((I*f*(c + d*x))
/d)^m + I*2^(2 + m)*d*E^(((2*I)*(d*e + c*f))/d)*(1 + m)*Gamma[1 + m, ((2*I)
*f*(c + d*x))/d] + I*d*E^(((4*I)*c*f)/d)*(1 + m)*Gamma[1 + m, ((4*I)*f*(c +
 d*x))/d])*Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^2)/(d*E^((2*I)*e)*f*(1 +
m)*((I*f*(c + d*x))/d)^m*(a + I*a*Tan[e + f*x])^2)
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{(a+ia\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((d*m + d)*integrate((d*x + c)^m*cos(4*f*x + 4*e), x) + 2*(d*m + d)*int
egrate((d*x + c)^m*cos(2*f*x + 2*e), x) - (I*d*m + I*d)*integrate((d*x + c)
```

```
^m*sin(4*f*x + 4*e), x) + 2*(-I*d*m - I*d)*integrate((d*x + c)^m*sin(2*f*x
+ 2*e), x) + e^(m*log(d*x + c) + log(d*x + c)))/(a^2*d*m + a^2*d)
```

Fricas [A]
time = 0.10, size = 146, normalized size = 0.85

$$-\frac{4\left(-i\,dm-i\,d\right)e^{\left(-\frac{dm\log\left(\frac{2i\,f}{d}\right)-2i\,cf+2i\,de}{d}\right)}\Gamma\left(m+1,-\frac{2\left(-i\,dfx-i\,cf\right)}{d}\right)-\left(i\,dm+i\,d\right)e^{\left(-\frac{dm\log\left(\frac{4i\,f}{d}\right)-4i\,cf+4i\,de}{d}\right)}\Gamma\left(m+1,-\frac{4\left(-i\,dfx-i\,cf\right)}{d}\right)-4\left(dfx+cf\right)\left(dx+c\right)^{m}}{16\left(a^{2}dfm+a^{2}df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(-I*d*m - I*d)*e^(-(d*m*log(2*I*f/d) - 2*I*c*f + 2*I*d*e)/d)*gamma
(m + 1, -2*(-I*d*f*x - I*c*f)/d) - (I*d*m + I*d)*e^(-(d*m*log(4*I*f/d) - 4*
I*c*f + 4*I*d*e)/d)*gamma(m + 1, -4*(-I*d*f*x - I*c*f)/d) - 4*(d*f*x + c*f)
*(d*x + c)^m)/(a^2*d*f*m + a^2*d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(c+dx)^m}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral((c + d*x)**m/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^m}{(a+a\tan(e+f\,x)\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] int((c + d*x)^m/(a + a*tan(e + f*x)*1i)^2, x)
```

3.38 $\int \frac{(c+dx)^m}{(a+ia\tan(e+fx))^3}\,dx$

Optimal. Leaf size=251

$$\frac{(c+dx)^{1+m}}{8a^3d(1+m)}+\frac{3i2^{-4-m}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(1+m,\frac{2if(c+dx)}{d}\right)}{a^3f}+\frac{3i2^{-5-2m}e^{-4i\left(e-\frac{cf}{d}\right)}(c$$

[Out] 1/8*(d*x+c)^(1+m)/a^3/d/(1+m)+3*I*2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*f*(d*x+c
)/d)/a^3/exp(2*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+3*I*2^(-5-2*m)*(d*x+c)^m*
GAMMA(1+m,4*I*f*(d*x+c)/d)/a^3/exp(4*I*(e-c*f/d))/f/((I*f*(d*x+c)/d)^m)+I*2
^(-4-m)*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,6*I*f*(d*x+c)/d)/a^3/exp(6*I*(e-c*f/d)
)/f/((I*f*(d*x+c)/d)^m)

Rubi [A]
time = 0.18, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3810, 2212}

$$\frac{3i2^{-m-4}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2if(c+dx)}{d}\right)}{a^3f}+\frac{3i2^{-2m-5}e^{-4i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{4if(c+dx)}{d}\right)}{a^3f}+\frac{i2^{-m-4}3^{-m-1}e^{-6i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{6if(c+dx)}{d}\right)}{a^3f}+\frac{(c+dx)^{m+1}}{8a^3d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^m/(a + I*a*Tan[e + f*x])^3,x]

[Out] (c + d*x)^(1 + m)/(8*a^3*d*(1 + m)) + ((3*I)*2^(-4 - m)*(c + d*x)^m*Gamma[1
 + m, ((2*I)*f*(c + d*x))/d])/(a^3*E^((2*I)*(e - (c*f)/d))*f*((I*f*(c + d*x
))/d)^m) + ((3*I)*2^(-5 - 2*m)*(c + d*x)^m*Gamma[1 + m, ((4*I)*f*(c + d*x))
/d])/(a^3*E^((4*I)*(e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m) + (I*2^(-4 - m)*
3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((6*I)*f*(c + d*x))/d])/(a^3*E^((6*I)*(
e - (c*f)/d))*f*((I*f*(c + d*x))/d)^m)

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^m}{(a+ia\tan(e+fx))^3}\,dx &= \int \left(\frac{(c+dx)^m}{8a^3}+\frac{3e^{-2ie-2ifx}(c+dx)^m}{8a^3}+\frac{3e^{-4ie-4ifx}(c+dx)^m}{8a^3}+\frac{e^{-6ie-6ifx}(c\\
&= \frac{(c+dx)^{1+m}}{8a^3d(1+m)}+\frac{\int e^{-6ie-6ifx}(c+dx)^m\,dx}{8a^3}+\frac{3\int e^{-2ie-2ifx}(c+dx)^m\,dx}{8a^3}+\frac{3\int e\\
&= \frac{(c+dx)^{1+m}}{8a^3d(1+m)}+\frac{3i2^{-4-m}e^{-2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2if(c+dx)}{d}\right)}{a^3f}
\end{aligned}
$$

Mathematica [A]
time = 119.59, size = 269, normalized size = 1.07

$$
\frac{2^{-5-2m}3^{-1-m}e^{-3ie}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^{-m}\left(12^{1+m}e^{6ie}f(c+dx)\left(\frac{if(c+dx)}{d}\right)^m+i2^{1+m}3^{2+m}de^{2i\left(2e+\frac{cf}{d}\right)}(1+m)\mathrm{Gamma}\left(1+m,\frac{2if(c+dx)}{d}\right)+i3^{2+m}de^{2ie+\frac{4icf}{d}}(1+m)\mathrm{Gamma}\left(1+m,\frac{4if(c+dx)}{d}\right)+i2^{1+m}de^{\frac{6icf}{d}}(1+m)\mathrm{Gamma}\left(1+m,\frac{6if(c+dx)}{d}\right)\right)\sec^3(e+fx)(\cos(fx)+i\sin(fx))^3}{df(1+m)(a+ia\tan(e+fx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (2^(-5 - 2*m)*3^(-1 - m)*(c + d*x)^m*(12^(1 + m)*E^((6*I)*e)*f*(c + d*x)*((
I*f*(c + d*x))/d)^m + I*2^(1 + m)*3^(2 + m)*d*E^((2*I)*(2*e + (c*f)/d))*(1
+ m)*Gamma[1 + m, ((2*I)*f*(c + d*x))/d] + I*3^(2 + m)*d*E^((2*I)*e + ((4*I
)*c*f)/d)*(1 + m)*Gamma[1 + m, ((4*I)*f*(c + d*x))/d] + I*2^(1 + m)*d*E^(((
6*I)*c*f)/d)*(1 + m)*Gamma[1 + m, ((6*I)*f*(c + d*x))/d])*Sec[e + f*x]^3*(C
os[f*x] + I*Sin[f*x])^3)/(d*E^((3*I)*e)*f*(1 + m)*((I*f*(c + d*x))/d)^m*(a
+ I*a*Tan[e + f*x])^3)
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{(a+ia\tan(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*tan(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m/(a+I*a*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/8*((d*m + d)*integrate((d*x + c)^m*cos(6*f*x + 6*e), x) + 3*(d*m + d)*int
egrate((d*x + c)^m*cos(4*f*x + 4*e), x) + 3*(d*m + d)*integrate((d*x + c)^m
*cos(2*f*x + 2*e), x) - (I*d*m + I*d)*integrate((d*x + c)^m*sin(6*f*x + 6*e
), x) + 3*(-I*d*m - I*d)*integrate((d*x + c)^m*sin(4*f*x + 4*e), x) + 3*(-I
*d*m - I*d)*integrate((d*x + c)^m*sin(2*f*x + 2*e), x) + e^(m*log(d*x + c)
+ log(d*x + c)))/(a^3*d*m + a^3*d)
```

Fricas [A]
time = 0.11, size = 201, normalized size = 0.80

$$-\frac{18\,(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(\frac{2i\,f}{d}\right)-2i\,cf+2i\,de}{d}\right)}\Gamma\left(m+1,-\frac{2\,(-i\,dfx-i\,cf)}{d}\right)+9\,(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(\frac{4i\,f}{d}\right)-4i\,cf+4i\,de}{d}\right)}\Gamma\left(m+1,-\frac{4\,(-i\,dfx-i\,cf)}{d}\right)+2\,(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(\frac{6i\,f}{d}\right)-6i\,cf+6i\,de}{d}\right)}\Gamma\left(m+1,-\frac{6\,(-i\,dfx-i\,cf)}{d}\right)-12\,(dfx+cf)(dx+c)^m}{96\,(a^3dfm+a^3df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/96*(18*(-I*d*m - I*d)*e^(-(d*m*log(2*I*f/d) - 2*I*c*f + 2*I*d*e)/d)*gamm
a(m + 1, -2*(-I*d*f*x - I*c*f)/d) + 9*(-I*d*m - I*d)*e^(-(d*m*log(4*I*f/d)
- 4*I*c*f + 4*I*d*e)/d)*gamma(m + 1, -4*(-I*d*f*x - I*c*f)/d) + 2*(-I*d*m -
 I*d)*e^(-(d*m*log(6*I*f/d) - 6*I*c*f + 6*I*d*e)/d)*gamma(m + 1, -6*(-I*d*f
*x - I*c*f)/d) - 12*(d*f*x + c*f)*(d*x + c)^m)/(a^3*d*f*m + a^3*d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{(c+dx)^m}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] I*Integral((c + d*x)**m/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e +
f*x) + I), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^m}{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*tan(e + f*x)*1i)^3,x)
[Out] int((c + d*x)^m/(a + a*tan(e + f*x)*1i)^3, x)
```

3.39 $\int (c+dx)^3(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=152

$$\frac{a(c+dx)^4}{4d}+\frac{ib(c+dx)^4}{4d}-\frac{b(c+dx)^3\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{3ibd(c+dx)^2\mathrm{PolyLog}\left(2,-e^{2i(e+fx)}\right)}{2f^2}-\frac{3bd^2(c+dx}{}$$

```
[Out] 1/4*a*(d*x+c)^4/d+1/4*I*b*(d*x+c)^4/d-b*(d*x+c)^3*ln(1+exp(2*I*(f*x+e)))/f+
3/2*I*b*d*(d*x+c)^2*polylog(2,-exp(2*I*(f*x+e)))/f^2-3/2*b*d^2*(d*x+c)*poly
log(3,-exp(2*I*(f*x+e)))/f^3-3/4*I*b*d^3*polylog(4,-exp(2*I*(f*x+e)))/f^4
```

Rubi [A]
time = 0.18, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3803, 3800, 2221, 2611, 6744, 2320, 6724}

$$\frac{a(c+dx)^4}{4d}-\frac{3bd^2(c+dx)\mathrm{Li}_3\left(-e^{2i(e+fx)}\right)}{2f^3}+\frac{3ibd(c+dx)^2\mathrm{Li}_2\left(-e^{2i(e+fx)}\right)}{2f^2}-\frac{b(c+dx)^3\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{ib(c+dx)^4}{4d}-\frac{3ibd^3\mathrm{Li}_4\left(-e^{2i(e+fx)}\right)}{4f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Tan[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + ((I/4)*b*(c + d*x)^4)/d - (b*(c + d*x)^3*Log[1 + E^
((2*I)*(e + f*x))])/f + (((3*I)/2)*b*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(e
+ f*x))])/f^2 - (3*b*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^3
) - (((3*I)/4)*b*d^3*PolyLog[4, -E^((2*I)*(e + f*x))])/f^4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3(a+b\tan(e+fx))\,dx &= \int \left(a(c+dx)^3 + b(c+dx)^3\tan(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^4}{4d} + b\int (c+dx)^3\tan(e+fx)\,dx \\
&= \frac{a(c+dx)^4}{4d} + \frac{ib(c+dx)^4}{4d} - (2ib)\int \frac{e^{2i(e+fx)}(c+dx)^3}{1+e^{2i(e+fx)}}\,dx \\
&= \frac{a(c+dx)^4}{4d} + \frac{ib(c+dx)^4}{4d} - \frac{b(c+dx)^3\log\left(1+e^{2i(e+fx)}\right)}{f} + \frac{(3bd)\int(c
\\
&= \frac{a(c+dx)^4}{4d} + \frac{ib(c+dx)^4}{4d} - \frac{b(c+dx)^3\log\left(1+e^{2i(e+fx)}\right)}{f} + \frac{3ibd(c+
\\
&= \frac{a(c+dx)^4}{4d} + \frac{ib(c+dx)^4}{4d} - \frac{b(c+dx)^3\log\left(1+e^{2i(e+fx)}\right)}{f} + \frac{3ibd(c+
\\
&= \frac{a(c+dx)^4}{4d} + \frac{ib(c+dx)^4}{4d} - \frac{b(c+dx)^3\log\left(1+e^{2i(e+fx)}\right)}{f} + \frac{3ibd(c+
\\
&= \frac{a(c+dx)^4}{4d} + \frac{ib(c+dx)^4}{4d} - \frac{b(c+dx)^3\log\left(1+e^{2i(e+fx)}\right)}{f} + \frac{3ibd(c+
\end{aligned}
$$

<u>**Mathematica**</u> [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 546 vs. $2(152) = 304$.
time = 7.08, size = 546, normalized size = 3.59

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Tan[e + f*x]),x]
```

```
[Out] (b*c*d^2*((2*I)*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(1 + E^((2*I)*e))*Log[1
+ E^((2*I)*(e + f*x))]) + (6*I)*(1 + E^((2*I)*e))*f*x*PolyLog[2, -E^((2*I)*
(e + f*x))] - 3*(1 + E^((2*I)*e))*PolyLog[3, -E^((2*I)*(e + f*x))])*Sec[e])
/(4*E^(I*e)*f^3) - (I/4)*b*d^3*E^(I*e)*(-x^4 + (1 + E^((-2*I)*e))*x^4 - ((1
 + E^((2*I)*e))*(2*f^4*x^4 + (4*I)*f^3*x^3*Log[1 + E^((2*I)*(e + f*x))] + 6
*f^2*x^2*PolyLog[2, -E^((2*I)*(e + f*x))] + (6*I)*f*x*PolyLog[3, -E^((2*I)*
(e + f*x))] - 3*PolyLog[4, -E^((2*I)*(e + f*x))]))/(2*E^((2*I)*e)*f^4))*Sec
[e] + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Sec[e]*(a*Cos[e] + b*S
in[e]))/4 - (b*c^3*Sec[e]*(Cos[e]*Log[Cos[e]*Cos[f*x] - Sin[e]*Sin[f*x]] +
f*x*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) - (3*b*c^2*d*Csc[e]*((f^2*x^2)/E^(I*
ArcTan[Cot[e]]) - (Cot[e]*(I*f*x*(-Pi - 2*ArcTan[Cot[e]]) - Pi*Log[1 + E^((
-2*I)*f*x)] - 2*(f*x - ArcTan[Cot[e]])*Log[1 - E^((2*I)*(f*x - ArcTan[Cot[e
]]))] + Pi*Log[Cos[f*x]] - 2*ArcTan[Cot[e]]*Log[Sin[f*x - ArcTan[Cot[e]]]]
```

```
+ I*PolyLog[2, E^((2*I)*(f*x - ArcTan[Cot[e]]))]))/Sqrt[1 + Cot[e]^2])*Sec[
e])/(2*f^2*Sqrt[Csc[e]^2*(Cos[e]^2 + Sin[e]^2)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(131) = 262$.
time = 0.34, size = 500, normalized size = 3.29

method	result
risch	$\frac{3ib\,c^2d\,\mathrm{polylog}(2,-e^{2i(fx+e)})}{2f^2}+\frac{2ib\,d^3e^3x}{f^3}+\frac{3ib\,c^2d\,e^2}{f^2}-\frac{4ibc\,d^2e^3}{f^3}+\frac{6bc\,d^2e^2\ln(e^{i(fx+e)})}{f^3}-\frac{b\,c^3\ln(e^{2i(fx+e)}+1)}{f}+\frac{2b\,c^3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*I*d*b*c^2*x^2+I*d^2*b*c*x^3-1/f*b*c^3*ln(exp(2*I*(f*x+e))+1)+2/f*b*c^3*
ln(exp(I*(f*x+e)))-3/f*b*c^2*d*ln(exp(2*I*(f*x+e))+1)*x+3/2*I/f^2*b*d^3*pol
ylog(2,-exp(2*I*(f*x+e)))*x^2+1/4*I*d^3*b*x^4-I*b*c^3*x-1/4*I/d*b*c^4+d^2*a
*c*x^3+3/2*d*a*c^2*x^2+a*c^3*x+1/4*d^3*a*x^4+1/4/d*a*c^4-3/2/f^3*b*c*d^2*po
lylog(3,-exp(2*I*(f*x+e)))+3/2*I/f^4*b*d^3*e^4-3/2/f^3*b*d^3*polylog(3,-exp
(2*I*(f*x+e)))*x-2/f^4*b*d^3*e^3*ln(exp(I*(f*x+e)))-6*I/f^2*b*c*d^2*e^2*x+3
*I/f^2*b*c*d^2*polylog(2,-exp(2*I*(f*x+e)))*x+6*I/f*b*c^2*d*e*x+2*I/f^3*b*d
^3*e^3*x+3/2*I/f^2*b*c^2*d*polylog(2,-exp(2*I*(f*x+e)))+3*I/f^2*b*c^2*d*e^2
-4*I/f^3*b*c*d^2*e^3-1/f*b*d^3*ln(exp(2*I*(f*x+e))+1)*x^3+6/f^3*b*c*d^2*e^2
*ln(exp(I*(f*x+e)))-6/f^2*b*c^2*d*e*ln(exp(I*(f*x+e)))-3/f*b*c*d^2*ln(exp(2
*I*(f*x+e))+1)*x^2-3/4*I*b*d^3*polylog(4,-exp(2*I*(f*x+e)))/f^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(131) = 262$.
time = 0.57, size = 710, normalized size = 4.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(f*x + e)*a*c^3 + 3*(f*x + e)^4*a*d^3/f^3 + 12*(f*x + e)^3*a*c*d^2
/f^2 + 18*(f*x + e)^2*a*c^2*d/f - 12*(f*x + e)^3*a*d^3*e/f^3 - 36*(f*x + e)
^2*a*c*d^2*e/f^2 - 36*(f*x + e)*a*c^2*d*e/f + 12*b*c^3*log(sec(f*x + e)) -
36*b*c^2*d*e*log(sec(f*x + e))/f + 18*(f*x + e)^2*a*d^3*e^2/f^3 + 36*(f*x +
 e)*a*c*d^2*e^2/f^2 + 36*b*c*d^2*e^2*log(sec(f*x + e))/f^2 - 12*(f*x + e)*a
*d^3*e^3/f^3 - 12*b*d^3*e^3*log(sec(f*x + e))/f^3 - (-3*I*(f*x + e)^4*b*d^3
 + 12*I*b*d^3*polylog(4, -e^(2*I*f*x + 2*I*e)) - 12*(I*b*c*d^2*f - I*b*d^3*
e)*(f*x + e)^3 - 18*(I*b*c^2*d*f^2 - 2*I*b*c*d^2*f*e + I*b*d^3*e^2)*(f*x +
e)^2 - 4*(-4*I*(f*x + e)^3*b*d^3 + 9*(-I*b*c*d^2*f + I*b*d^3*e)*(f*x + e)^2
 + 9*(-I*b*c^2*d*f^2 + 2*I*b*c*d^2*f*e - I*b*d^3*e^2)*(f*x + e))*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 6*(4*I*(f*x + e)^2*b*d^3 + 3*I*b*c^
```

```
2*d*f^2 - 6*I*b*c*d^2*f*e + 3*I*b*d^3*e^2 + 6*(I*b*c*d^2*f - I*b*d^3*e)*(f*
x + e))*dilog(-e^(2*I*f*x + 2*I*e)) + 2*(4*(f*x + e)^3*b*d^3 + 9*(b*c*d^2*f
 - b*d^3*e)*(f*x + e)^2 + 9*(b*c^2*d*f^2 - 2*b*c*d^2*f*e + b*d^3*e^2)*(f*x
+ e))*log(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
 + 6*(4*(f*x + e)*b*d^3 + 3*b*c*d^2*f - 3*b*d^3*e)*polylog(3, -e^(2*I*f*x +
 2*I*e)))/f^3)/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(131) = 262$.
time = 0.37, size = 520, normalized size = 3.42

$$\frac{2\,ad^3f^4x^4+8\,acd^2f^4x^3+12\,ac^2df^4x^2+8\,ac^3f^4x+3i\,bd^3\mathrm{polylog}\left(4,\frac{\tan(fx+e)^2+2i\,\tan(fx+e)-1}{\tan(fx+e)^2+1}\right)-3i\,bd^3\mathrm{polylog}\left(4,\frac{\tan(fx+e)^2-2i\,\tan(fx+e)-1}{\tan(fx+e)^2+1}\right)-6\left(i\,bd^3f^2x^2+2i\,bcd^2f^2x+i\,bc^2df^2\right)\mathrm{Li}_2\left(\frac{2\left(i\,\tan(fx+e)-1\right)}{\tan(fx+e)^2+1}+1\right)-6\left(-i\,bd^3f^2x^2-2i\,bcd^2f^2x-i\,bc^2df^2\right)\mathrm{Li}_2\left(\frac{2\left(-i\,\tan(fx+e)-1\right)}{\tan(fx+e)^2+1}+1\right)-4\left(bd^3f^3x^3+3\,bcd^2f^3x^2+3\,bc^2df^3x+bc^3f^3\right)\log\left(-\frac{2\left(i\,\tan(fx+e)-1\right)}{\tan(fx+e)^2+1}\right)-4\left(bd^3f^3x^3+3\,bcd^2f^3x^2+3\,bc^2df^3x+bc^3f^3\right)\log\left(-\frac{2\left(-i\,\tan(fx+e)-1\right)}{\tan(fx+e)^2+1}\right)-6\left(bd^3fx+bcd^2f\right)\mathrm{polylog}\left(3,\frac{\tan(fx+e)^2+2i\,\tan(fx+e)-1}{\tan(fx+e)^2+1}\right)-6\left(bd^3fx+bcd^2f\right)\mathrm{polylog}\left(3,\frac{\tan(fx+e)^2-2i\,\tan(fx+e)-1}{\tan(fx+e)^2+1}\right)}{8\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*d^3*f^4*x^4 + 8*a*c*d^2*f^4*x^3 + 12*a*c^2*d*f^4*x^2 + 8*a*c^3*f^4
*x + 3*I*b*d^3*polylog(4, (tan(f*x + e)^2 + 2*I*tan(f*x + e) - 1)/(tan(f*x
+ e)^2 + 1)) - 3*I*b*d^3*polylog(4, (tan(f*x + e)^2 - 2*I*tan(f*x + e) - 1)
/(tan(f*x + e)^2 + 1)) - 6*(I*b*d^3*f^2*x^2 + 2*I*b*c*d^2*f^2*x + I*b*c^2*d
*f^2)*dilog(2*(I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1) + 1) - 6*(-I*b*d^3*
f^2*x^2 - 2*I*b*c*d^2*f^2*x - I*b*c^2*d*f^2)*dilog(2*(-I*tan(f*x + e) - 1)/
(tan(f*x + e)^2 + 1) + 1) - 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*
d*f^3*x + b*c^3*f^3)*log(-2*(I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 4*
(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*c^3*f^3)*log(-2*(-
I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 6*(b*d^3*f*x + b*c*d^2*f)*polyl
og(3, (tan(f*x + e)^2 + 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 6*(b*
d^3*f*x + b*c*d^2*f)*polylog(3, (tan(f*x + e)^2 - 2*I*tan(f*x + e) - 1)/(ta
n(f*x + e)^2 + 1)))/f^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(e + fx)})\,(c + dx)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\tan(e+f\,x))\,\left(c+d\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*x)^3,x)
```

```
[Out] int((a + b*tan(e + f*x))*(c + d*x)^3, x)
```

3.40 $\int (c+dx)^2 (a+b\tan(e+fx))\,dx$

Optimal. Leaf size=115

$$\frac{a(c+dx)^3}{3d}+\frac{ib(c+dx)^3}{3d}-\frac{b(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{ibd(c+dx)\text{PolyLog}\left(2,-e^{2i(e+fx)}\right)}{f^2}-\frac{bd^2\text{PolyLog}(3}{2f}$$

```
[Out] 1/3*a*(d*x+c)^3/d+1/3*I*b*(d*x+c)^3/d-b*(d*x+c)^2*ln(1+exp(2*I*(f*x+e)))/f+
I*b*d*(d*x+c)*polylog(2,-exp(2*I*(f*x+e)))/f^2-1/2*b*d^2*polylog(3,-exp(2*I
*(f*x+e)))/f^3
```

Rubi [A]
time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3803, 3800, 2221, 2611, 2320, 6724}

$$\frac{a(c+dx)^3}{3d}+\frac{ibd(c+dx)\text{Li}_2\left(-e^{2i(e+fx)}\right)}{f^2}-\frac{b(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{ib(c+dx)^3}{3d}-\frac{bd^2\text{Li}_3\left(-e^{2i(e+fx)}\right)}{2f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Tan[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + ((I/3)*b*(c + d*x)^3)/d - (b*(c + d*x)^2*Log[1 + E^
((2*I)*(e + f*x))])/f + (I*b*d*(c + d*x)*PolyLog[2, -E^((2*I)*(e + f*x))])/
f^2 - (b*d^2*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2(a+b\tan(e+fx))\,dx &= \int \left(a(c+dx)^2+b(c+dx)^2\tan(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^3}{3d}+b\int (c+dx)^2\tan(e+fx)\,dx \\
&= \frac{a(c+dx)^3}{3d}+\frac{ib(c+dx)^3}{3d}-(2ib)\int \frac{e^{2i(e+fx)}(c+dx)^2}{1+e^{2i(e+fx)}}\,dx \\
&= \frac{a(c+dx)^3}{3d}+\frac{ib(c+dx)^3}{3d}-\frac{b(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{(2bd)\int(c+}{} \\
&= \frac{a(c+dx)^3}{3d}+\frac{ib(c+dx)^3}{3d}-\frac{b(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{ibd(c+dx)}{} \\
&= \frac{a(c+dx)^3}{3d}+\frac{ib(c+dx)^3}{3d}-\frac{b(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{ibd(c+dx)}{} \\
&= \frac{a(c+dx)^3}{3d}+\frac{ib(c+dx)^3}{3d}-\frac{b(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{ibd(c+dx)}{}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 324 vs. $2(115) = 230$.

time = 6.32, size = 324, normalized size = 2.82

$$\frac{1}{12}\sec(e)\left(\frac{bd^2e^{-ie}\left(2if^2x^2\left(2e^{2ie}fx+3i(1+e^{2ie})\log\left(1+e^{2i(e+fx)}\right)\right)+6i(1+e^{2ie})\,fx\mathrm{PolyLog}\left(2,-e^{2i(e+fx)}\right)-3(1+e^{2ie})\,\mathrm{PolyLog}\left(3,-e^{2i(e+fx)}\right)\right)}{f^3}+4x(3c^2+3cdx+d^2x^2)\,(a\cos(e)+b\sin(e))-\frac{12bc^2(\cos(e)\log(\cos(e+fx))+fx\sin(e))}{f}+12bcd\csc(e)\left(-\frac{e^{-i\mathrm{ArcTan}(\cot(e))}x^2}{\sqrt{\csc^2(e)}}+\frac{\cos(e)\left(-ifx(\pi+2\mathrm{ArcTan}(\cot(e)))-\pi\log\left(1+e^{-2ifx}\right)-2(fx-\mathrm{ArcTan}(\cot(e)))\log\left(1-e^{2i(fx-\mathrm{ArcTan}(\cot(e)))}\right)+\pi\log(\cos(fx))-2\mathrm{ArcTan}(\cot(e))\log(\sin(fx-\mathrm{ArcTan}(\cot(e))))+i\mathrm{PolyLog}\left(2,e^{2i(fx-\mathrm{ArcTan}(\cot(e)))}\right)\right)\sin(e)}{f^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Tan[e + f*x]),x]
```

```
[Out] (Sec[e]*((b*d^2*((2*I)*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(1 + E^((2*I)*e))
*Log[1 + E^((2*I)*(e + f*x))]) + (6*I)*(1 + E^((2*I)*e))*f*x*PolyLog[2, -E^
((2*I)*(e + f*x))] - 3*(1 + E^((2*I)*e))*PolyLog[3, -E^((2*I)*(e + f*x))]))
/(E^(I*e)*f^3) + 4*x*(3*c^2 + 3*c*d*x + d^2*x^2)*(a*Cos[e] + b*Sin[e]) - (1
2*b*c^2*(Cos[e]*Log[Cos[e + f*x]] + f*x*Sin[e]))/f + 12*b*c*d*Csc[e]*(-(x^2
/(E^(I*ArcTan[Cot[e]])*Sqrt[Csc[e]^2])) + (Cos[e]*((-I)*f*x*(Pi + 2*ArcTan[
Cot[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x - ArcTan[Cot[e]])*Log[1 - E^
((2*I)*(f*x - ArcTan[Cot[e]]))] + Pi*Log[Cos[f*x]] - 2*ArcTan[Cot[e]]*Log[S
in[f*x - ArcTan[Cot[e]]]] + I*PolyLog[2, E^((2*I)*(f*x - ArcTan[Cot[e]]))])
*Sin[e])/f^2)))/12
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(101) = 202$.
time = 0.21, size = 314, normalized size = 2.73

method	result
risch	$-\frac{ib\,c^3}{3d}-ib\,c^2x+\frac{d^2a\,x^3}{3}+\frac{2ibcd\,e^2}{f^2}+dac\,x^2+\frac{ib\,d^2\,\mathrm{polylog}\left(2,-e^{2i(fx+e)}\right)x}{f^2}+a\,c^2x+\frac{a\,c^3}{3d}+\frac{2b\,c^2\ln\left(e^{i(fx+e)}\right)}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/d*b*c^3-I*b*c^2*x+1/3*d^2*a*x^3+2*I/f^2*b*c*d*e^2+d*a*c*x^2+I/f^2*b*
d^2*polylog(2,-exp(2*I*(f*x+e)))*x+a*c^2*x+1/3/d*a*c^3+2/f*b*c^2*ln(exp(I*(
f*x+e)))-1/f*b*c^2*ln(exp(2*I*(f*x+e))+1)+2/f^3*b*d^2*e^2*ln(exp(I*(f*x+e))
)-4/3*I/f^3*b*d^2*e^3-2/f*b*c*d*ln(exp(2*I*(f*x+e))+1)*x+4*I/f*b*c*d*e*x-1/
f*b*d^2*ln(exp(2*I*(f*x+e))+1)*x^2-2*I/f^2*b*d^2*e^2*x-1/2*b*d^2*polylog(3,
-exp(2*I*(f*x+e)))/f^3-4/f^2*b*c*d*e*ln(exp(I*(f*x+e)))+I*d*b*c*x^2+I/f^2*b
*c*d*polylog(2,-exp(2*I*(f*x+e)))+1/3*I*d^2*b*x^3
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(101) = 202$.
time = 0.55, size = 401, normalized size = 3.49

$$\frac{6\,(fx+e)ac^2+\frac{2\,(fx+e)^3ad^2}{f^2}+\frac{6\,(fx+e)^2acd}{f}-\frac{6\,(fx+e)^2ad^2e}{f^2}-\frac{12\,(fx+e)acde}{f}+6\,bc^2\log\left(\sec\left(fx+e\right)\right)-\frac{12\,bcde\log(\sec(fx+e))}{f}+\frac{6\,(fx+e)ad^2e^2}{f^2}+\frac{6\,bd^2e^2\log(\sec(fx+e))}{f^2}-\frac{-2i\,(fx+e)^3bd^2+3\,bd^2\mathrm{Li}_3(-e^{(2i\,fx+2i\,e)})-6\,(i\,bcdf-i\,bd^2e)\,(fx+e)^2-6\left(-i\,(fx+e)^2bd^2+2\,(-i\,bcdf+i\,bd^2e)\,(fx+e)\right)\arctan(\sin(2\,fx+2\,e),\cos(2\,fx+2\,e)+1)-6\,(i\,(fx+e)bd^2+i\,bcdf-i\,bd^2e)\mathrm{Li}_2\left(-e^{(2i\,fx+2i\,e)}\right)+3\left((fx+e)^2bd^2+2\,(bcdf-bd^2e)\,(fx+e)\right)\log\left(\cos(2\,fx+2\,e)^2+\sin(2\,fx+2\,e)^2+2\,\cos(2\,fx+2\,e)+1\right)}{f^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(f*x + e)*a*c^2 + 2*(f*x + e)^3*a*d^2/f^2 + 6*(f*x + e)^2*a*c*d/f -
6*(f*x + e)^2*a*d^2*e/f^2 - 12*(f*x + e)*a*c*d*e/f + 6*b*c^2*log(sec(f*x +
e)) - 12*b*c*d*e*log(sec(f*x + e))/f + 6*(f*x + e)*a*d^2*e^2/f^2 + 6*b*d^2*
e^2*log(sec(f*x + e))/f^2 - (-2*I*(f*x + e)^3*b*d^2 + 3*b*d^2*polylog(3, -e
^(2*I*f*x + 2*I*e)) - 6*(I*b*c*d*f - I*b*d^2*e)*(f*x + e)^2 - 6*(-I*(f*x +
e)^2*b*d^2 + 2*(-I*b*c*d*f + I*b*d^2*e)*(f*x + e))*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1) - 6*(I*(f*x + e)*b*d^2 + I*b*c*d*f - I*b*d^2*e)*dil
og(-e^(2*I*f*x + 2*I*e)) + 3*((f*x + e)^2*b*d^2 + 2*(b*c*d*f - b*d^2*e)*(f*
x + e))*log(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1))/f^2)/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(101) = 202$.
time = 0.40, size = 331, normalized size = 2.88

$$\frac{4\,ad^2f^3x^3 + 12\,acdf^3x^2 + 12\,ac^2f^3x - 3\,bd^2\mathrm{polylog}\left(3, \frac{\tan(fx+e)^2+2i\,\tan(fx+e)-1}{\tan(fx+e)^2+1}\right) - 3\,bd^2\mathrm{polylog}\left(3, \frac{\tan(fx+e)^2-2i\,\tan(fx+e)-1}{\tan(fx+e)^2+1}\right) - 6\,(i\,bd^2fx + i\,bcdf)\mathrm{Li}_2\left(\frac{2\,(i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1} + 1\right) - 6\,(-i\,bd^2fx - i\,bcdf)\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1} + 1\right) - 6\,(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2)\log\left(-\frac{2\,(i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}\right) - 6\,(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2)\log\left(-\frac{2\,(-i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}\right)}{12\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*d^2*f^3*x^3 + 12*a*c*d*f^3*x^2 + 12*a*c^2*f^3*x - 3*b*d^2*polylog
(3, (tan(f*x + e)^2 + 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 3*b*d^2
*polylog(3, (tan(f*x + e)^2 - 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) -
 6*(I*b*d^2*f*x + I*b*c*d*f)*dilog(2*(I*tan(f*x + e) - 1)/(tan(f*x + e)^2 +
 1) + 1) - 6*(-I*b*d^2*f*x - I*b*c*d*f)*dilog(2*(-I*tan(f*x + e) - 1)/(tan(
f*x + e)^2 + 1) + 1) - 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*log(-2
*(I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^
2*x + b*c^2*f^2)*log(-2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)))/f^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan\left(e + fx\right)\right)\left(c + dx\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\tan(e + f\,x))\;(c + d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*x)^2,x)
```

```
[Out] int((a + b*tan(e + f*x))*(c + d*x)^2, x)
```

3.41 $\int (c+dx)(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=84

$$\frac{a(c+dx)^2}{2d}+\frac{ib(c+dx)^2}{2d}-\frac{b(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{ibd\text{PolyLog}\left(2,-e^{2i(e+fx)}\right)}{2f^2}$$

```
[Out] 1/2*a*(d*x+c)^2/d+1/2*I*b*(d*x+c)^2/d-b*(d*x+c)*ln(1+exp(2*I*(f*x+e)))/f+1/
2*I*b*d*polylog(2,-exp(2*I*(f*x+e)))/f^2
```

Rubi [A]
time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3803, 3800, 2221, 2317, 2438}

$$\frac{a(c+dx)^2}{2d}-\frac{b(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{ib(c+dx)^2}{2d}+\frac{ibd\text{Li}_2\left(-e^{2i(e+fx)}\right)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) + ((I/2)*b*(c + d*x)^2)/d - (b*(c + d*x)*Log[1 + E^((
2*I)*(e + f*x))])/f + ((I/2)*b*d*PolyLog[2, -E^((2*I)*(e + f*x))])/f^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)(a+b\tan(e+fx))\,dx &= \int (a(c+dx)+b(c+dx)\tan(e+fx))\,dx\\
&= \frac{a(c+dx)^2}{2d}+b\int (c+dx)\tan(e+fx)\,dx\\
&= \frac{a(c+dx)^2}{2d}+\frac{ib(c+dx)^2}{2d}-(2ib)\int \frac{e^{2i(e+fx)}(c+dx)}{1+e^{2i(e+fx)}}\,dx\\
&= \frac{a(c+dx)^2}{2d}+\frac{ib(c+dx)^2}{2d}-\frac{b(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{(bd)\int \log(1}{}\\
&= \frac{a(c+dx)^2}{2d}+\frac{ib(c+dx)^2}{2d}-\frac{b(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f}-\frac{(ibd)\text{Subst}(}{}\\
&= \frac{a(c+dx)^2}{2d}+\frac{ib(c+dx)^2}{2d}-\frac{b(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{ibd\text{Li}_2(-e^{2i}}{2f^2}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 87, normalized size = 1.04

$$
acx+\frac{1}{2}adx^2+\frac{1}{2}ibdx^2-\frac{bdx\log\left(1+e^{2i(e+fx)}\right)}{f}-\frac{bc\log(\cos(e+fx))}{f}+\frac{ibd\text{PolyLog}\left(2,-e^{2i(e+fx)}\right)}{2f^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Tan[e + f*x]),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (I/2)*b*d*x^2 - (b*d*x*Log[1 + E^((2*I)*(e + f*x))])/
f - (b*c*Log[Cos[e + f*x]])/f + ((I/2)*b*d*PolyLog[2, -E^((2*I)*(e + f*x))]
)/f^2
```

Maple [A]
time = 0.15, size = 143, normalized size = 1.70

method	result
risch	$\frac{ibd\,x^2}{2}+\frac{ad\,x^2}{2}-ibcx+acx+\frac{2bc\ln(\mathrm{e}^{i(fx+e)})}{f}-\frac{bc\ln(\mathrm{e}^{2i(fx+e)}+1)}{f}+\frac{2ibdex}{f}+\frac{ibd\,e^2}{f^2}-\frac{bd\ln(\mathrm{e}^{2i(fx+e)}+1)x}{f}+\frac{ibd}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*b*d*x^2+1/2*a*d*x^2-I*b*c*x+a*c*x+2/f*b*c*ln(exp(I*(f*x+e)))-1/f*b*c*
ln(exp(2*I*(f*x+e))+1)+2*I/f*b*d*e*x+I/f^2*b*d*e^2-1/f*b*d*ln(exp(2*I*(f*x+
e))+1)*x+1/2*I*b*d*polylog(2,-exp(2*I*(f*x+e)))/f^2-2/f^2*b*d*e*ln(exp(I*(f
*x+e)))
```

Maxima [A]
time = 0.55, size = 136, normalized size = 1.62

$$\frac{(a+ib)df^2x^2+2(a+ib)cf^2x+ibd\mathrm{Li}_2\left(-e^{(2ifx+2ie)}\right)+2(-ibdfx-ibcf)\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)-(bdfx+bcf)\log\left(\cos\left(2fx+2e\right)^2+\sin\left(2fx+2e\right)^2+2\cos\left(2fx+2e\right)+1\right)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a + I*b)*d*f^2*x^2 + 2*(a + I*b)*c*f^2*x + I*b*d*dilog(-e^(2*I*f*x +
2*I*e)) + 2*(-I*b*d*f*x - I*b*c*f)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1) - (b*d*f*x + b*c*f)*log(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1))/f^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(71) = 142$.
time = 0.37, size = 168, normalized size = 2.00

$$\frac{2adf^2x^2+4acf^2x-ibd\mathrm{Li}_2\left(\frac{2(i\tan(fx+e)-1)}{\tan(fx+e)^2+1}+1\right)+ibd\mathrm{Li}_2\left(\frac{2(-i\tan(fx+e)-1)}{\tan(fx+e)^2+1}+1\right)-2(bdfx+bcf)\log\left(-\frac{2(i\tan(fx+e)-1)}{\tan(fx+e)^2+1}\right)-2(bdfx+bcf)\log\left(-\frac{2(-i\tan(fx+e)-1)}{\tan(fx+e)^2+1}\right)}{4f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d*f^2*x^2 + 4*a*c*f^2*x - I*b*d*dilog(2*(I*tan(f*x + e) - 1)/(tan(
f*x + e)^2 + 1) + 1) + I*b*d*dilog(2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2
+ 1) + 1) - 2*(b*d*f*x + b*c*f)*log(-2*(I*tan(f*x + e) - 1)/(tan(f*x + e)^2
 + 1)) - 2*(b*d*f*x + b*c*f)*log(-2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 +
 1)))/f^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tan(e+fx))(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*tan(f*x + e) + a), x)
```

Mupad [B]
time = 3.22, size = 161, normalized size = 1.92

$$\frac{a\,x\,\left(2\,c+d\,x\right)}{2}-\frac{b\,d\,\left(\pi \,\ln \left(\cos \left(f\,x\right)\right)-\pi \,\ln \left({\mathrm{e}}^{f\,x\,2\mathrm{i}}+1\right)+\mathrm{polylog}\left(2,-{\mathrm{e}}^{-e\,2\mathrm{i}}\,{\mathrm{e}}^{-f\,x\,2\mathrm{i}}\right)\,1\mathrm{i}-\pi \,\ln \left({\mathrm{e}}^{-e\,2\mathrm{i}}\,{\mathrm{e}}^{-f\,x\,2\mathrm{i}}+1\right)+2\,e\,\ln \left({\mathrm{e}}^{-e\,2\mathrm{i}}\,{\mathrm{e}}^{-f\,x\,2\mathrm{i}}+1\right)-\ln \left(\cos \left(e+f\,x\right)\right)\,\left(2\,e-\pi \right)+f^2\,x^2\,1\mathrm{i}+2\,f\,x\,\ln \left({\mathrm{e}}^{-e\,2\mathrm{i}}\,{\mathrm{e}}^{-f\,x\,2\mathrm{i}}+1\right)+e\,f\,x\,2\mathrm{i}\right)}{2\,f^2}+\frac{b\,c\,\ln \left({\mathrm{tan}\left(e+f\,x\right)}^2+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*x),x)
```

```
[Out] (a*x*(2*c + d*x))/2 - (b*d*(polylog(2, -exp(-e*2i)*exp(-f*x*2i))*1i - pi*lo
g(exp(f*x*2i) + 1) - log(cos(e + f*x))*(2*e - pi) - pi*log(exp(-e*2i)*exp(-
f*x*2i) + 1) + 2*e*log(exp(-e*2i)*exp(-f*x*2i) + 1) + pi*log(cos(f*x)) + f^
2*x^2*1i + 2*f*x*log(exp(-e*2i)*exp(-f*x*2i) + 1) + e*f*x*2i))/(2*f^2) + (b
*c*log(tan(e + f*x)^2 + 1))/(2*f)
```

3.42 $\int \frac{a+b\tan(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{a+b\tan(e+fx)}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*tan(f*x+e))/(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\tan(e+fx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Tan[e + f*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*Tan[e + f*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{c+dx}\,dx = \int \frac{a+b\tan(e+fx)}{c+dx}\,dx$$

Mathematica [A]
time = 2.09, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan(e+fx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])/(c + d*x),x]
```

```
[Out] Integrate[(a + b*Tan[e + f*x])/(c + d*x), x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan(fx+e)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*x+c),x)
```

```
[Out] int((a+b*tan(f*x+e))/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*b*d*integrate(sin(2*f*x + 2*e)/((d*x + c)*cos(2*f*x + 2*e)^2 + (d*x + c)
*sin(2*f*x + 2*e)^2 + d*x + 2*(d*x + c)*cos(2*f*x + 2*e) + c), x) + a*log(d
*x + c))/d
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e) + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan{\left(e + fx \right)}}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a+b\tan(e+f\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + b*tan(e + f*x))/(c + d*x), x)
```

3.43 $\int \frac{a+b\tan(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{a+b\tan(e+fx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*tan(f*x+e))/(d*x+c)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\tan(e+fx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Tan[e + f*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Tan[e + f*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\tan(e+fx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 6.37, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan(e+fx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*Tan[e + f*x])/(c + d*x)^2, x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan\left(fx+e\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*x+c)^2,x)
```

```
[Out] int((a+b*tan(f*x+e))/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (2*(b*d^2*x + b*c*d)*integrate(sin(2*f*x + 2*e)/(d^2*x^2 + 2*c*d*x + (d^2*x
^2 + 2*c*d*x + c^2)*cos(2*f*x + 2*e)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*f*
x + 2*e)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(2*f*x + 2*e)), x) - a)/(
d^2*x + c*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e) + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan\left(e + fx\right)}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a+b\tan(e+f\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(c + d*x)^2,x)
```

```
[Out] int((a + b*tan(e + f*x))/(c + d*x)^2, x)
```

3.44 $\int (c+dx)^3(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=300

$$-\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}+\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}+\frac{3b^2d(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f^2}-\frac{2ab(c+dx)^3\log}{f}$$

```
[Out] -I*b^2*(d*x+c)^3/f+1/4*a^2*(d*x+c)^4/d+1/2*I*a*b*(d*x+c)^4/d-1/4*b^2*(d*x+c
)^4/d+3*b^2*d*(d*x+c)^2*ln(1+exp(2*I*(f*x+e)))/f^2-2*a*b*(d*x+c)^3*ln(1+exp
(2*I*(f*x+e)))/f-3*I*b^2*d^2*(d*x+c)*polylog(2,-exp(2*I*(f*x+e)))/f^3+3*I*a
*b*d*(d*x+c)^2*polylog(2,-exp(2*I*(f*x+e)))/f^2+3/2*b^2*d^3*polylog(3,-exp(
2*I*(f*x+e)))/f^4-3*a*b*d^2*(d*x+c)*polylog(3,-exp(2*I*(f*x+e)))/f^3-3/2*I*
a*b*d^3*polylog(4,-exp(2*I*(f*x+e)))/f^4+b^2*(d*x+c)^3*tan(f*x+e)/f
```

Rubi [A]
time = 0.37, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3803, 3800, 2221, 2611, 6744, 2320, 6724, 3801, 32}

$$\frac{a^2(c+dx)^4}{4d}-\frac{3abd^2(c+dx)\text{Li}_3\left(-e^{2i(e+fx)}\right)}{f^3}+\frac{3iabd(c+dx)^2\text{Li}_2\left(-e^{2i(e+fx)}\right)}{f^2}-\frac{2ab(c+dx)^3\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{iab(c+dx)^4}{2d}-\frac{3iabd^3\text{Li}_4\left(-e^{2i(e+fx)}\right)}{2f^4}-\frac{3ib^2d^2(c+dx)\text{Li}_2\left(-e^{2i(e+fx)}\right)}{f^3}+\frac{3b^2d(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f^2}+\frac{b^2(c+dx)^3\tan(e+fx)}{f}-\frac{ib^2(c+dx)^3}{f}-\frac{b^2(c+dx)^4}{4d}+\frac{3b^2d^3\text{Li}_3\left(-e^{2i(e+fx)}\right)}{2f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((-I)*b^2*(c + d*x)^3)/f + (a^2*(c + d*x)^4)/(4*d) + ((I/2)*a*b*(c + d*x)^4
)/d - (b^2*(c + d*x)^4)/(4*d) + (3*b^2*d*(c + d*x)^2*Log[1 + E^((2*I)*(e +
f*x))])/f^2 - (2*a*b*(c + d*x)^3*Log[1 + E^((2*I)*(e + f*x))])/f - ((3*I)*b
^2*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(e + f*x))])/f^3 + ((3*I)*a*b*d*(c +
d*x)^2*PolyLog[2, -E^((2*I)*(e + f*x))])/f^2 + (3*b^2*d^3*PolyLog[3, -E^((2
*I)*(e + f*x))])/(2*f^4) - (3*a*b*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(e + f
*x))])/f^3 - (((3*I)/2)*a*b*d^3*PolyLog[4, -E^((2*I)*(e + f*x))])/f^4 + (b^
2*(c + d*x)^3*Tan[e + f*x])/f
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3(a+b\tan(e+fx))^2\,dx &= \int \left(a^2(c+dx)^3+2ab(c+dx)^3\tan(e+fx)+b^2(c+dx)^3\tan^2(e+fx)\right.\\
&= \frac{a^2(c+dx)^4}{4d}+(2ab)\int (c+dx)^3\tan(e+fx)\,dx+b^2\int (c+dx)^3\tan^2(\\
&= \frac{a^2(c+dx)^4}{4d}+\frac{iab(c+dx)^4}{2d}+\frac{b^2(c+dx)^3\tan(e+fx)}{f}-(4iab)\int \frac{e^{2i(}}{1}\\
&= -\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}+\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}-\frac{2ab(c+}{}\\
&= -\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}+\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}+\frac{3b^2d(c+}{}\\
&= -\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}+\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}+\frac{3b^2d(c+}{}\\
&= -\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}+\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}+\frac{3b^2d(c+}{}\\
&= -\frac{ib^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}+\frac{iab(c+dx)^4}{2d}-\frac{b^2(c+dx)^4}{4d}+\frac{3b^2d(c+}{}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1347 vs. $2(300) = 600$.
time = 6.97, size = 1347, normalized size = 4.49

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -1/4*(b^2*d^3*((2*I)*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(1 + E^((2*I)*e))*L
og[1 + E^((2*I)*(e + f*x))]) + (6*I)*(1 + E^((2*I)*e))*f*x*PolyLog[2, -E^((
2*I)*(e + f*x))] - 3*(1 + E^((2*I)*e))*PolyLog[3, -E^((2*I)*(e + f*x))])*Se
c[e])/(E^(I*e)*f^4) + (a*b*c*d^2*((2*I)*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*
(1 + E^((2*I)*e))*Log[1 + E^((2*I)*(e + f*x))]) + (6*I)*(1 + E^((2*I)*e))*f
*x*PolyLog[2, -E^((2*I)*(e + f*x))] - 3*(1 + E^((2*I)*e))*PolyLog[3, -E^((2
*I)*(e + f*x))])*Sec[e])/(2*E^(I*e)*f^3) - (I/2)*a*b*d^3*E^(I*e)*(-x^4 + (1
```

```
 + E^((-2*I)*e))*x^4 - ((1 + E^((2*I)*e))*(2*f^4*x^4 + (4*I)*f^3*x^3*Log[1
+ E^((2*I)*(e + f*x))] + 6*f^2*x^2*PolyLog[2, -E^((2*I)*(e + f*x))] + (6*I)
*f*x*PolyLog[3, -E^((2*I)*(e + f*x))] - 3*PolyLog[4, -E^((2*I)*(e + f*x))])
)/(2*E^((2*I)*e)*f^4))*Sec[e] + (3*b^2*c^2*d*Sec[e]*(Cos[e]*Log[Cos[e]*Cos[
f*x] - Sin[e]*Sin[f*x]] + f*x*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)) - (2*a*b
*c^3*Sec[e]*(Cos[e]*Log[Cos[e]*Cos[f*x] - Sin[e]*Sin[f*x]] + f*x*Sin[e]))/(
f*(Cos[e]^2 + Sin[e]^2)) + (3*b^2*c*d^2*Csc[e]*((f^2*x^2)/E^(I*ArcTan[Cot[e
]]) - (Cot[e]*(I*f*x*(-Pi - 2*ArcTan[Cot[e]]) - Pi*Log[1 + E^((-2*I)*f*x)]
- 2*(f*x - ArcTan[Cot[e]])*Log[1 - E^((2*I)*(f*x - ArcTan[Cot[e]]))] + Pi*L
og[Cos[f*x]] - 2*ArcTan[Cot[e]]*Log[Sin[f*x - ArcTan[Cot[e]]]] + I*PolyLog[
2, E^((2*I)*(f*x - ArcTan[Cot[e]]))]))/Sqrt[1 + Cot[e]^2])*Sec[e])/(f^3*Sqr
t[Csc[e]^2*(Cos[e]^2 + Sin[e]^2)]) - (3*a*b*c^2*d*Csc[e]*((f^2*x^2)/E^(I*Ar
cTan[Cot[e]]) - (Cot[e]*(I*f*x*(-Pi - 2*ArcTan[Cot[e]]) - Pi*Log[1 + E^((-2
*I)*f*x)] - 2*(f*x - ArcTan[Cot[e]])*Log[1 - E^((2*I)*(f*x - ArcTan[Cot[e]]
))] + Pi*Log[Cos[f*x]] - 2*ArcTan[Cot[e]]*Log[Sin[f*x - ArcTan[Cot[e]]]] +
I*PolyLog[2, E^((2*I)*(f*x - ArcTan[Cot[e]]))]))/Sqrt[1 + Cot[e]^2])*Sec[e]
)/(f^2*Sqrt[Csc[e]^2*(Cos[e]^2 + Sin[e]^2)]) + (Sec[e]*Sec[e + f*x]*(4*a^2*
c^3*f*x*Cos[f*x] - 4*b^2*c^3*f*x*Cos[f*x] + 6*a^2*c^2*d*f*x^2*Cos[f*x] - 6*
b^2*c^2*d*f*x^2*Cos[f*x] + 4*a^2*c*d^2*f*x^3*Cos[f*x] - 4*b^2*c*d^2*f*x^3*C
os[f*x] + a^2*d^3*f*x^4*Cos[f*x] - b^2*d^3*f*x^4*Cos[f*x] + 4*a^2*c^3*f*x*C
os[2*e + f*x] - 4*b^2*c^3*f*x*Cos[2*e + f*x] + 6*a^2*c^2*d*f*x^2*Cos[2*e +
f*x] - 6*b^2*c^2*d*f*x^2*Cos[2*e + f*x] + 4*a^2*c*d^2*f*x^3*Cos[2*e + f*x]
- 4*b^2*c*d^2*f*x^3*Cos[2*e + f*x] + a^2*d^3*f*x^4*Cos[2*e + f*x] - b^2*d^3
*f*x^4*Cos[2*e + f*x] + 8*b^2*c^3*Sin[f*x] + 24*b^2*c^2*d*x*Sin[f*x] - 8*a*
b*c^3*f*x*Sin[f*x] + 24*b^2*c*d^2*x^2*Sin[f*x] - 12*a*b*c^2*d*f*x^2*Sin[f*x
] + 8*b^2*d^3*x^3*Sin[f*x] - 8*a*b*c*d^2*f*x^3*Sin[f*x] - 2*a*b*d^3*f*x^4*S
in[f*x] + 8*a*b*c^3*f*x*Sin[2*e + f*x] + 12*a*b*c^2*d*f*x^2*Sin[2*e + f*x]
+ 8*a*b*c*d^2*f*x^3*Sin[2*e + f*x] + 2*a*b*d^3*f*x^4*Sin[2*e + f*x]))/(8*f)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(271) = 542$.
time = 0.37, size = 952, normalized size = 3.17

method	result
risch	$\frac{12bac\,d^2e^2\ln(\mathrm{e}^{i(fx+e)})}{f^3}-\frac{6bac\,d^2\ln(\mathrm{e}^{2i(fx+e)}+1)x^2}{f}-\frac{6b\ln(\mathrm{e}^{2i(fx+e)}+1)a\,c^2dx}{f}+\frac{d^3a^2x^4}{4}+\frac{a^2c^4}{4d}-\frac{d^3b^2x^4}{4}-b^2c^3x$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 12/f^3*b*a*c*d^2*e^2*ln(exp(I*(f*x+e)))-2/f*b*a*d^3*ln(exp(2*I*(f*x+e))+1)*
x^3-6/f*b*a*c*d^2*ln(exp(2*I*(f*x+e))+1)*x^2-6/f*b*ln(exp(2*I*(f*x+e))+1)*a
*c^2*d*x-12*I/f^2*b^2*c*d^2*e*x-8*I/f^3*b*a*c*d^2*e^3+6*I/f^2*b*a*c^2*d*e^2
+3*I/f^2*b*a*c^2*d*polylog(2,-exp(2*I*(f*x+e)))+3*I/f^2*b*a*d^3*polylog(2,-
exp(2*I*(f*x+e)))*x^2+4*I/f^3*b*a*d^3*e^3*x+2*I*b^2*(d^3*x^3+3*c*d^2*x^2+3*
```

```
c^2*d*x+c^3)/f/(exp(2*I*(f*x+e))+1)+1/4*d^3*a^2*x^4+1/4/d*a^2*c^4-1/4*d^3*b
^2*x^4-b^2*c^3*x-1/4/d*b^2*c^4+2*I*d^2*a*b*c*x^3+3*I*d*a*b*c^2*x^2-d^2*b^2*
c*x^3-3/2*d*b^2*c^2*x^2-3/f^3*b*a*d^3*polylog(3,-exp(2*I*(f*x+e)))*x-4/f^4*
b*a*d^3*e^3*ln(exp(I*(f*x+e)))-3/f^3*b*a*c*d^2*polylog(3,-exp(2*I*(f*x+e)))
+12/f^3*b^2*c*d^2*e*ln(exp(I*(f*x+e)))+6/f^2*b^2*c*d^2*ln(exp(2*I*(f*x+e))+
1)*x-3/2*I*a*b*d^3*polylog(4,-exp(2*I*(f*x+e)))/f^4+1/2*I*d^3*a*b*x^4-2*I*a
*b*c^3*x-1/2*I/d*a*b*c^4+d^2*a^2*c*x^3+3/2*d*a^2*c^2*x^2+a^2*c^3*x+3/f^2*b^
2*d^3*ln(exp(2*I*(f*x+e))+1)*x^2-2/f*b*a*c^3*ln(exp(2*I*(f*x+e))+1)+4/f*b*a
*c^3*ln(exp(I*(f*x+e)))+3/f^2*b^2*c^2*d*ln(exp(2*I*(f*x+e))+1)-6/f^2*b^2*c^
2*d*ln(exp(I*(f*x+e)))-6/f^4*b^2*d^3*e^2*ln(exp(I*(f*x+e)))-2*I/f*b^2*d^3*x
^3+4*I/f^4*b^2*d^3*e^3-12/f^2*b*a*c^2*d*e*ln(exp(I*(f*x+e)))+6*I/f^2*b*a*c*
d^2*polylog(2,-exp(2*I*(f*x+e)))*x-12*I/f^2*b*a*c*d^2*e^2*x+12*I/f*b*a*c^2*
d*e*x+3*I/f^4*b*a*d^3*e^4-3*I/f^3*b^2*d^3*polylog(2,-exp(2*I*(f*x+e)))*x-6*
I/f*b^2*c*d^2*x^2-6*I/f^3*b^2*c*d^2*e^2+6*I/f^3*b^2*d^3*e^2*x-3*I/f^3*b^2*c
*d^2*polylog(2,-exp(2*I*(f*x+e)))+3/2*b^2*d^3*polylog(3,-exp(2*I*(f*x+e)))/
f^4
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2500 vs. $2(272) = 544$.
time = 1.26, size = 2500, normalized size = 8.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a^2*c^3 + (f*x + e)^4*a^2*d^3/f^3 + 4*(f*x + e)^3*a^2*c*d^
2/f^2 + 6*(f*x + e)^2*a^2*c^2*d/f - 4*(f*x + e)^3*a^2*d^3*e/f^3 - 12*(f*x +
 e)^2*a^2*c*d^2*e/f^2 - 12*(f*x + e)*a^2*c^2*d*e/f + 8*a*b*c^3*log(sec(f*x
+ e)) - 24*a*b*c^2*d*e*log(sec(f*x + e))/f + 6*(f*x + e)^2*a^2*d^3*e^2/f^3
+ 12*(f*x + e)*a^2*c*d^2*e^2/f^2 + 24*a*b*c*d^2*e^2*log(sec(f*x + e))/f^2 -
 4*(f*x + e)*a^2*d^3*e^3/f^3 - 8*a*b*d^3*e^3*log(sec(f*x + e))/f^3 + 4*(3*(
2*a*b + I*b^2)*(f*x + e)^4*d^3 + 24*b^2*c^3*f^3 - 72*b^2*c^2*d*f^2*e + 72*b
^2*c*d^2*f*e^2 - 24*b^2*d^3*e^3 + 12*((2*a*b + I*b^2)*c*d^2*f - (2*a*b*e +
I*b^2*e)*d^3)*(f*x + e)^3 + 18*((2*a*b + I*b^2)*c^2*d*f^2 - 2*(2*a*b*e + I*
b^2*e)*c*d^2*f + (2*a*b*e^2 + I*b^2*e^2)*d^3)*(f*x + e)^2 + 12*(I*b^2*c^3*f
^3 - 3*I*b^2*c^2*d*f^2*e + 3*I*b^2*c*d^2*f*e^2 - I*b^2*d^3*e^3)*(f*x + e) -
 4*(8*(f*x + e)^3*a*b*d^3 - 9*b^2*c^2*d*f^2 + 18*b^2*c*d^2*f*e - 9*b^2*d^3*
e^2 + 9*(2*a*b*c*d^2*f - (2*a*b*e + b^2)*d^3)*(f*x + e)^2 + 18*(a*b*c^2*d*f
^2 - (2*a*b*e + b^2)*c*d^2*f + (a*b*e^2 + b^2*e)*d^3)*(f*x + e) + (8*(f*x +
 e)^3*a*b*d^3 - 9*b^2*c^2*d*f^2 + 18*b^2*c*d^2*f*e - 9*b^2*d^3*e^2 + 9*(2*a
*b*c*d^2*f - (2*a*b*e + b^2)*d^3)*(f*x + e)^2 + 18*(a*b*c^2*d*f^2 - (2*a*b*
e + b^2)*c*d^2*f + (a*b*e^2 + b^2*e)*d^3)*(f*x + e))*cos(2*f*x + 2*e) - (-8
*I*(f*x + e)^3*a*b*d^3 + 9*I*b^2*c^2*d*f^2 - 18*I*b^2*c*d^2*f*e + 9*I*b^2*d
^3*e^2 + 9*(-2*I*a*b*c*d^2*f + (2*I*a*b*e + I*b^2)*d^3)*(f*x + e)^2 + 18*(-
```

```
I*a*b*c^2*d*f^2 + (2*I*a*b*e + I*b^2)*c*d^2*f + (-I*a*b*e^2 - I*b^2*e)*d^3)
*(f*x + e))*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1) + 3*((2*a*b + I*b^2)*(f*x + e)^4*d^3 + 4*((2*a*b + I*b^2)*c*d^2*f + (b^2
*(-I*e - 2) - 2*a*b*e)*d^3)*(f*x + e)^3 + 6*((2*a*b + I*b^2)*c^2*d*f^2 + 2*
(b^2*(-I*e - 2) - 2*a*b*e)*c*d^2*f + (b^2*(I*e^2 + 4*e) + 2*a*b*e^2)*d^3)*(
f*x + e)^2 + 4*(I*b^2*c^3*f^3 + 3*b^2*c^2*d*f^2*(-I*e - 2) + 3*b^2*c*d^2*f*
(I*e^2 + 4*e) + b^2*d^3*(-I*e^3 - 6*e^2))*(f*x + e))*cos(2*f*x + 2*e) + 12*
(4*(f*x + e)^2*a*b*d^3 + 3*a*b*c^2*d*f^2 - 3*(2*a*b*e + b^2)*c*d^2*f + 3*(a
*b*e^2 + b^2*e)*d^3 + 3*(2*a*b*c*d^2*f - (2*a*b*e + b^2)*d^3)*(f*x + e) + (
4*(f*x + e)^2*a*b*d^3 + 3*a*b*c^2*d*f^2 - 3*(2*a*b*e + b^2)*c*d^2*f + 3*(a*
b*e^2 + b^2*e)*d^3 + 3*(2*a*b*c*d^2*f - (2*a*b*e + b^2)*d^3)*(f*x + e))*cos
(2*f*x + 2*e) + (4*I*(f*x + e)^2*a*b*d^3 + 3*I*a*b*c^2*d*f^2 + 3*(-2*I*a*b*
e - I*b^2)*c*d^2*f + 3*(I*a*b*e^2 + I*b^2*e)*d^3 + 3*(2*I*a*b*c*d^2*f + (-2
*I*a*b*e - I*b^2)*d^3)*(f*x + e))*sin(2*f*x + 2*e))*dilog(-e^(2*I*f*x + 2*I
*e)) + 2*(8*I*(f*x + e)^3*a*b*d^3 - 9*I*b^2*c^2*d*f^2 + 18*I*b^2*c*d^2*f*e
- 9*I*b^2*d^3*e^2 + 9*(2*I*a*b*c*d^2*f + (-2*I*a*b*e - I*b^2)*d^3)*(f*x + e
)^2 + 18*(I*a*b*c^2*d*f^2 + (-2*I*a*b*e - I*b^2)*c*d^2*f + (I*a*b*e^2 + I*b
^2*e)*d^3)*(f*x + e) + (8*I*(f*x + e)^3*a*b*d^3 - 9*I*b^2*c^2*d*f^2 + 18*I*
b^2*c*d^2*f*e - 9*I*b^2*d^3*e^2 + 9*(2*I*a*b*c*d^2*f + (-2*I*a*b*e - I*b^2)
*d^3)*(f*x + e)^2 + 18*(I*a*b*c^2*d*f^2 + (-2*I*a*b*e - I*b^2)*c*d^2*f + (I
*a*b*e^2 + I*b^2*e)*d^3)*(f*x + e))*cos(2*f*x + 2*e) - (8*(f*x + e)^3*a*b*d
^3 - 9*b^2*c^2*d*f^2 + 18*b^2*c*d^2*f*e - 9*b^2*d^3*e^2 + 9*(2*a*b*c*d^2*f
- (2*a*b*e + b^2)*d^3)*(f*x + e)^2 + 18*(a*b*c^2*d*f^2 - (2*a*b*e + b^2)*c*
d^2*f + (a*b*e^2 + b^2*e)*d^3)*(f*x + e))*sin(2*f*x + 2*e))*log(cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1) - 24*(a*b*d^3*cos(2*
f*x + 2*e) + I*a*b*d^3*sin(2*f*x + 2*e) + a*b*d^3)*polylog(4, -e^(2*I*f*x +
 2*I*e)) + 6*(8*I*(f*x + e)*a*b*d^3 + 6*I*a*b*c*d^2*f + 3*(-2*I*a*b*e - I*b
^2)*d^3 + (8*I*(f*x + e)*a*b*d^3 + 6*I*a*b*c*d^2*f + 3*(-2*I*a*b*e - I*b^2)
*d^3)*cos(2*f*x + 2*e) - (8*(f*x + e)*a*b*d^3 + 6*a*b*c*d^2*f - 3*(2*a*b*e
+ b^2)*d^3)*sin(2*f*x + 2*e))*polylog(3, -e^(2*I*f*x + 2*I*e)) + 3*((2*I*a*
b - b^2)*(f*x + e)^4*d^3 + 4*((2*I*a*b - b^2)*c*d^2*f + (b^2*(e - 2*I) - 2*
I*a*b*e)*d^3)*(f*x + e)^3 + 6*((2*I*a*b - b^2)*c^2*d*f^2 + 2*(b^2*(e - 2*I)
 - 2*I*a*b*e)*c*d^2*f - (b^2*(e^2 - 4*I*e) - 2*I*a*b*e^2)*d^3)*(f*x + e)^2
- 4*(b^2*c^3*f^3 - 3*b^2*c^2*d*f^2*(e - 2*I) + 3*b^2*c*d^2*f*(e^2 - 4*I*e)
- b^2*d^3*(e^3 - 6*I*e^2))*(f*x + e))*sin(2*f*x + 2*e))/(-12*I*f^3*cos(2*f*
x + 2*e) + 12*f^3*sin(2*f*x + 2*e) - 12*I*f^3))/f
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(272) = 544$.
time = 0.39, size = 780, normalized size = 2.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - b^2)*d^3*f^4*x^4 + 4*(a^2 - b^2)*c*d^2*f^4*x^3 + 6*(a^2 - b^2)*
c^2*d*f^4*x^2 + 4*(a^2 - b^2)*c^3*f^4*x + 3*I*a*b*d^3*polylog(4, (tan(f*x +
 e)^2 + 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 3*I*a*b*d^3*polylog(4
, (tan(f*x + e)^2 - 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 6*(I*a*b*
d^3*f^2*x^2 + I*a*b*c^2*d*f^2 - I*b^2*c*d^2*f + I*(2*a*b*c*d^2*f^2 - b^2*d^
3*f)*x)*dilog(2*(I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1) + 1) - 6*(-I*a*b*
d^3*f^2*x^2 - I*a*b*c^2*d*f^2 + I*b^2*c*d^2*f - I*(2*a*b*c*d^2*f^2 - b^2*d^
3*f)*x)*dilog(2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1) + 1) - 2*(2*a*b*
d^3*f^3*x^3 + 2*a*b*c^3*f^3 - 3*b^2*c^2*d*f^2 + 3*(2*a*b*c*d^2*f^3 - b^2*d^
3*f^2)*x^2 + 6*(a*b*c^2*d*f^3 - b^2*c*d^2*f^2)*x)*log(-2*(I*tan(f*x + e) -
1)/(tan(f*x + e)^2 + 1)) - 2*(2*a*b*d^3*f^3*x^3 + 2*a*b*c^3*f^3 - 3*b^2*c^2
*d*f^2 + 3*(2*a*b*c*d^2*f^3 - b^2*d^3*f^2)*x^2 + 6*(a*b*c^2*d*f^3 - b^2*c*d
^2*f^2)*x)*log(-2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 3*(2*a*b*d^
3*f*x + 2*a*b*c*d^2*f - b^2*d^3)*polylog(3, (tan(f*x + e)^2 + 2*I*tan(f*x +
 e) - 1)/(tan(f*x + e)^2 + 1)) - 3*(2*a*b*d^3*f*x + 2*a*b*c*d^2*f - b^2*d^3
)*polylog(3, (tan(f*x + e)^2 - 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1))
+ 4*(b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*f^3*x^2 + 3*b^2*c^2*d*f^3*x + b^2*c^3*f^
3)*tan(f*x + e))/f^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(e + fx)})^2 (c + dx)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*tan(f*x+e))**2,x)
[Out] Integral((a + b*tan(e + f*x))**2*(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^3*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^2\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*x)^3,x)
[Out] int((a + b*tan(e + f*x))^2*(c + d*x)^3, x)
```

3.45 $\int (c+dx)^2(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=229

$$-\frac{ib^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}+\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{3d}+\frac{2b^2d(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f^2}-\frac{2ab(c+dx)^2\,\mathrm{lo}}{}$$

```
[Out] -I*b^2*(d*x+c)^2/f+1/3*a^2*(d*x+c)^3/d+2/3*I*a*b*(d*x+c)^3/d-1/3*b^2*(d*x+c
)^3/d+2*b^2*d*(d*x+c)*ln(1+exp(2*I*(f*x+e)))/f^2-2*a*b*(d*x+c)^2*ln(1+exp(2
*I*(f*x+e)))/f-I*b^2*d^2*polylog(2,-exp(2*I*(f*x+e)))/f^3+2*I*a*b*d*(d*x+c)
*polylog(2,-exp(2*I*(f*x+e)))/f^2-a*b*d^2*polylog(3,-exp(2*I*(f*x+e)))/f^3+
b^2*(d*x+c)^2*tan(f*x+e)/f
```

Rubi [A]
time = 0.26, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3803, 3800, 2221, 2611, 2320, 6724, 3801, 2317, 2438, 32}

$$\frac{a^2(c+dx)^3}{3d}+\frac{2iabd(c+dx)\mathrm{Li}_2\left(-e^{2i(e+fx)}\right)}{f^2}-\frac{2ab(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{2iab(c+dx)^3}{3d}-\frac{abd^2\mathrm{Li}_3\left(-e^{2i(e+fx)}\right)}{f^3}+\frac{2b^2d(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f^2}+\frac{b^2(c+dx)^2\tan(e+fx)}{f}-\frac{ib^2(c+dx)^2}{f}-\frac{b^2(c+dx)^3}{3d}-\frac{ib^2d^2\mathrm{Li}_2\left(-e^{2i(e+fx)}\right)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((-I)*b^2*(c + d*x)^2)/f + (a^2*(c + d*x)^3)/(3*d) + (((2*I)/3)*a*b*(c + d*
x)^3)/d - (b^2*(c + d*x)^3)/(3*d) + (2*b^2*d*(c + d*x)*Log[1 + E^((2*I)*(e
+ f*x))])/f^2 - (2*a*b*(c + d*x)^2*Log[1 + E^((2*I)*(e + f*x))])/f - (I*b^2
*d^2*PolyLog[2, -E^((2*I)*(e + f*x))])/f^3 + ((2*I)*a*b*d*(c + d*x)*PolyLog
[2, -E^((2*I)*(e + f*x))])/f^2 - (a*b*d^2*PolyLog[3, -E^((2*I)*(e + f*x))])
/f^3 + (b^2*(c + d*x)^2*Tan[e + f*x])/f
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2(a+b\tan(e+fx))^2\,dx &= \int \left(a^2(c+dx)^2+2ab(c+dx)^2\tan(e+fx)+b^2(c+dx)^2\tan^2(e+f\right.\\ &= \frac{a^2(c+dx)^3}{3d}+(2ab)\int (c+dx)^2\tan(e+fx)\,dx+b^2\int (c+dx)^2\tan^2\\ &= \frac{a^2(c+dx)^3}{3d}+\frac{2iab(c+dx)^3}{3d}+\frac{b^2(c+dx)^2\tan(e+fx)}{f}-(4iab)\int \\ &= -\frac{ib^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}+\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{3d}-\frac{2ab(c}{}\\ &= -\frac{ib^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}+\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{3d}+\frac{2b^2d(}{}\\ &= -\frac{ib^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}+\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{3d}+\frac{2b^2d(}{}\\ &= -\frac{ib^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}+\frac{2iab(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{3d}+\frac{2b^2d(}{}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 656 vs. 2(229) = 458.
time = 6.95, size = 656, normalized size = 2.86

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (a*b*d^2*((2*I)*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(1 + E^((2*I)*e))*Log[1
+ E^((2*I)*(e + f*x))]) + (6*I)*(1 + E^((2*I)*e))*f*x*PolyLog[2, -E^((2*I)*
(e + f*x))] - 3*(1 + E^((2*I)*e))*PolyLog[3, -E^((2*I)*(e + f*x))])*Sec[e])
/(6*E^(I*e)*f^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Sec[e]*(a^2*Cos[e] - b^2*
Cos[e] + 2*a*b*Sin[e]))/3 + (2*b^2*c*d*Sec[e]*(Cos[e]*Log[Cos[e]*Cos[f*x] -
 Sin[e]*Sin[f*x]] + f*x*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)) - (2*a*b*c^2*S
ec[e]*(Cos[e]*Log[Cos[e]*Cos[f*x] - Sin[e]*Sin[f*x]] + f*x*Sin[e]))/(f*(Cos
[e]^2 + Sin[e]^2)) + (b^2*d^2*Csc[e]*((f^2*x^2)/E^(I*ArcTan[Cot[e]]) - (Cot
[e]*(I*f*x*(-Pi - 2*ArcTan[Cot[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x -
```

```
 ArcTan[Cot[e]])*Log[1 - E^((2*I)*(f*x - ArcTan[Cot[e]]))] + Pi*Log[Cos[f*x
]] - 2*ArcTan[Cot[e]]*Log[Sin[f*x - ArcTan[Cot[e]]]] + I*PolyLog[2, E^((2*I
)*(f*x - ArcTan[Cot[e]]))]))/Sqrt[1 + Cot[e]^2])*Sec[e])/(f^3*Sqrt[Csc[e]^2
*(Cos[e]^2 + Sin[e]^2)]) - (2*a*b*c*d*Csc[e]*((f^2*x^2)/E^(I*ArcTan[Cot[e]]
) - (Cot[e]*(I*f*x*(-Pi - 2*ArcTan[Cot[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] -
2*(f*x - ArcTan[Cot[e]])*Log[1 - E^((2*I)*(f*x - ArcTan[Cot[e]]))] + Pi*Log
[Cos[f*x]] - 2*ArcTan[Cot[e]]*Log[Sin[f*x - ArcTan[Cot[e]]]] + I*PolyLog[2,
 E^((2*I)*(f*x - ArcTan[Cot[e]]))]))/Sqrt[1 + Cot[e]^2])*Sec[e])/(f^2*Sqrt[
Csc[e]^2*(Cos[e]^2 + Sin[e]^2)]) + (Sec[e]*Sec[e + f*x]*(b^2*c^2*Sin[f*x] +
 2*b^2*c*d*x*Sin[f*x] + b^2*d^2*x^2*Sin[f*x]))/f
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(209) = 418$.
time = 0.27, size = 575, normalized size = 2.51

method	result
risch	$-\frac{ab\,d^2\,\mathrm{polylog}\left(3,-\mathrm{e}^{2i(fx+e)}\right)}{f^3}-\frac{ib^2d^2\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(fx+e)}\right)}{f^3}-\frac{2ib^2d^2x^2}{f}-\frac{2ib^2d^2e^2}{f^3}+\frac{2id^2ab\,x^3}{3}-2iab\,c^2x-\frac{2iab\,c^3}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*a^2*c*x^2+a^2*c^2*x-d*b^2*c*x^2-2/f*b*a*d^2*ln(exp(2*I*(f*x+e))+1)*x^2+4/
f^3*b*a*d^2*e^2*ln(exp(I*(f*x+e)))-8/3*I/f^3*b*a*d^2*e^3-4*I/f^2*b^2*d^2*e*
x-2*I/f*b^2*d^2*x^2-2*I/f^3*b^2*d^2*e^2+2/3*I*d^2*a*b*x^3-8/f^2*b*a*c*d*e*l
n(exp(I*(f*x+e)))-4/f*b*ln(exp(2*I*(f*x+e))+1)*a*c*d*x+8*I/f*b*a*c*d*e*x-2*
I*a*b*c^2*x-2/3*I/d*a*b*c^3-4*I/f^2*b*a*d^2*e^2*x+4*I/f^2*b*a*c*d*e^2+2*I/f
^2*b*a*d^2*polylog(2,-exp(2*I*(f*x+e)))*x+2*I/f^2*b*a*c*d*polylog(2,-exp(2*
I*(f*x+e)))+2*I*b^2*(d^2*x^2+2*c*d*x+c^2)/f/(exp(2*I*(f*x+e))+1)+2/f^2*b^2*
d^2*ln(exp(2*I*(f*x+e))+1)*x+4/f^3*b^2*d^2*e*ln(exp(I*(f*x+e)))-2/f*b*a*c^2
*ln(exp(2*I*(f*x+e))+1)+4/f*b*a*c^2*ln(exp(I*(f*x+e)))+2/f^2*b^2*c*d*ln(exp
(2*I*(f*x+e))+1)-4/f^2*b^2*c*d*ln(exp(I*(f*x+e)))+1/3*d^2*a^2*x^3+1/3/d*a^2
*c^3-1/3*d^2*b^2*x^3-b^2*c^2*x-1/3/d*b^2*c^3+2*I*d*a*b*c*x^2-a*b*d^2*polylo
g(3,-exp(2*I*(f*x+e)))/f^3-I*b^2*d^2*polylog(2,-exp(2*I*(f*x+e)))/f^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1295 vs. $2(209) = 418$.
time = 0.72, size = 1295, normalized size = 5.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^2*c^2 + (f*x + e)^3*a^2*d^2/f^2 + 3*(f*x + e)^2*a^2*c*d/
f - 3*(f*x + e)^2*a^2*d^2*e/f^2 - 6*(f*x + e)*a^2*c*d*e/f + 6*a*b*c^2*log(s
```

```
ec(f*x + e)) - 12*a*b*c*d*e*log(sec(f*x + e))/f + 3*(f*x + e)*a^2*d^2*e^2/f
^2 + 6*a*b*d^2*e^2*log(sec(f*x + e))/f^2 + 3*((2*a*b + I*b^2)*(f*x + e)^3*d
^2 + 6*b^2*c^2*f^2 - 12*b^2*c*d*f*e + 6*b^2*d^2*e^2 + 3*((2*a*b + I*b^2)*c*
d*f - (2*a*b*e + I*b^2*e)*d^2)*(f*x + e)^2 + 3*(I*b^2*c^2*f^2 - 2*I*b^2*c*d
*f*e + I*b^2*d^2*e^2)*(f*x + e) - 6*((f*x + e)^2*a*b*d^2 - b^2*c*d*f + b^2*
d^2*e + (2*a*b*c*d*f - (2*a*b*e + b^2)*d^2)*(f*x + e) + ((f*x + e)^2*a*b*d^
2 - b^2*c*d*f + b^2*d^2*e + (2*a*b*c*d*f - (2*a*b*e + b^2)*d^2)*(f*x + e))*
cos(2*f*x + 2*e) - (-I*(f*x + e)^2*a*b*d^2 + I*b^2*c*d*f - I*b^2*d^2*e + (-
2*I*a*b*c*d*f + (2*I*a*b*e + I*b^2)*d^2)*(f*x + e))*sin(2*f*x + 2*e))*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + ((2*a*b + I*b^2)*(f*x + e)^3*d
^2 + 3*((2*a*b + I*b^2)*c*d*f + (b^2*(-I*e - 2) - 2*a*b*e)*d^2)*(f*x + e)^2
 + 3*(I*b^2*c^2*f^2 + 2*b^2*c*d*f*(-I*e - 2) + b^2*d^2*(I*e^2 + 4*e))*(f*x
+ e))*cos(2*f*x + 2*e) + 3*(2*(f*x + e)*a*b*d^2 + 2*a*b*c*d*f - (2*a*b*e +
b^2)*d^2 + (2*(f*x + e)*a*b*d^2 + 2*a*b*c*d*f - (2*a*b*e + b^2)*d^2)*cos(2*
f*x + 2*e) + (2*I*(f*x + e)*a*b*d^2 + 2*I*a*b*c*d*f + (-2*I*a*b*e - I*b^2)*
d^2)*sin(2*f*x + 2*e))*dilog(-e^(2*I*f*x + 2*I*e)) + 3*(I*(f*x + e)^2*a*b*d
^2 - I*b^2*c*d*f + I*b^2*d^2*e + (2*I*a*b*c*d*f + (-2*I*a*b*e - I*b^2)*d^2)
*(f*x + e) + (I*(f*x + e)^2*a*b*d^2 - I*b^2*c*d*f + I*b^2*d^2*e + (2*I*a*b*
c*d*f + (-2*I*a*b*e - I*b^2)*d^2)*(f*x + e))*cos(2*f*x + 2*e) - ((f*x + e)^
2*a*b*d^2 - b^2*c*d*f + b^2*d^2*e + (2*a*b*c*d*f - (2*a*b*e + b^2)*d^2)*(f*
x + e))*sin(2*f*x + 2*e))*log(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1) + 3*(I*a*b*d^2*cos(2*f*x + 2*e) - a*b*d^2*sin(2*f*x +
2*e) + I*a*b*d^2)*polylog(3, -e^(2*I*f*x + 2*I*e)) - ((-2*I*a*b + b^2)*(f*x
 + e)^3*d^2 - 3*((2*I*a*b - b^2)*c*d*f + (b^2*(e - 2*I) - 2*I*a*b*e)*d^2)*(
f*x + e)^2 + 3*(b^2*c^2*f^2 - 2*b^2*c*d*f*(e - 2*I) + b^2*d^2*(e^2 - 4*I*e)
)*(f*x + e))*sin(2*f*x + 2*e))/(-3*I*f^2*cos(2*f*x + 2*e) + 3*f^2*sin(2*f*x
 + 2*e) - 3*I*f^2))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(209) = 418$.
time = 0.39, size = 465, normalized size = 2.03

$$\frac{2\,(a^2-b^2)d^2f^3x^3+6\,(a^2-b^2)cdf^3x^2+6\,(a^2-b^2)c^2f^3x-3\,abd^2\mathrm{polylog}\left(3,\frac{\tan(fx+e)^2+2i\,\tan(fx+e)-1}{\tan(fx+e)^2+1}\right)-3\,abd^2\mathrm{polylog}\left(3,\frac{\tan(fx+e)^2-2i\,\tan(fx+e)-1}{\tan(fx+e)^2+1}\right)-3\,(2i\,abd^2fx+2i\,abcdf-i\,b^2d^2)\mathrm{Li}_2\left(\frac{2\,(i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}+1\right)-3\,(-2i\,abd^2fx-2i\,abcdf+i\,b^2d^2)\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}+1\right)-6\,(abd^2f^2x^2+abc^2f^2-b^2cdf+(2\,abcdf^2-b^2d^2f)x)\log\left(-\frac{2\,(i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}\right)-6\,(abd^2f^2x^2+abc^2f^2-b^2cdf+(2\,abcdf^2-b^2d^2f)x)\log\left(-\frac{2\,(-i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}\right)+6\,(b^2d^2f^2x^2+2\,b^2cdf^2x+b^2c^2f^2)\tan\left(fx+e\right)}{6\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^2 - b^2)*d^2*f^3*x^3 + 6*(a^2 - b^2)*c*d*f^3*x^2 + 6*(a^2 - b^2)*
c^2*f^3*x - 3*a*b*d^2*polylog(3, (tan(f*x + e)^2 + 2*I*tan(f*x + e) - 1)/(t
an(f*x + e)^2 + 1)) - 3*a*b*d^2*polylog(3, (tan(f*x + e)^2 - 2*I*tan(f*x +
e) - 1)/(tan(f*x + e)^2 + 1)) - 3*(2*I*a*b*d^2*f*x + 2*I*a*b*c*d*f - I*b^2*
d^2)*dilog(2*(I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1) + 1) - 3*(-2*I*a*b*d
^2*f*x - 2*I*a*b*c*d*f + I*b^2*d^2)*dilog(2*(-I*tan(f*x + e) - 1)/(tan(f*x
+ e)^2 + 1) + 1) - 6*(a*b*d^2*f^2*x^2 + a*b*c^2*f^2 - b^2*c*d*f + (2*a*b*c*
d*f^2 - b^2*d^2*f)*x)*log(-2*(I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 6
```

```
*(a*b*d^2*f^2*x^2 + a*b*c^2*f^2 - b^2*c*d*f + (2*a*b*c*d*f^2 - b^2*d^2*f)*x
)*log(-2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) + 6*(b^2*d^2*f^2*x^2 +
 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*tan(f*x + e))/f^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(e + fx)})^2 (c + dx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*tan(f*x+e))**2,x)
[Out] Integral((a + b*tan(e + f*x))**2*(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\tan(e + f\,x))^2\,(c + d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*x)^2,x)
[Out] int((a + b*tan(e + f*x))^2*(c + d*x)^2, x)
```

3.46 $\int (c+dx)(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=136

$$-b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{iab(c+dx)^2}{d}-\frac{2ab(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{b^2d\log(\cos(e+fx))}{f^2}+\frac{iabd\mathrm{Po}}{}$$

```
[Out] -b^2*c*x-1/2*b^2*d*x^2+1/2*a^2*(d*x+c)^2/d+I*a*b*(d*x+c)^2/d-2*a*b*(d*x+c)*
ln(1+exp(2*I*(f*x+e)))/f+b^2*d*ln(cos(f*x+e))/f^2+I*a*b*d*polylog(2,-exp(2*
I*(f*x+e)))/f^2+b^2*(d*x+c)*tan(f*x+e)/f
```

Rubi [A]
time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3803, 3800, 2221, 2317, 2438, 3801, 3556}

$$\frac{a^2(c+dx)^2}{2d}-\frac{2ab(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{iab(c+dx)^2}{d}+\frac{iabd\mathrm{Li}_2\left(-e^{2i(e+fx)}\right)}{f^2}+\frac{b^2(c+dx)\tan(e+fx)}{f}-b^2cx+\frac{b^2d\log(\cos(e+fx))}{f^2}-\frac{1}{2}b^2dx^2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -(b^2*c*x) - (b^2*d*x^2)/2 + (a^2*(c + d*x)^2)/(2*d) + (I*a*b*(c + d*x)^2)/
d - (2*a*b*(c + d*x)*Log[1 + E^((2*I)*(e + f*x))])/f + (b^2*d*Log[Cos[e + f
*x]])/f^2 + (I*a*b*d*PolyLog[2, -E^((2*I)*(e + f*x))])/f^2 + (b^2*(c + d*x)
*Tan[e + f*x])/f
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+b\tan(e+fx))^2\,dx &= \int \left(a^2(c+dx)+2ab(c+dx)\tan(e+fx)+b^2(c+dx)\tan^2(e+fx)\right)\,dx \\
&= \frac{a^2(c+dx)^2}{2d}+(2ab)\int (c+dx)\tan(e+fx)\,dx+b^2\int (c+dx)\tan^2(e+ \\
&= \frac{a^2(c+dx)^2}{2d}+\frac{iab(c+dx)^2}{d}+\frac{b^2(c+dx)\tan(e+fx)}{f}-(4iab)\int \frac{e^{2i(e+}}{1+} \\
&= -b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{iab(c+dx)^2}{d}-\frac{2ab(c+dx)\log\left(1+e\right.}{f} \\
&= -b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{iab(c+dx)^2}{d}-\frac{2ab(c+dx)\log\left(1+e\right.}{f} \\
&= -b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{iab(c+dx)^2}{d}-\frac{2ab(c+dx)\log\left(1+e\right.}{f}
\end{aligned}$$

Mathematica [A]
time = 2.21, size = 200, normalized size = 1.47

$$\frac{\cos(e+fx)\left(\cos(e+fx)\left(-\left((e+fx)\left(-2iabd(e+fx)+a^2(de-2cf-dfx)+b^2(-de+2cf+dfx)\right)\right)-4abd(e+fx)\log\left(1+e^{2i(e+fx)}\right)+2b(bd+2ade-2acf)\log(\cos(e+fx))\right)+2iabd\cos(e+fx)\text{PolyLog}\left(2,-e^{2i(e+fx)}\right)+2b^2f(c+dx)\sin(e+fx)\right)(a+b\tan(e+fx))^2}{2f^2(a\cos(e+fx)+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(Cos[e + f*x]*(-((e + f*x)*((-2*I)*a*b*d*(e + f*x) + a^2*(d*e
 - 2*c*f - d*f*x) + b^2*(-(d*e) + 2*c*f + d*f*x))) - 4*a*b*d*(e + f*x)*Log[
1 + E^((2*I)*(e + f*x))] + 2*b*(b*d + 2*a*d*e - 2*a*c*f)*Log[Cos[e + f*x]])
 + (2*I)*a*b*d*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))] + 2*b^2*f*(c +
 d*x)*Sin[e + f*x])*(a + b*Tan[e + f*x])^2)/(2*f^2*(a*Cos[e + f*x] + b*Sin[
e + f*x])^2)
```

Maple [A]
time = 0.23, size = 238, normalized size = 1.75

method	result
risch	$-\frac{b^2 d x^2}{2}+\frac{2ib^2(dx+c)}{f\left(e^{2i(fx+e)}+1\right)}+\frac{a^2 d x^2}{2}-b^2cx+iabd\,x^2+a^2cx+\frac{4ibadex}{f}+\frac{b^2 d\ln\left(e^{2i(fx+e)}+1\right)}{f^2}-\frac{2b^2 d\ln\left(e^{i(fx+e}\right.}{f^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*d*x^2+2*I*b^2*(d*x+c)/f/(exp(2*I*(f*x+e))+1)+1/2*a^2*d*x^2-b^2*c*x
+I*a*b*d*x^2+a^2*c*x+4*I/f*b*a*d*e*x+1/f^2*b^2*d*ln(exp(2*I*(f*x+e))+1)-2/f
^2*b^2*d*ln(exp(I*(f*x+e)))-2/f*b*a*c*ln(exp(2*I*(f*x+e))+1)+4/f*b*a*c*ln(e
xp(I*(f*x+e)))-4/f^2*b*a*d*e*ln(exp(I*(f*x+e)))+I*a*b*d*polylog(2,-exp(2*I*
(f*x+e)))/f^2-2*I*a*b*c*x+2*I/f^2*b*a*d*e^2-2/f*b*ln(exp(2*I*(f*x+e))+1)*a*
d*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(127) = 254.
time = 0.58, size = 559, normalized size = 4.11

$$\frac{2(fx+e)a^2c+\frac{(fx+e)^2a^2d}{f}-\frac{2(fx+e)a^2de}{f}+4abc\log(\sec(fx+e))-\frac{4abde\log(\sec(fx+e))}{f}+\frac{2\left(\left(2ab+ib^2\right)(fx+e)^2d+4b^2cf-4b^2de+2\left(ib^2cf-ib^2de\right)(fx+e)-2\left(2(fx+e)abd-b^2d+\left(2(fx+e)abd-b^2d\right)\cos(2fx+2e)-\left(-2i(fx+e)abd+ib^2d\right)\sin(2fx+2e)\right)\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)+\left(\left(2ab+ib^2\right)(fx+e)^2d+2\left(ib^2cf+b^2d(-ie-2)\right)(fx+e)\right)\cos(2fx+2e)+2\left(abd\cos(2fx+2e)+iabd\sin(2fx+2e)+abd\right)\mathrm{Li}_2\left(-e^{(2ifx+2ie)}\right)-\left(-2i(fx+e)abd+ib^2d+\left(-2i(fx+e)abd+ib^2d\right)\cos(2fx+2e)+\left(2(fx+e)abd-b^2d\right)\sin(2fx+2e)\right)\log\left(\cos(2fx+2e)^2+\sin(2fx+2e)^2+2\cos(2fx+2e)+1\right)-\left(\left(-2iab+b^2\right)(fx+e)^2d+2\left(b^2cf-b^2d(e-2i)\right)(fx+e)\right)\sin(2fx+2e)\right)}{-2if\cos(2fx+2e)+2f\sin(2fx+2e)-2if}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a^2*c + (f*x + e)^2*a^2*d/f - 2*(f*x + e)*a^2*d*e/f + 4*a*
b*c*log(sec(f*x + e)) - 4*a*b*d*e*log(sec(f*x + e))/f + 2*((2*a*b + I*b^2)*
(f*x + e)^2*d + 4*b^2*c*f - 4*b^2*d*e + 2*(I*b^2*c*f - I*b^2*d*e)*(f*x + e)
 - 2*(2*(f*x + e)*a*b*d - b^2*d + (2*(f*x + e)*a*b*d - b^2*d)*cos(2*f*x + 2
*e) - (-2*I*(f*x + e)*a*b*d + I*b^2*d)*sin(2*f*x + 2*e))*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1) + ((2*a*b + I*b^2)*(f*x + e)^2*d + 2*(I*b^2*c
*f + b^2*d*(-I*e - 2))*(f*x + e))*cos(2*f*x + 2*e) + 2*(a*b*d*cos(2*f*x + 2
*e) + I*a*b*d*sin(2*f*x + 2*e) + a*b*d)*dilog(-e^(2*I*f*x + 2*I*e)) - (-2*I
*(f*x + e)*a*b*d + I*b^2*d + (-2*I*(f*x + e)*a*b*d + I*b^2*d)*cos(2*f*x + 2
```

```
*e) + (2*(f*x + e)*a*b*d - b^2*d)*sin(2*f*x + 2*e))*log(cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1) - ((-2*I*a*b + b^2)*(f*x + e
)^2*d + 2*(b^2*c*f - b^2*d*(e - 2*I))*(f*x + e))*sin(2*f*x + 2*e))/(-2*I*f*
cos(2*f*x + 2*e) + 2*f*sin(2*f*x + 2*e) - 2*I*f))/f
```

Fricas [A]
time = 0.37, size = 228, normalized size = 1.68

$$\frac{(a^2-b^2)df^2x^2+2\,(a^2-b^2)cf^2x-i\,abd\mathrm{Li}_2\left(\frac{2\,(i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}+1\right)+i\,abd\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}+1\right)-(2\,abdfx+2\,abcf-b^2d)\log\left(-\frac{2\,(i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}\right)-(2\,abdfx+2\,abcf-b^2d)\log\left(-\frac{2\,(-i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}\right)+2\,(b^2dfx+b^2cf)\tan\left(fx+e\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - b^2)*d*f^2*x^2 + 2*(a^2 - b^2)*c*f^2*x - I*a*b*d*dilog(2*(I*tan
(f*x + e) - 1)/(tan(f*x + e)^2 + 1) + 1) + I*a*b*d*dilog(2*(-I*tan(f*x + e)
 - 1)/(tan(f*x + e)^2 + 1) + 1) - (2*a*b*d*f*x + 2*a*b*c*f - b^2*d)*log(-2*
(I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - (2*a*b*d*f*x + 2*a*b*c*f - b^2
*d)*log(-2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) + 2*(b^2*d*f*x + b^2
*c*f)*tan(f*x + e))/f^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(e+fx\right)\right)^2\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\tan\left(e+f\,x\right)\right)}^2\,\left(c+d\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*x),x)
```

```
[Out] int((a + b*tan(e + f*x))^2*(c + d*x), x)
```

3.47 $\int \frac{(a+b\tan(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{(a+b\tan(e+fx))^2}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*tan(f*x+e))^2/(d*x+c),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\tan(e+fx))^2}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Tan[e + f*x])^2/(c + d*x),x]
[Out] Defer[Int][(a + b*Tan[e + f*x])^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\tan(e+fx))^2}{c+dx}\,dx$$

Mathematica [A]
time = 19.31, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^2}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(c + d*x),x]
[Out] Integrate[(a + b*Tan[e + f*x])^2/(c + d*x), x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*x+c),x)
```

```
[Out] int((a+b*tan(f*x+e))^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] (((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2*e)^2*log(d*x + c) + 2*
b^2*d*sin(2*f*x + 2*e) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(d*x + c)
*sin(2*f*x + 2*e)^2 + 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2
*e)*log(d*x + c) + (d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2
+ (d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 2*(d^2*f*x + c*d*f)*cos(2*f*x + 2*
e))*integrate(2*(2*a*b*d*f*x + 2*a*b*c*f + b^2*d)*sin(2*f*x + 2*e)/(d^2*f*x
^2 + 2*c*d*f*x + c^2*f + (d^2*f*x^2 + 2*c*d*f*x + c^2*f)*cos(2*f*x + 2*e)^2
 + (d^2*f*x^2 + 2*c*d*f*x + c^2*f)*sin(2*f*x + 2*e)^2 + 2*(d^2*f*x^2 + 2*c*
d*f*x + c^2*f)*cos(2*f*x + 2*e)), x) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f
)*log(d*x + c))/(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + (
d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 2*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^2}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*x+c),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\tan(e+f\,x))^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(c + d*x),x)
```

```
[Out] int((a + b*tan(e + f*x))^2/(c + d*x), x)
```

3.48 $\int \frac{(a+b\tan(e+fx))^2}{(c+dx)^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\tan(e+fx))^2}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*tan(f*x+e))^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\tan(e+fx))^2}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Tan[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Tan[e + f*x])^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(c+dx)^2} dx = \int \frac{(a+b\tan(e+fx))^2}{(c+dx)^2} dx$$

Mathematica [A]
time = 14.75, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^2}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*Tan[e + f*x])^2/(c + d*x)^2, x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] int((a+b*tan(f*x+e))^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((a^2 - b^2)*d*f*x - 2*b^2*d*sin(2*f*x + 2*e) + (a^2 - b^2)*c*f + ((a^2 -
b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2*e)^2 + ((a^2 - b^2)*d*f*x + (a^
2 - b^2)*c*f)*sin(2*f*x + 2*e)^2 + 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*
cos(2*f*x + 2*e) - (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2 + 2*c*d^
2*f*x + c^2*d*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*s
in(2*f*x + 2*e)^2 + 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e))
*integrate(4*(a*b*d*f*x + a*b*c*f + b^2*d)*sin(2*f*x + 2*e)/(d^3*f*x^3 + 3*
c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*
x + c^3*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x +
c^3*f)*sin(2*f*x + 2*e)^2 + 2*(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^
3*f)*cos(2*f*x + 2*e)), x))/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2
 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c
^2*d*f)*sin(2*f*x + 2*e)^2 + 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*
x + 2*e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(d^2*x^2 + 2*c*d*x
 + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^2}{(c + dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\tan(e+f\,x))^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + b*tan(e + f*x))^2/(c + d*x)^2, x)
```

3.49 $\int (c+dx)^3 (a+b\tan(e+fx))^3 \, dx$

Optimal. Leaf size=612

$$\frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}+\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}+\frac{3ia^2b(c+dx)^4}{4d}-\frac{3ab^2(c+dx)^4}{4d}-\frac{ib^3(c+dx)^4}{4d}-$$

```
[Out] 3/2*I*b^3*d*(d*x+c)^2/f^2-3/2*I*b^3*d*(d*x+c)^2*polylog(2,-exp(2*I*(f*x+e))
)/f^2+1/2*b^3*(d*x+c)^3/f+1/4*a^3*(d*x+c)^4/d-3*I*a*b^2*(d*x+c)^3/f-3/4*a*b
^2*(d*x+c)^4/d+3/4*I*a^2*b*(d*x+c)^4/d-3*b^3*d^2*(d*x+c)*ln(1+exp(2*I*(f*x+
e)))/f^3+9*a*b^2*d*(d*x+c)^2*ln(1+exp(2*I*(f*x+e)))/f^2-3*a^2*b*(d*x+c)^3*l
n(1+exp(2*I*(f*x+e)))/f+b^3*(d*x+c)^3*ln(1+exp(2*I*(f*x+e)))/f+9/2*I*a^2*b*
d*(d*x+c)^2*polylog(2,-exp(2*I*(f*x+e)))/f^2+3/2*I*b^3*d^3*polylog(2,-exp(2
*I*(f*x+e)))/f^4+3/4*I*b^3*d^3*polylog(4,-exp(2*I*(f*x+e)))/f^4-9/4*I*a^2*b
*d^3*polylog(4,-exp(2*I*(f*x+e)))/f^4+9/2*a*b^2*d^3*polylog(3,-exp(2*I*(f*x
+e)))/f^4-9/2*a^2*b*d^2*(d*x+c)*polylog(3,-exp(2*I*(f*x+e)))/f^3+3/2*b^3*d^
2*(d*x+c)*polylog(3,-exp(2*I*(f*x+e)))/f^3-9*I*a*b^2*d^2*(d*x+c)*polylog(2,
-exp(2*I*(f*x+e)))/f^3-1/4*I*b^3*(d*x+c)^4/d-3/2*b^3*d*(d*x+c)^2*tan(f*x+e)
/f^2+3*a*b^2*(d*x+c)^3*tan(f*x+e)/f+1/2*b^3*(d*x+c)^3*tan(f*x+e)^2/f
```

Rubi [A]

time = 0.67, antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3803, 3800, 2221, 2611, 6744, 2320, 6724, 3801, 32, 2317, 2438}

$$\frac{a^3(c+dx)^4}{4d}-\frac{9a^2bd^2(c+dx)\text{Li}_3\left(-e^{2i(e+fx)}\right)}{2f^3}+\frac{9ia^2bd(c+dx)^2\text{Li}_2\left(-e^{2i(e+fx)}\right)}{2f^2}-\frac{3a^2b(c+dx)^3\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{3ia^2b(c+dx)^4}{4d}-\frac{9ia^2bd^3\text{Li}_4\left(-e^{2i(e+fx)}\right)}{4f^4}-\frac{9iab^2d^2(c+dx)\text{Li}_2\left(-e^{2i(e+fx)}\right)}{f^3}+\frac{9ab^2d(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f^2}+\frac{3ab^2(c+dx)^3\tan(e+fx)}{f}-\frac{3iab^2(c+dx)^3}{f}-\frac{3ab^2(c+dx)^4}{4d}+\frac{9ab^2d^3\text{Li}_3\left(-e^{2i(e+fx)}\right)}{2f^4}+\frac{3b^3d^2(c+dx)\text{Li}_3\left(-e^{2i(e+fx)}\right)}{2f^3}-\frac{3b^3d^2(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f^3}-\frac{3ib^3d(c+dx)^2\text{Li}_2\left(-e^{2i(e+fx)}\right)}{2f^2}-\frac{3b^3d(c+dx)^2\tan(e+fx)}{2f^2}+\frac{b^3(c+dx)^3\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{b^3(c+dx)^3\tan^2(e+fx)}{2f}+\frac{3ib^3d(c+dx)^2}{2f^2}+\frac{b^3(c+dx)^3}{2f}-\frac{ib^3(c+dx)^4}{4d}+\frac{3ib^3d^3\text{Li}_2\left(-e^{2i(e+fx)}\right)}{2f^4}+\frac{3ib^3d^3\text{Li}_4\left(-e^{2i(e+fx)}\right)}{4f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (((3*I)/2)*b^3*d*(c + d*x)^2)/f^2 - ((3*I)*a*b^2*(c + d*x)^3)/f + (b^3*(c +
 d*x)^3)/(2*f) + (a^3*(c + d*x)^4)/(4*d) + (((3*I)/4)*a^2*b*(c + d*x)^4)/d
- (3*a*b^2*(c + d*x)^4)/(4*d) - ((I/4)*b^3*(c + d*x)^4)/d - (3*b^3*d^2*(c +
 d*x)*Log[1 + E^((2*I)*(e + f*x))])/f^3 + (9*a*b^2*d*(c + d*x)^2*Log[1 + E^
((2*I)*(e + f*x))])/f^2 - (3*a^2*b*(c + d*x)^3*Log[1 + E^((2*I)*(e + f*x))]
)/f + (b^3*(c + d*x)^3*Log[1 + E^((2*I)*(e + f*x))])/f + (((3*I)/2)*b^3*d^3
*PolyLog[2, -E^((2*I)*(e + f*x))])/f^4 - ((9*I)*a*b^2*d^2*(c + d*x)*PolyLog
[2, -E^((2*I)*(e + f*x))])/f^3 + (((9*I)/2)*a^2*b*d*(c + d*x)^2*PolyLog[2,
-E^((2*I)*(e + f*x))])/f^2 - (((3*I)/2)*b^3*d*(c + d*x)^2*PolyLog[2, -E^((2
*I)*(e + f*x))])/f^2 + (9*a*b^2*d^3*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^
4) - (9*a^2*b*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^3) + (3*
b^3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^3) - (((9*I)/4)*a^
2*b*d^3*PolyLog[4, -E^((2*I)*(e + f*x))])/f^4 + (((3*I)/4)*b^3*d^3*PolyLog[
4, -E^((2*I)*(e + f*x))])/f^4 - (3*b^3*d*(c + d*x)^2*Tan[e + f*x])/(2*f^2)
+ (3*a*b^2*(c + d*x)^3*Tan[e + f*x])/f + (b^3*(c + d*x)^3*Tan[e + f*x]^2)/(
2*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3(a+b\tan(e+fx))^3\,dx = \int \left(a^3(c+dx)^3+3a^2b(c+dx)^3\tan(e+fx)+3ab^2(c+dx)^3\tan^2(e+\right.$$
$$= \frac{a^3(c+dx)^4}{4d}+\left(3a^2b\right)\int (c+dx)^3\tan(e+fx)\,dx+\left(3ab^2\right)\int (c+dx)^3$$
$$= \frac{a^3(c+dx)^4}{4d}+\frac{3ia^2b(c+dx)^4}{4d}+\frac{3ab^2(c+dx)^3\tan(e+fx)}{f}+\frac{b^3(c+d}{}$$
$$= -\frac{3iab^2(c+dx)^3}{f}+\frac{a^3(c+dx)^4}{4d}+\frac{3ia^2b(c+dx)^4}{4d}-\frac{3ab^2(c+dx)^4}{4d}-\frac{i}{}$$
$$= \frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}+\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}+\frac{3ia^2b}{}$$
$$= \frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}+\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}+\frac{3ia^2b}{}$$
$$= \frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}+\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}+\frac{3ia^2b}{}$$
$$= \frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}+\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}+\frac{3ia^2b}{}$$
$$= \frac{3ib^3d(c+dx)^2}{2f^2}-\frac{3iab^2(c+dx)^3}{f}+\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}+\frac{3ia^2b}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2607 vs. $2(612) = 1224$.
time = 7.86, size = 2607, normalized size = 4.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-3*a*b^2*d^3*((2*I)*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(1 + E^((2*I)*e))*L
og[1 + E^((2*I)*(e + f*x))]) + (6*I)*(1 + E^((2*I)*e))*f*x*PolyLog[2, -E^((
2*I)*(e + f*x))] - 3*(1 + E^((2*I)*e))*PolyLog[3, -E^((2*I)*(e + f*x))])*Se
c[e])/(4*E^(I*e)*f^4) + (3*a^2*b*c*d^2*((2*I)*f^2*x^2*(2*E^((2*I)*e)*f*x +
(3*I)*(1 + E^((2*I)*e))*Log[1 + E^((2*I)*(e + f*x))]) + (6*I)*(1 + E^((2*I)
*e))*f*x*PolyLog[2, -E^((2*I)*(e + f*x))] - 3*(1 + E^((2*I)*e))*PolyLog[3,
-E^((2*I)*(e + f*x))])*Sec[e])/(4*E^(I*e)*f^3) - (b^3*c*d^2*((2*I)*f^2*x^2*
(2*E^((2*I)*e)*f*x + (3*I)*(1 + E^((2*I)*e))*Log[1 + E^((2*I)*(e + f*x))])
+ (6*I)*(1 + E^((2*I)*e))*f*x*PolyLog[2, -E^((2*I)*(e + f*x))] - 3*(1 + E^(
(2*I)*e))*PolyLog[3, -E^((2*I)*(e + f*x))])*Sec[e])/(4*E^(I*e)*f^3) - ((3*I
```

```
)/4)*a^2*b*d^3*E^(I*e)*(-x^4 + (1 + E^((-2*I)*e))*x^4 - ((1 + E^((2*I)*e))*
(2*f^4*x^4 + (4*I)*f^3*x^3*Log[1 + E^((2*I)*(e + f*x))] + 6*f^2*x^2*PolyLog
[2, -E^((2*I)*(e + f*x))] + (6*I)*f*x*PolyLog[3, -E^((2*I)*(e + f*x))] - 3*
PolyLog[4, -E^((2*I)*(e + f*x))]))/(2*E^((2*I)*e)*f^4))*Sec[e] + (I/4)*b^3*
d^3*E^(I*e)*(-x^4 + (1 + E^((-2*I)*e))*x^4 - ((1 + E^((2*I)*e))*(2*f^4*x^4
+ (4*I)*f^3*x^3*Log[1 + E^((2*I)*(e + f*x))] + 6*f^2*x^2*PolyLog[2, -E^((2*
I)*(e + f*x))] + (6*I)*f*x*PolyLog[3, -E^((2*I)*(e + f*x))] - 3*PolyLog[4,
-E^((2*I)*(e + f*x))]))/(2*E^((2*I)*e)*f^4))*Sec[e] + ((b^3*c^3 + 3*b^3*c^2
*d*x + 3*b^3*c*d^2*x^2 + b^3*d^3*x^3)*Sec[e + f*x]^2)/(2*f) - (3*b^3*c*d^2*
Sec[e]*(Cos[e]*Log[Cos[e]*Cos[f*x] - Sin[e]*Sin[f*x]] + f*x*Sin[e]))/(f^3*(
Cos[e]^2 + Sin[e]^2)) + (9*a*b^2*c^2*d*Sec[e]*(Cos[e]*Log[Cos[e]*Cos[f*x] -
 Sin[e]*Sin[f*x]] + f*x*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)) - (3*a^2*b*c^3
*Sec[e]*(Cos[e]*Log[Cos[e]*Cos[f*x] - Sin[e]*Sin[f*x]] + f*x*Sin[e]))/(f*(C
os[e]^2 + Sin[e]^2)) + (b^3*c^3*Sec[e]*(Cos[e]*Log[Cos[e]*Cos[f*x] - Sin[e]
*Sin[f*x]] + f*x*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) - (3*b^3*d^3*Csc[e]*((f
^2*x^2)/E^(I*ArcTan[Cot[e]]) - (Cot[e]*(I*f*x*(-Pi - 2*ArcTan[Cot[e]]) - Pi
*Log[1 + E^((-2*I)*f*x)] - 2*(f*x - ArcTan[Cot[e]])*Log[1 - E^((2*I)*(f*x -
 ArcTan[Cot[e]]))] + Pi*Log[Cos[f*x]] - 2*ArcTan[Cot[e]]*Log[Sin[f*x - ArcT
an[Cot[e]]]] + I*PolyLog[2, E^((2*I)*(f*x - ArcTan[Cot[e]]))]))/Sqrt[1 + Co
t[e]^2])*Sec[e])/(2*f^4*Sqrt[Csc[e]^2*(Cos[e]^2 + Sin[e]^2)]) + (9*a*b^2*c*
d^2*Csc[e]*((f^2*x^2)/E^(I*ArcTan[Cot[e]]) - (Cot[e]*(I*f*x*(-Pi - 2*ArcTan
[Cot[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x - ArcTan[Cot[e]])*Log[1 - E
^((2*I)*(f*x - ArcTan[Cot[e]]))] + Pi*Log[Cos[f*x]] - 2*ArcTan[Cot[e]]*Log[
Sin[f*x - ArcTan[Cot[e]]]] + I*PolyLog[2, E^((2*I)*(f*x - ArcTan[Cot[e]]))]
))/Sqrt[1 + Cot[e]^2])*Sec[e])/(f^3*Sqrt[Csc[e]^2*(Cos[e]^2 + Sin[e]^2)]) -
 (9*a^2*b*c^2*d*Csc[e]*((f^2*x^2)/E^(I*ArcTan[Cot[e]]) - (Cot[e]*(I*f*x*(-P
i - 2*ArcTan[Cot[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x - ArcTan[Cot[e]
])*Log[1 - E^((2*I)*(f*x - ArcTan[Cot[e]]))] + Pi*Log[Cos[f*x]] - 2*ArcTan[
Cot[e]]*Log[Sin[f*x - ArcTan[Cot[e]]]] + I*PolyLog[2, E^((2*I)*(f*x - ArcTa
n[Cot[e]]))]))/Sqrt[1 + Cot[e]^2])*Sec[e])/(2*f^2*Sqrt[Csc[e]^2*(Cos[e]^2 +
 Sin[e]^2)]) + (3*b^3*c^2*d*Csc[e]*((f^2*x^2)/E^(I*ArcTan[Cot[e]]) - (Cot[e
]*(I*f*x*(-Pi - 2*ArcTan[Cot[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x - A
rcTan[Cot[e]])*Log[1 - E^((2*I)*(f*x - ArcTan[Cot[e]]))] + Pi*Log[Cos[f*x]]
 - 2*ArcTan[Cot[e]]*Log[Sin[f*x - ArcTan[Cot[e]]]] + I*PolyLog[2, E^((2*I)*
(f*x - ArcTan[Cot[e]]))]))/Sqrt[1 + Cot[e]^2])*Sec[e])/(2*f^2*Sqrt[Csc[e]^2
*(Cos[e]^2 + Sin[e]^2)]) + (3*x^2*(a^3*c^2*d + (3*I)*a^2*b*c^2*d - 3*a*b^2*
c^2*d - I*b^3*c^2*d + a^3*c^2*d*Cos[2*e] - (3*I)*a^2*b*c^2*d*Cos[2*e] - 3*a
*b^2*c^2*d*Cos[2*e] + I*b^3*c^2*d*Cos[2*e] + I*a^3*c^2*d*Sin[2*e] + 3*a^2*b
*c^2*d*Sin[2*e] - (3*I)*a*b^2*c^2*d*Sin[2*e] - b^3*c^2*d*Sin[2*e]))/(2*(1 +
 Cos[2*e] + I*Sin[2*e])) + (x^3*(a^3*c*d^2 + (3*I)*a^2*b*c*d^2 - 3*a*b^2*c*
d^2 - I*b^3*c*d^2 + a^3*c*d^2*Cos[2*e] - (3*I)*a^2*b*c*d^2*Cos[2*e] - 3*a*b
^2*c*d^2*Cos[2*e] + I*b^3*c*d^2*Cos[2*e] + I*a^3*c*d^2*Sin[2*e] + 3*a^2*b*c
*d^2*Sin[2*e] - (3*I)*a*b^2*c*d^2*Sin[2*e] - b^3*c*d^2*Sin[2*e]))/(1 + Cos[
2*e] + I*Sin[2*e]) + (x^4*(a^3*d^3 + (3*I)*a^2*b*d^3 - 3*a*b^2*d^3 - I*b^3*
d^3 + a^3*d^3*Cos[2*e] - (3*I)*a^2*b*d^3*Cos[2*e] - 3*a*b^2*d^3*Cos[2*e] +
```

```
I*b^3*d^3*Cos[2*e] + I*a^3*d^3*Sin[2*e] + 3*a^2*b*d^3*Sin[2*e] - (3*I)*a*b^
2*d^3*Sin[2*e] - b^3*d^3*Sin[2*e]))/(4*(1 + Cos[2*e] + I*Sin[2*e])) + x*(a^
3*c^3 - 3*a*b^2*c^3 + ((3*I)*a^2*b*c^3)/(1 + Cos[2*e] + I*Sin[2*e]) + ((-3*
I)*a^2*b*c^3*Cos[2*e] + 3*a^2*b*c^3*Sin[2*e])/(1 + Cos[2*e] + I*Sin[2*e]) +
 ((2*I)*b^3*c^3*Cos[2*e] - 2*b^3*c^3*Sin[2*e])/((1 + Cos[2*e] + I*Sin[2*e])
*(1 - Cos[2*e] + Cos[4*e] - I*Sin[2*e] + I*Sin[4*e])) + ((-2*I)*b^3*c^3*Cos
[4*e] + 2*b^3*c^3*Sin[4*e])/((1 + Cos[2*e] + I*Sin[2*e])*(1 - Cos[2*e] + Co
s[4*e] - I*Sin[2*e] + I*Sin[4*e])) - (I*b^3*c^3)/(1 + Cos[6*e] + I*Sin[6*e]
) + (I*b^3*c^3*Cos[6*e] - b^3*c^3*Sin[6*e])/(1 + Cos[6*e] + I*Sin[6*e])) +
(3*Sec[e]*Sec[e + f*x]*(-(b^3*c^2*d*Sin[f*x]) + 2*a*b^2*c^3*f*Sin[f*x] - 2*
b^3*c*d^2*x*Sin[f*x] + 6*a*b^2*c^2*d*f*x*Sin[f*x] - b^3*d^3*x^2*Sin[f*x] +
6*a*b^2*c*d^2*f*x^2*Sin[f*x] + 2*a*b^2*d^3*f*x^...
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1929 vs. 2(544) = 1088.
time = 0.44, size = 1930, normalized size = 3.15

method	result	size
risch	Expression too large to display	1930

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*I*d^3*a^2*b*x^4-3/2*I*b^3*c^2*d*x^2+b^2*(18*I*a*c^2*d*f*x-3*I*b*c^2*d*e
xp(2*I*(f*x+e))+2*b*d^3*f*x^3*exp(2*I*(f*x+e))+6*I*a*d^3*f*x^3*exp(2*I*(f*x
+e))+6*I*a*d^3*f*x^3-6*I*b*c*d^2*x+6*b*c*d^2*f*x^2*exp(2*I*(f*x+e))+6*I*a*c
^3*f*exp(2*I*(f*x+e))-3*I*b*d^3*x^2*exp(2*I*(f*x+e))-3*I*b*c^2*d+6*b*c^2*d*
f*x*exp(2*I*(f*x+e))+18*I*a*c*d^2*f*x^2+18*I*a*c*d^2*f*x^2*exp(2*I*(f*x+e))
+6*I*a*c^3*f+2*b*c^3*f*exp(2*I*(f*x+e))-6*I*b*c*d^2*x*exp(2*I*(f*x+e))+18*I
*a*c^2*d*f*x*exp(2*I*(f*x+e))-3*I*b*d^3*x^2)/f^2/(exp(2*I*(f*x+e))+1)^2+3/2
*I*b^3*d^3*polylog(2,-exp(2*I*(f*x+e)))/f^4+3/4*I*b^3*d^3*polylog(4,-exp(2*
I*(f*x+e)))/f^4+a^3*c^3*x-3/4*d^3*a*b^2*x^4-3*a*b^2*c^3*x-3/4/d*a*b^2*c^4+I
*b^3*c^3*x+1/4*I/d*b^3*c^4-9/f*b*ln(exp(2*I*(f*x+e))+1)*a^2*c^2*d*x+18/f^2*
b^2*ln(exp(2*I*(f*x+e))+1)*a*c*d^2*x-9/f*b*ln(exp(2*I*(f*x+e))+1)*a^2*c*d^2
*x^2-9*I/f^3*b^2*a*d^3*polylog(2,-exp(2*I*(f*x+e)))*x-3*I/f^2*b^3*polylog(2
,-exp(2*I*(f*x+e)))*c*d^2*x+9/2*I/f^2*b*a^2*d^3*polylog(2,-exp(2*I*(f*x+e))
)*x^2+6*I/f^2*b^3*c*d^2*e^2*x-6*I/f*b^3*c^2*d*e*x-18*I/f*b^2*a*c*d^2*x^2-12
*I/f^3*b*a^2*c*d^2*e^3+6*I/f^3*b*a^2*d^3*e^3*x+9/2*I/f^2*b*a^2*c^2*d*polylo
g(2,-exp(2*I*(f*x+e)))-9*I/f^3*b^2*a*c*d^2*polylog(2,-exp(2*I*(f*x+e)))+9*I
/f^2*b*a^2*c^2*d*e^2-18*I/f^3*b^2*a*c*d^2*e^2+18*I/f^3*b^2*d^3*a*e^2*x+18*I
/f*b*a^2*c^2*d*e*x-36*I/f^2*b^2*a*c*d^2*e*x+9*I/f^2*b*polylog(2,-exp(2*I*(f
*x+e)))*a^2*c*d^2*x-18*I/f^2*b*a^2*c*d^2*e^2*x+1/4*d^3*a^3*x^4+1/4/d*a^3*c^
4+18/f^3*b*a^2*c*d^2*e^2*ln(exp(I*(f*x+e)))+36/f^3*b^2*a*c*d^2*e*ln(exp(I*(
f*x+e)))-18/f^2*b*a^2*c^2*d*e*ln(exp(I*(f*x+e)))-3/f*b*a^2*d^3*ln(exp(2*I*(
f*x+e))+1)*x^3-3/2*I/f^4*b^3*d^3*e^4+3*I/f^2*b^3*d^3*x^2+3*I/f^4*b^3*d^3*e^
```

```
2+1/f*b^3*c^3*ln(exp(2*I*(f*x+e))+1)-2/f*b^3*c^3*ln(exp(I*(f*x+e)))-1/4*I*b
^3*d^3*x^4+d^2*a^3*c*x^3+3/2*d*a^3*c^2*x^2-I*d^2*b^3*c*x^3-3*d^2*a*b^2*c*x^
3-9/2*d*a*b^2*c^2*x^2-3*I*a^2*b*c^3*x-3/4*I/d*a^2*b*c^4-6/f^4*b^3*d^3*e*ln(
exp(I*(f*x+e)))-3/f*b*a^2*c^3*ln(exp(2*I*(f*x+e))+1)+6/f*b*a^2*c^3*ln(exp(I
*(f*x+e)))-3/f^3*b^3*c*d^2*ln(exp(2*I*(f*x+e))+1)+6/f^3*b^3*c*d^2*ln(exp(I*
(f*x+e)))+3/2/f^3*b^3*c*d^2*polylog(3,-exp(2*I*(f*x+e)))+2/f^4*b^3*d^3*e^3*
ln(exp(I*(f*x+e)))+3/2/f^3*b^3*d^3*polylog(3,-exp(2*I*(f*x+e)))*x-3/f^3*b^3
*d^3*ln(exp(2*I*(f*x+e))+1)*x-3/2*I/f^2*b^3*d^3*polylog(2,-exp(2*I*(f*x+e))
)*x^2+9/2*I/f^4*b*a^2*d^3*e^4+4*I/f^3*b^3*c*d^2*e^3+6*I/f^3*b^3*d^3*e*x-2*I
/f^3*b^3*d^3*e^3*x-3/2*I/f^2*b^3*c^2*d*polylog(2,-exp(2*I*(f*x+e)))-3*I/f^2
*b^3*c^2*d*e^2-6*I/f*b^2*d^3*a*x^3+12*I/f^4*b^2*d^3*a*e^3+9/f^2*b^2*a*d^3*l
n(exp(2*I*(f*x+e))+1)*x^2+3/f*b^3*ln(exp(2*I*(f*x+e))+1)*c^2*d*x+3/f*b^3*ln
(exp(2*I*(f*x+e))+1)*c*d^2*x^2-9/2/f^3*b*a^2*d^3*polylog(3,-exp(2*I*(f*x+e)
))*x+1/f*b^3*d^3*ln(exp(2*I*(f*x+e))+1)*x^3-6/f^3*b^3*c*d^2*e^2*ln(exp(I*(f
*x+e)))-6/f^4*b*a^2*d^3*e^3*ln(exp(I*(f*x+e)))-9/2/f^3*b*a^2*c*d^2*polylog(
3,-exp(2*I*(f*x+e)))-18/f^4*b^2*a*d^3*e^2*ln(exp(I*(f*x+e)))+6/f^2*b^3*c^2*
d*e*ln(exp(I*(f*x+e)))+9/f^2*b^2*a*c^2*d*ln(exp(2*I*(f*x+e))+1)-18/f^2*b^2*
a*c^2*d*ln(exp(I*(f*x+e)))+9/2*a*b^2*d^3*polylog(3,-exp(2*I*(f*x+e)))/f^4+3
*I*d^2*a^2*b*c*x^3+9/2*I*d*a^2*b*c^2*x^2-9/4*I*a^2*b*d^3*polylog(4,-exp(2*I
*(f*x+e)))/f^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6424 vs. $2(546) = 1092$.
time = 9.42, size = 6424, normalized size = 10.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a^3*c^3 + (f*x + e)^4*a^3*d^3/f^3 + 4*(f*x + e)^3*a^3*c*d^
2/f^2 + 6*(f*x + e)^2*a^3*c^2*d/f - 4*(f*x + e)^3*a^3*d^3*e/f^3 - 12*(f*x +
 e)^2*a^3*c*d^2*e/f^2 - 12*(f*x + e)*a^3*c^2*d*e/f + 12*a^2*b*c^3*log(sec(f
*x + e)) - 36*a^2*b*c^2*d*e*log(sec(f*x + e))/f + 6*(f*x + e)^2*a^3*d^3*e^2
/f^3 + 12*(f*x + e)*a^3*c*d^2*e^2/f^2 + 36*a^2*b*c*d^2*e^2*log(sec(f*x + e)
)/f^2 - 4*(f*x + e)*a^3*d^3*e^3/f^3 - 12*a^2*b*d^3*e^3*log(sec(f*x + e))/f^
3 + 4*(72*a*b^2*c^3*f^3 + 3*(3*a^2*b + 3*I*a*b^2 - b^3)*(f*x + e)^4*d^3 - 3
6*(6*a*b^2*e + b^3)*c^2*d*f^2 + 72*(3*a*b^2*e^2 + b^3*e)*c*d^2*f + 12*((3*a
^2*b + 3*I*a*b^2 - b^3)*c*d^2*f - (3*a^2*b*e + 3*I*a*b^2*e - b^3*e)*d^3)*(f
*x + e)^3 - 36*(2*a*b^2*e^3 + b^3*e^2)*d^3 + 18*((3*a^2*b + 3*I*a*b^2 - b^3
)*c^2*d*f^2 - 2*(3*a^2*b*e + 3*I*a*b^2*e - b^3*e)*c*d^2*f + (3*a^2*b*e^2 +
3*I*a*b^2*e^2 - b^3*e^2)*d^3)*(f*x + e)^2 - 12*((-3*I*a*b^2 + b^3)*c^3*f^3
+ 3*(3*I*a*b^2*e - b^3*e)*c^2*d*f^2 + 3*(-3*I*a*b^2*e^2 + b^3*e^2)*c*d^2*f
+ (3*I*a*b^2*e^3 - b^3*e^3)*d^3)*(f*x + e) + 4*(3*b^3*c^3*f^3 - 4*(3*a^2*b
- b^3)*(f*x + e)^3*d^3 - 9*(b^3*e - 3*a*b^2)*c^2*d*f^2 + 9*(b^3*(e^2 - 1) -
```

```
 6*a*b^2*e)*c*d^2*f - 3*(b^3*(e^3 - 3*e) - 9*a*b^2*e^2)*d^3 - 9*((3*a^2*b -
 b^3)*c*d^2*f - (3*a^2*b*e - b^3*e + 3*a*b^2)*d^3)*(f*x + e)^2 - 9*((3*a^2*
b - b^3)*c^2*d*f^2 - 2*(3*a^2*b*e - b^3*e + 3*a*b^2)*c*d^2*f - (b^3*(e^2 -
1) - 3*a^2*b*e^2 - 6*a*b^2*e)*d^3)*(f*x + e) + (3*b^3*c^3*f^3 - 4*(3*a^2*b
- b^3)*(f*x + e)^3*d^3 - 9*(b^3*e - 3*a*b^2)*c^2*d*f^2 + 9*(b^3*(e^2 - 1) -
 6*a*b^2*e)*c*d^2*f - 3*(b^3*(e^3 - 3*e) - 9*a*b^2*e^2)*d^3 - 9*((3*a^2*b -
 b^3)*c*d^2*f - (3*a^2*b*e - b^3*e + 3*a*b^2)*d^3)*(f*x + e)^2 - 9*((3*a^2*
b - b^3)*c^2*d*f^2 - 2*(3*a^2*b*e - b^3*e + 3*a*b^2)*c*d^2*f - (b^3*(e^2 -
1) - 3*a^2*b*e^2 - 6*a*b^2*e)*d^3)*(f*x + e))*cos(4*f*x + 4*e) + 2*(3*b^3*c
^3*f^3 - 4*(3*a^2*b - b^3)*(f*x + e)^3*d^3 - 9*(b^3*e - 3*a*b^2)*c^2*d*f^2
+ 9*(b^3*(e^2 - 1) - 6*a*b^2*e)*c*d^2*f - 3*(b^3*(e^3 - 3*e) - 9*a*b^2*e^2)
*d^3 - 9*((3*a^2*b - b^3)*c*d^2*f - (3*a^2*b*e - b^3*e + 3*a*b^2)*d^3)*(f*x
 + e)^2 - 9*((3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a^2*b*e - b^3*e + 3*a*b^2)*c*
d^2*f - (b^3*(e^2 - 1) - 3*a^2*b*e^2 - 6*a*b^2*e)*d^3)*(f*x + e))*cos(2*f*x
 + 2*e) - (-3*I*b^3*c^3*f^3 + 4*(3*I*a^2*b - I*b^3)*(f*x + e)^3*d^3 + 9*(I*
b^3*e - 3*I*a*b^2)*c^2*d*f^2 + 9*(b^3*(-I*e^2 + I) + 6*I*a*b^2*e)*c*d^2*f +
 3*(b^3*(I*e^3 - 3*I*e) - 9*I*a*b^2*e^2)*d^3 + 9*((3*I*a^2*b - I*b^3)*c*d^2
*f + (-3*I*a^2*b*e + I*b^3*e - 3*I*a*b^2)*d^3)*(f*x + e)^2 + 9*((3*I*a^2*b
- I*b^3)*c^2*d*f^2 + 2*(-3*I*a^2*b*e + I*b^3*e - 3*I*a*b^2)*c*d^2*f + (b^3*
(-I*e^2 + I) + 3*I*a^2*b*e^2 + 6*I*a*b^2*e)*d^3)*(f*x + e))*sin(4*f*x + 4*e
) - 2*(-3*I*b^3*c^3*f^3 + 4*(3*I*a^2*b - I*b^3)*(f*x + e)^3*d^3 + 9*(I*b^3*
e - 3*I*a*b^2)*c^2*d*f^2 + 9*(b^3*(-I*e^2 + I) + 6*I*a*b^2*e)*c*d^2*f + 3*(
b^3*(I*e^3 - 3*I*e) - 9*I*a*b^2*e^2)*d^3 + 9*((3*I*a^2*b - I*b^3)*c*d^2*f +
 (-3*I*a^2*b*e + I*b^3*e - 3*I*a*b^2)*d^3)*(f*x + e)^2 + 9*((3*I*a^2*b - I*
b^3)*c^2*d*f^2 + 2*(-3*I*a^2*b*e + I*b^3*e - 3*I*a*b^2)*c*d^2*f + (b^3*(-I*
e^2 + I) + 3*I*a^2*b*e^2 + 6*I*a*b^2*e)*d^3)*(f*x + e))*sin(2*f*x + 2*e))*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 3*((3*a^2*b + 3*I*a*b^2 -
b^3)*(f*x + e)^4*d^3 + 4*((3*a^2*b + 3*I*a*b^2 - b^3)*c*d^2*f - (3*a*b^2*(I
*e + 2) + 3*a^2*b*e - b^3*e)*d^3)*(f*x + e)^3 + 6*((3*a^2*b + 3*I*a*b^2 - b
^3)*c^2*d*f^2 - 2*(3*a*b^2*(I*e + 2) + 3*a^2*b*e - b^3*e)*c*d^2*f - (b^3*(e
^2 - 2) + 3*a*b^2*(-I*e^2 - 4*e) - 3*a^2*b*e^2)*d^3)*(f*x + e)^2 - 4*((-3*I
*a*b^2 + b^3)*c^3*f^3 + 3*(3*a*b^2*(I*e + 2) - b^3*e)*c^2*d*f^2 + 3*(b^3*(e
^2 - 2) + 3*a*b^2*(-I*e^2 - 4*e))*c*d^2*f - (b^3*(e^3 - 6*e) - 3*a*b^2*(I*e
^3 + 6*e^2))*d^3)*(f*x + e))*cos(4*f*x + 4*e) + 6*((3*a^2*b + 3*I*a*b^2 - b
^3)*(f*x + e)^4*d^3 + 4*(3*a*b^2 - I*b^3)*c^3*f^3 - 6*(b^3*(-2*I*e + 1) + 6
*a*b^2*e)*c^2*d*f^2 - 12*(b^3*(I*e^2 - e) - 3*a*b^2*e^2)*c*d^2*f + 4*((3*a^
2*b + 3*I*a*b^2 - b^3)*c*d^2*f + (b^3*(e - I) - 3*a*b^2*(I*e + 1) - 3*a^2*b
*e)*d^3)*(f*x + e)^3 - 2*(b^3*(-2*I*e^3 + 3*e^2) + 6*a*b^2*e^3)*d^3 + 6*((3
*a^2*b + 3*I*a*b^2 - b^3)*c^2*d*f^2 + 2*(b^3*(e - I) - 3*a*b^2*(I*e + 1) -
3*a^2*b*e)*c*d^2*f - (b^3*(e^2 - 2*I*e - 1) + 3*a*b^2*(-I*e^2 - 2*e) - 3*a^
2*b*e^2)*d^3)*(f*x + e)^2 - 4*((-3*I*a*b^2 + b^3)*c^3*f^3 - 3*(b^3*(e - I)
- 3*a*b^2*(I*e + 1))*c^2*d*f^2 + 3*(b^3*(e^2 - 2*I*e - 1) + 3*a*b^2*(-I*e^2
 - 2*e))*c*d^2*f - (b^3*(e^3 - 3*I*e^2 - 3*e) - 3*a*b^2*(I*e^3 + 3*e^2))*d^
3)*(f*x + e))*cos(2*f*x + 2*e) + 6*(4*(3*a^2*b - b^3)*(f*x + e)^2*d^3 + 3*(
3*a^2*b - b^3)*c^2*d*f^2 - 6*(3*a^2*b*e - b^3*e + 3*a*b^2)*c*d^2*f - 3*(b^3
```

```
*(e^2 - 1) - 3*a^2*b*e^2 - 6*a*b^2*e)*d^3 + 6*((3*a^2*b - b^3)*c*d^2*f - (3
*a^2*b*e - b^3*e + 3*a*b^2)*d^3)*(f*x + e) + (4*(3*a^2*b - b^3)*(f*x + e)^2
*d^3 + 3*(3*a^2*b - b^3)*c^2*d*f^2 - 6*(3*a^2*b*e - b^3*e + 3*a*b^2)*c*d^2*
f - 3*(b^3*(e^2 - 1) - 3*a^2*b*e^2 - 6*a*b^2*e)*d^3 + 6*((3*a^2*b - b^3)*c*
d^2*f - (3*a^2*b*e - b^3*e + 3*a*b^2)*d^3)*(f*x + e))*cos(4*f*x + 4*e) + 2*
(4*(3*a^2*b - b^3)*(f*x + e)^2*d^3 + 3*(3*a^2*b - b^3)*c^2*d*f^2 - 6*(3*a^2
*b*e - b^3*e + 3*a*b^2)*c*d^2*f - 3*(b^3*(e^2 -...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(546) = 1092$.
time = 0.40, size = 1199, normalized size = 1.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(a^3 - 3*a*b^2)*d^3*f^4*x^4 + 3*I*(3*a^2*b - b^3)*d^3*polylog(4, (ta
n(f*x + e)^2 + 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 3*I*(3*a^2*b -
 b^3)*d^3*polylog(4, (tan(f*x + e)^2 - 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^
2 + 1)) + 4*(b^3*d^3*f^3 + 2*(a^3 - 3*a*b^2)*c*d^2*f^4)*x^3 + 12*(b^3*c*d^2
*f^3 + (a^3 - 3*a*b^2)*c^2*d*f^4)*x^2 + 4*(b^3*d^3*f^3*x^3 + 3*b^3*c*d^2*f^
3*x^2 + 3*b^3*c^2*d*f^3*x + b^3*c^3*f^3)*tan(f*x + e)^2 + 4*(3*b^3*c^2*d*f^
3 + 2*(a^3 - 3*a*b^2)*c^3*f^4)*x - 6*(I*(3*a^2*b - b^3)*d^3*f^2*x^2 - 6*I*a
*b^2*c*d^2*f + I*b^3*d^3 + I*(3*a^2*b - b^3)*c^2*d*f^2 - 2*I*(3*a*b^2*d^3*f
 - (3*a^2*b - b^3)*c*d^2*f^2)*x)*dilog(2*(I*tan(f*x + e) - 1)/(tan(f*x + e)
^2 + 1) + 1) - 6*(-I*(3*a^2*b - b^3)*d^3*f^2*x^2 + 6*I*a*b^2*c*d^2*f - I*b^
3*d^3 - I*(3*a^2*b - b^3)*c^2*d*f^2 + 2*I*(3*a*b^2*d^3*f - (3*a^2*b - b^3)*
c*d^2*f^2)*x)*dilog(2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1) + 1) - 4*(
(3*a^2*b - b^3)*d^3*f^3*x^3 - 9*a*b^2*c^2*d*f^2 + 3*b^3*c*d^2*f + (3*a^2*b
- b^3)*c^3*f^3 - 3*(3*a*b^2*d^3*f^2 - (3*a^2*b - b^3)*c*d^2*f^3)*x^2 - 3*(6
*a*b^2*c*d^2*f^2 - b^3*d^3*f - (3*a^2*b - b^3)*c^2*d*f^3)*x)*log(-2*(I*tan(
f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 4*((3*a^2*b - b^3)*d^3*f^3*x^3 - 9*a*
b^2*c^2*d*f^2 + 3*b^3*c*d^2*f + (3*a^2*b - b^3)*c^3*f^3 - 3*(3*a*b^2*d^3*f^
2 - (3*a^2*b - b^3)*c*d^2*f^3)*x^2 - 3*(6*a*b^2*c*d^2*f^2 - b^3*d^3*f - (3*
a^2*b - b^3)*c^2*d*f^3)*x)*log(-2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1
)) + 6*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*f*x - (3*a^2*b - b^3)*c*d^2*f)*po
lylog(3, (tan(f*x + e)^2 + 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) + 6*
(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*f*x - (3*a^2*b - b^3)*c*d^2*f)*polylog(3
, (tan(f*x + e)^2 - 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) + 12*(2*a*b
^2*d^3*f^3*x^3 + 2*a*b^2*c^3*f^3 - b^3*c^2*d*f^2 + (6*a*b^2*c*d^2*f^3 - b^3
*d^3*f^2)*x^2 + 2*(3*a*b^2*c^2*d*f^3 - b^3*c*d^2*f^2)*x)*tan(f*x + e))/f^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (e + fx))^3 (c + dx)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(c + d*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*tan(f*x + e) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\tan\left(e+f\,x\right)\right)}^3\,{\left(c+d\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*x)^3,x)
```

```
[Out] int((a + b*tan(e + f*x))^3*(c + d*x)^3, x)
```

3.50 $\int (c+dx)^2(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=436

$$\frac{b^3cdx}{f}+\frac{b^3d^2x^2}{2f}-\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}+\frac{ia^2b(c+dx)^3}{d}-\frac{ab^2(c+dx)^3}{d}-\frac{ib^3(c+dx)^3}{3d}+\frac{6ab^2d(c+dx)\,\mathrm{l}}{}$$

```
[Out] b^3*c*d*x/f+1/2*b^3*d^2*x^2/f-1/3*I*b^3*(d*x+c)^3/d+1/3*a^3*(d*x+c)^3/d-3*I
*a*b^2*d^2*polylog(2,-exp(2*I*(f*x+e)))/f^3-a*b^2*(d*x+c)^3/d+3*I*a^2*b*d*(
d*x+c)*polylog(2,-exp(2*I*(f*x+e)))/f^2+6*a*b^2*d*(d*x+c)*ln(1+exp(2*I*(f*x
+e)))/f^2-3*a^2*b*(d*x+c)^2*ln(1+exp(2*I*(f*x+e)))/f+b^3*(d*x+c)^2*ln(1+exp
(2*I*(f*x+e)))/f-b^3*d^2*ln(cos(f*x+e))/f^3-3*I*a*b^2*(d*x+c)^2/f+I*a^2*b*(
d*x+c)^3/d-I*b^3*d*(d*x+c)*polylog(2,-exp(2*I*(f*x+e)))/f^2-3/2*a^2*b*d^2*p
olylog(3,-exp(2*I*(f*x+e)))/f^3+1/2*b^3*d^2*polylog(3,-exp(2*I*(f*x+e)))/f^
3-b^3*d*(d*x+c)*tan(f*x+e)/f^2+3*a*b^2*(d*x+c)^2*tan(f*x+e)/f+1/2*b^3*(d*x+
c)^2*tan(f*x+e)^2/f
```

Rubi [A]
time = 0.44, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3803, 3800, 2221, 2611, 2320, 6724, 3801, 2317, 2438, 32, 3556}

$$\frac{a^3(c+dx)^3}{3d}+\frac{3ia^2bd(c+dx)\mathrm{Li}_2\left(-e^{2i(e+fx)}\right)}{f^2}-\frac{3a^2b(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{ia^2b(c+dx)^3}{d}-\frac{3a^2bd^2\mathrm{Li}_3\left(-e^{2i(e+fx)}\right)}{2f^3}+\frac{6ab^2d(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f^2}+\frac{3ab^2(c+dx)^2\tan(e+fx)}{f}-\frac{3iab^2(c+dx)^2}{f}-\frac{ab^2(c+dx)^3}{d}-\frac{3iab^2d^2\mathrm{Li}_2\left(-e^{2i(e+fx)}\right)}{f^3}-\frac{ib^3d(c+dx)\mathrm{Li}_2\left(-e^{2i(e+fx)}\right)}{f^2}-\frac{b^3d(c+dx)\tan(e+fx)}{f^2}+\frac{b^3(c+dx)^2\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{b^3(c+dx)^2\tan^2(e+fx)}{2f}+\frac{b^3cdx}{f}-\frac{ib^3(c+dx)^3}{3d}+\frac{b^3d^2\mathrm{Li}_3\left(-e^{2i(e+fx)}\right)}{2f^3}-\frac{b^3d^2\log(\cos(e+fx))}{f^3}+\frac{b^3d^2x^2}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (b^3*c*d*x)/f + (b^3*d^2*x^2)/(2*f) - ((3*I)*a*b^2*(c + d*x)^2)/f + (a^3*(c
 + d*x)^3)/(3*d) + (I*a^2*b*(c + d*x)^3)/d - (a*b^2*(c + d*x)^3)/d - ((I/3)
*b^3*(c + d*x)^3)/d + (6*a*b^2*d*(c + d*x)*Log[1 + E^((2*I)*(e + f*x))])/f^
2 - (3*a^2*b*(c + d*x)^2*Log[1 + E^((2*I)*(e + f*x))])/f + (b^3*(c + d*x)^2
*Log[1 + E^((2*I)*(e + f*x))])/f - (b^3*d^2*Log[Cos[e + f*x]])/f^3 - ((3*I)
*a*b^2*d^2*PolyLog[2, -E^((2*I)*(e + f*x))])/f^3 + ((3*I)*a^2*b*d*(c + d*x)
*PolyLog[2, -E^((2*I)*(e + f*x))])/f^2 - (I*b^3*d*(c + d*x)*PolyLog[2, -E^(
(2*I)*(e + f*x))])/f^2 - (3*a^2*b*d^2*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*
f^3) + (b^3*d^2*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^3) - (b^3*d*(c + d*x
)*Tan[e + f*x])/f^2 + (3*a*b^2*(c + d*x)^2*Tan[e + f*x])/f + (b^3*(c + d*x)
^2*Tan[e + f*x]^2)/(2*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+b\tan(e+fx))^3\,dx &= \int \left(a^3(c+dx)^2+3a^2b(c+dx)^2\tan(e+fx)+3ab^2(c+dx)^2\tan^2(e+\right.\\
&= \frac{a^3(c+dx)^3}{3d}+\left(3a^2b\right)\int (c+dx)^2\tan(e+fx)\,dx+\left(3ab^2\right)\int (c+dx)\\
&= \frac{a^3(c+dx)^3}{3d}+\frac{ia^2b(c+dx)^3}{d}+\frac{3ab^2(c+dx)^2\tan(e+fx)}{f}+\frac{b^3(c+d}{}\\
&= -\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}+\frac{ia^2b(c+dx)^3}{d}-\frac{ab^2(c+dx)^3}{d}-\frac{ib}{}\\
&= \frac{b^3cdx}{f}+\frac{b^3d^2x^2}{2f}-\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}+\frac{ia^2b(c+dx)^3}{d}-\\
&= \frac{b^3cdx}{f}+\frac{b^3d^2x^2}{2f}-\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}+\frac{ia^2b(c+dx)^3}{d}-\\
&= \frac{b^3cdx}{f}+\frac{b^3d^2x^2}{2f}-\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}+\frac{ia^2b(c+dx)^3}{d}-\\
&= \frac{b^3cdx}{f}+\frac{b^3d^2x^2}{2f}-\frac{3iab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}+\frac{ia^2b(c+dx)^3}{d}-
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1860 vs. 2(436) = 872.

time = 7.37, size = 1860, normalized size = 4.27

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (a^2*b*d^2*((2*I)*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(1 + E^((2*I)*e))*Log[
1 + E^((2*I)*(e + f*x))]) + (6*I)*(1 + E^((2*I)*e))*f*x*PolyLog[2, -E^((2*I
)*(e + f*x))] - 3*(1 + E^((2*I)*e))*PolyLog[3, -E^((2*I)*(e + f*x))])*Sec[e
])/(4*E^(I*e)*f^3) - (b^3*d^2*((2*I)*f^2*x^2*(2*E^((2*I)*e)*f*x + (3*I)*(1
+ E^((2*I)*e))*Log[1 + E^((2*I)*(e + f*x))]) + (6*I)*(1 + E^((2*I)*e))*f*x*
PolyLog[2, -E^((2*I)*(e + f*x))] - 3*(1 + E^((2*I)*e))*PolyLog[3, -E^((2*I)
*(e + f*x))])*Sec[e])/(12*E^(I*e)*f^3) - (b^3*d^2*Sec[e]*(Cos[e]*Log[Cos[e]
*Cos[f*x] - Sin[e]*Sin[f*x]] + f*x*Sin[e]))/(f^3*(Cos[e]^2 + Sin[e]^2)) + (
6*a*b^2*c*d*Sec[e]*(Cos[e]*Log[Cos[e]*Cos[f*x] - Sin[e]*Sin[f*x]] + f*x*Sin
[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)) - (3*a^2*b*c^2*Sec[e]*(Cos[e]*Log[Cos[e]*
Cos[f*x] - Sin[e]*Sin[f*x]] + f*x*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) + (b^3
*c^2*Sec[e]*(Cos[e]*Log[Cos[e]*Cos[f*x] - Sin[e]*Sin[f*x]] + f*x*Sin[e]))/(
f*(Cos[e]^2 + Sin[e]^2)) + (3*a*b^2*d^2*Csc[e]*((f^2*x^2)/E^(I*ArcTan[Cot[e
]]) - (Cot[e]*(I*f*x*(-Pi - 2*ArcTan[Cot[e]]) - Pi*Log[1 + E^((-2*I)*f*x)]
- 2*(f*x - ArcTan[Cot[e]])*Log[1 - E^((2*I)*(f*x - ArcTan[Cot[e]]))] + Pi*L
og[Cos[f*x]] - 2*ArcTan[Cot[e]]*Log[Sin[f*x - ArcTan[Cot[e]]]] + I*PolyLog[
2, E^((2*I)*(f*x - ArcTan[Cot[e]]))]))/Sqrt[1 + Cot[e]^2])*Sec[e])/(f^3*Sqr
t[Csc[e]^2*(Cos[e]^2 + Sin[e]^2)]) - (3*a^2*b*c*d*Csc[e]*((f^2*x^2)/E^(I*Ar
cTan[Cot[e]]) - (Cot[e]*(I*f*x*(-Pi - 2*ArcTan[Cot[e]]) - Pi*Log[1 + E^((-2
*I)*f*x)] - 2*(f*x - ArcTan[Cot[e]])*Log[1 - E^((2*I)*(f*x - ArcTan[Cot[e]]
))] + Pi*Log[Cos[f*x]] - 2*ArcTan[Cot[e]]*Log[Sin[f*x - ArcTan[Cot[e]]]] +
I*PolyLog[2, E^((2*I)*(f*x - ArcTan[Cot[e]]))]))/Sqrt[1 + Cot[e]^2])*Sec[e]
)/(f^2*Sqrt[Csc[e]^2*(Cos[e]^2 + Sin[e]^2)]) + (b^3*c*d*Csc[e]*((f^2*x^2)/E
^(I*ArcTan[Cot[e]]) - (Cot[e]*(I*f*x*(-Pi - 2*ArcTan[Cot[e]]) - Pi*Log[1 +
E^((-2*I)*f*x)] - 2*(f*x - ArcTan[Cot[e]])*Log[1 - E^((2*I)*(f*x - ArcTan[C
ot[e]]))] + Pi*Log[Cos[f*x]] - 2*ArcTan[Cot[e]]*Log[Sin[f*x - ArcTan[Cot[e]
]]] + I*PolyLog[2, E^((2*I)*(f*x - ArcTan[Cot[e]]))]))/Sqrt[1 + Cot[e]^2])*
Sec[e])/(f^2*Sqrt[Csc[e]^2*(Cos[e]^2 + Sin[e]^2)]) + (Sec[e]*Sec[e + f*x]^2
*(6*b^3*c^2*f*Cos[e] + 12*b^3*c*d*f*x*Cos[e] + 6*a^3*c^2*f^2*x*Cos[e] - 18*
a*b^2*c^2*f^2*x*Cos[e] + 6*b^3*d^2*f*x^2*Cos[e] + 6*a^3*c*d*f^2*x^2*Cos[e]
- 18*a*b^2*c*d*f^2*x^2*Cos[e] + 2*a^3*d^2*f^2*x^3*Cos[e] - 6*a*b^2*d^2*f^2*
x^3*Cos[e] + 3*a^3*c^2*f^2*x*Cos[e + 2*f*x] - 9*a*b^2*c^2*f^2*x*Cos[e + 2*f
*x] + 3*a^3*c*d*f^2*x^2*Cos[e + 2*f*x] - 9*a*b^2*c*d*f^2*x^2*Cos[e + 2*f*x]
 + a^3*d^2*f^2*x^3*Cos[e + 2*f*x] - 3*a*b^2*d^2*f^2*x^3*Cos[e + 2*f*x] + 3*
a^3*c^2*f^2*x*Cos[3*e + 2*f*x] - 9*a*b^2*c^2*f^2*x*Cos[3*e + 2*f*x] + 3*a^3
*c*d*f^2*x^2*Cos[3*e + 2*f*x] - 9*a*b^2*c*d*f^2*x^2*Cos[3*e + 2*f*x] + a^3*
d^2*f^2*x^3*Cos[3*e + 2*f*x] - 3*a*b^2*d^2*f^2*x^3*Cos[3*e + 2*f*x] + 6*b^3
*c*d*Sin[e] - 18*a*b^2*c^2*f*Sin[e] + 6*b^3*d^2*x*Sin[e] - 36*a*b^2*c*d*f*x
```

```
*Sin[e] + 18*a^2*b*c^2*f^2*x*Sin[e] - 6*b^3*c^2*f^2*x*Sin[e] - 18*a*b^2*d^2
*f*x^2*Sin[e] + 18*a^2*b*c*d*f^2*x^2*Sin[e] - 6*b^3*c*d*f^2*x^2*Sin[e] + 6*
a^2*b*d^2*f^2*x^3*Sin[e] - 2*b^3*d^2*f^2*x^3*Sin[e] - 6*b^3*c*d*Sin[e + 2*f
*x] + 18*a*b^2*c^2*f*Sin[e + 2*f*x] - 6*b^3*d^2*x*Sin[e + 2*f*x] + 36*a*b^2
*c*d*f*x*Sin[e + 2*f*x] - 9*a^2*b*c^2*f^2*x*Sin[e + 2*f*x] + 3*b^3*c^2*f^2*
x*Sin[e + 2*f*x] + 18*a*b^2*d^2*f*x^2*Sin[e + 2*f*x] - 9*a^2*b*c*d*f^2*x^2*
Sin[e + 2*f*x] + 3*b^3*c*d*f^2*x^2*Sin[e + 2*f*x] - 3*a^2*b*d^2*f^2*x^3*Sin
[e + 2*f*x] + b^3*d^2*f^2*x^3*Sin[e + 2*f*x] + 9*a^2*b*c^2*f^2*x*Sin[3*e +
2*f*x] - 3*b^3*c^2*f^2*x*Sin[3*e + 2*f*x] + 9*a^2*b*c*d*f^2*x^2*Sin[3*e + 2
*f*x] - 3*b^3*c*d*f^2*x^2*Sin[3*e + 2*f*x] + 3*a^2*b*d^2*f^2*x^3*Sin[3*e +
2*f*x] - b^3*d^2*f^2*x^3*Sin[3*e + 2*f*x]))/(12*f^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(402) = 804$.
time = 0.38, size = 1138, normalized size = 2.61

method	result	size
risch	Expression too large to display	1138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 6/f^3*b*a^2*d^2*e^2*ln(exp(I*(f*x+e)))-12/f^2*b*a^2*c*d*e*ln(exp(I*(f*x+e))
)-2/f^3*b^3*d^2*e^2*ln(exp(I*(f*x+e)))-3/f*b*a^2*c^2*ln(exp(2*I*(f*x+e))+1)
+4/3*I/f^3*b^3*d^2*e^3-1/3*I*b^3*d^2*x^3-2/f*b^3*c^2*ln(exp(I*(f*x+e)))+1/f
*b^3*c^2*ln(exp(2*I*(f*x+e))+1)-1/f^3*b^3*d^2*ln(exp(2*I*(f*x+e))+1)+2/f^3*
b^3*d^2*ln(exp(I*(f*x+e)))+2*b^2*(3*I*a*d^2*f*x^2*exp(2*I*(f*x+e))+6*I*a*c*
d*f*x*exp(2*I*(f*x+e))+b*d^2*f*x^2*exp(2*I*(f*x+e))+3*I*a*c^2*f*exp(2*I*(f*
x+e))+3*I*a*d^2*f*x^2-I*b*d^2*x*exp(2*I*(f*x+e))+2*b*c*d*f*x*exp(2*I*(f*x+e
))+6*I*a*c*d*f*x-I*b*c*d*exp(2*I*(f*x+e))+b*c^2*f*exp(2*I*(f*x+e))+3*I*a*c^
2*f-I*b*d^2*x-I*b*c*d)/f^2/(exp(2*I*(f*x+e))+1)^2-4*I/f^3*b*a^2*d^2*e^3+3*I
*d*a^2*b*c*x^2+d*a^3*c*x^2+a^3*c^2*x-d^2*a*b^2*x^3-3*a*b^2*c^2*x-1/d*a*b^2*
c^3+I*b^3*c^2*x+1/3*I/d*b^3*c^3-I*d*b^3*c*x^2-3*d*a*b^2*c*x^2+I*d^2*a^2*b*x
^3-3*I*a^2*b*c^2*x-I/d*a^2*b*c^3+1/3*d^2*a^3*x^3+1/3/d*a^3*c^3+12/f^3*b^2*a
*d^2*e*ln(exp(I*(f*x+e)))+6/f^2*b^2*a*c*d*ln(exp(2*I*(f*x+e))+1)-12/f^2*b^2
*a*c*d*ln(exp(I*(f*x+e)))+4/f^2*b^3*c*d*e*ln(exp(I*(f*x+e)))+2/f*b^3*ln(exp
(2*I*(f*x+e))+1)*c*d*x+6/f^2*b^2*ln(exp(2*I*(f*x+e))+1)*a*d^2*x-3/f*b*ln(ex
p(2*I*(f*x+e))+1)*a^2*d^2*x^2+2*I/f^2*b^3*d^2*e^2*x-6*I/f^3*b^2*a*d^2*e^2-I
/f^2*b^3*c*d*polylog(2,-exp(2*I*(f*x+e)))-2*I/f^2*b^3*c*d*e^2-6*I/f*b^2*a*d
^2*x^2-I/f^2*b^3*polylog(2,-exp(2*I*(f*x+e)))*d^2*x-6/f*b*ln(exp(2*I*(f*x+e
))+1)*a^2*c*d*x-12*I/f^2*b^2*a*d^2*e*x-4*I/f*b^3*c*d*e*x+3*I/f^2*b*a^2*c*d*
polylog(2,-exp(2*I*(f*x+e)))-6*I/f^2*b*a^2*d^2*e^2*x+6*I/f^2*b*a^2*c*d*e^2+
3*I/f^2*b*polylog(2,-exp(2*I*(f*x+e)))*a^2*d^2*x+12*I/f*b*a^2*c*d*e*x+1/f*b
^3*ln(exp(2*I*(f*x+e))+1)*d^2*x^2+6/f*b*a^2*c^2*ln(exp(I*(f*x+e)))+1/2*b^3*
```

```
d^2*polylog(3,-exp(2*I*(f*x+e)))/f^3-3/2*a^2*b*d^2*polylog(3,-exp(2*I*(f*x+
e)))/f^3-3*I*a*b^2*d^2*polylog(2,-exp(2*I*(f*x+e)))/f^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3327 vs. $2(405) = 810$.
time = 2.52, size = 3327, normalized size = 7.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^3*c^2 + (f*x + e)^3*a^3*d^2/f^2 + 3*(f*x + e)^2*a^3*c*d/
f - 3*(f*x + e)^2*a^3*d^2*e/f^2 - 6*(f*x + e)*a^3*c*d*e/f + 9*a^2*b*c^2*log
(sec(f*x + e)) - 18*a^2*b*c*d*e*log(sec(f*x + e))/f + 3*(f*x + e)*a^3*d^2*e
^2/f^2 + 9*a^2*b*d^2*e^2*log(sec(f*x + e))/f^2 + 3*(36*a*b^2*c^2*f^2 + 2*(3
*a^2*b + 3*I*a*b^2 - b^3)*(f*x + e)^3*d^2 - 12*(6*a*b^2*e + b^3)*c*d*f + 6*
((3*a^2*b + 3*I*a*b^2 - b^3)*c*d*f - (3*a^2*b*e + 3*I*a*b^2*e - b^3*e)*d^2)
*(f*x + e)^2 + 12*(3*a*b^2*e^2 + b^3*e)*d^2 - 6*((-3*I*a*b^2 + b^3)*c^2*f^2
 + 2*(3*I*a*b^2*e - b^3*e)*c*d*f + (-3*I*a*b^2*e^2 + b^3*e^2)*d^2)*(f*x + e
) + 6*(b^3*c^2*f^2 - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - 2*(b^3*e - 3*a*b^2)*
c*d*f + (b^3*(e^2 - 1) - 6*a*b^2*e)*d^2 - 2*((3*a^2*b - b^3)*c*d*f - (3*a^2
*b*e - b^3*e + 3*a*b^2)*d^2)*(f*x + e) + (b^3*c^2*f^2 - (3*a^2*b - b^3)*(f*
x + e)^2*d^2 - 2*(b^3*e - 3*a*b^2)*c*d*f + (b^3*(e^2 - 1) - 6*a*b^2*e)*d^2
- 2*((3*a^2*b - b^3)*c*d*f - (3*a^2*b*e - b^3*e + 3*a*b^2)*d^2)*(f*x + e))*
cos(4*f*x + 4*e) + 2*(b^3*c^2*f^2 - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - 2*(b^
3*e - 3*a*b^2)*c*d*f + (b^3*(e^2 - 1) - 6*a*b^2*e)*d^2 - 2*((3*a^2*b - b^3)
*c*d*f - (3*a^2*b*e - b^3*e + 3*a*b^2)*d^2)*(f*x + e))*cos(2*f*x + 2*e) - (
-I*b^3*c^2*f^2 + (3*I*a^2*b - I*b^3)*(f*x + e)^2*d^2 + 2*(I*b^3*e - 3*I*a*b
^2)*c*d*f + (b^3*(-I*e^2 + I) + 6*I*a*b^2*e)*d^2 + 2*((3*I*a^2*b - I*b^3)*c
*d*f + (-3*I*a^2*b*e + I*b^3*e - 3*I*a*b^2)*d^2)*(f*x + e))*sin(4*f*x + 4*e
) - 2*(-I*b^3*c^2*f^2 + (3*I*a^2*b - I*b^3)*(f*x + e)^2*d^2 + 2*(I*b^3*e -
3*I*a*b^2)*c*d*f + (b^3*(-I*e^2 + I) + 6*I*a*b^2*e)*d^2 + 2*((3*I*a^2*b - I
*b^3)*c*d*f + (-3*I*a^2*b*e + I*b^3*e - 3*I*a*b^2)*d^2)*(f*x + e))*sin(2*f*
x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*((3*a^2*b + 3
*I*a*b^2 - b^3)*(f*x + e)^3*d^2 + 3*((3*a^2*b + 3*I*a*b^2 - b^3)*c*d*f - (3
*a*b^2*(I*e + 2) + 3*a^2*b*e - b^3*e)*d^2)*(f*x + e)^2 - 3*((-3*I*a*b^2 + b
^3)*c^2*f^2 + 2*(3*a*b^2*(I*e + 2) - b^3*e)*c*d*f + (b^3*(e^2 - 2) + 3*a*b^
2*(-I*e^2 - 4*e))*d^2)*(f*x + e))*cos(4*f*x + 4*e) + 4*((3*a^2*b + 3*I*a*b^
2 - b^3)*(f*x + e)^3*d^2 + 3*(3*a*b^2 - I*b^3)*c^2*f^2 - 3*(b^3*(-2*I*e + 1
) + 6*a*b^2*e)*c*d*f + 3*((3*a^2*b + 3*I*a*b^2 - b^3)*c*d*f + (b^3*(e - I)
- 3*a*b^2*(I*e + 1) - 3*a^2*b*e)*d^2)*(f*x + e)^2 - 3*(b^3*(I*e^2 - e) - 3*
a*b^2*e^2)*d^2 - 3*((-3*I*a*b^2 + b^3)*c^2*f^2 - 2*(b^3*(e - I) - 3*a*b^2*(
I*e + 1))*c*d*f + (b^3*(e^2 - 2*I*e - 1) + 3*a*b^2*(-I*e^2 - 2*e))*d^2)*(f*
x + e))*cos(2*f*x + 2*e) + 6*((3*a^2*b - b^3)*(f*x + e)*d^2 + (3*a^2*b - b^
```

```
3)*c*d*f - (3*a^2*b*e - b^3*e + 3*a*b^2)*d^2 + ((3*a^2*b - b^3)*(f*x + e)*d
^2 + (3*a^2*b - b^3)*c*d*f - (3*a^2*b*e - b^3*e + 3*a*b^2)*d^2)*cos(4*f*x +
 4*e) + 2*((3*a^2*b - b^3)*(f*x + e)*d^2 + (3*a^2*b - b^3)*c*d*f - (3*a^2*b
*e - b^3*e + 3*a*b^2)*d^2)*cos(2*f*x + 2*e) - ((-3*I*a^2*b + I*b^3)*(f*x +
e)*d^2 + (-3*I*a^2*b + I*b^3)*c*d*f + (3*I*a^2*b*e - I*b^3*e + 3*I*a*b^2)*d
^2)*sin(4*f*x + 4*e) - 2*((-3*I*a^2*b + I*b^3)*(f*x + e)*d^2 + (-3*I*a^2*b
+ I*b^3)*c*d*f + (3*I*a^2*b*e - I*b^3*e + 3*I*a*b^2)*d^2)*sin(2*f*x + 2*e))
*dilog(-e^(2*I*f*x + 2*I*e)) - 3*(I*b^3*c^2*f^2 + (-3*I*a^2*b + I*b^3)*(f*x
 + e)^2*d^2 + 2*(-I*b^3*e + 3*I*a*b^2)*c*d*f + (b^3*(I*e^2 - I) - 6*I*a*b^2
*e)*d^2 + 2*((-3*I*a^2*b + I*b^3)*c*d*f + (3*I*a^2*b*e - I*b^3*e + 3*I*a*b^
2)*d^2)*(f*x + e) + (I*b^3*c^2*f^2 + (-3*I*a^2*b + I*b^3)*(f*x + e)^2*d^2 +
 2*(-I*b^3*e + 3*I*a*b^2)*c*d*f + (b^3*(I*e^2 - I) - 6*I*a*b^2*e)*d^2 + 2*(
(-3*I*a^2*b + I*b^3)*c*d*f + (3*I*a^2*b*e - I*b^3*e + 3*I*a*b^2)*d^2)*(f*x
+ e))*cos(4*f*x + 4*e) + 2*(I*b^3*c^2*f^2 + (-3*I*a^2*b + I*b^3)*(f*x + e)^
2*d^2 + 2*(-I*b^3*e + 3*I*a*b^2)*c*d*f + (b^3*(I*e^2 - I) - 6*I*a*b^2*e)*d^
2 + 2*((-3*I*a^2*b + I*b^3)*c*d*f + (3*I*a^2*b*e - I*b^3*e + 3*I*a*b^2)*d^2
)*(f*x + e))*cos(2*f*x + 2*e) - (b^3*c^2*f^2 - (3*a^2*b - b^3)*(f*x + e)^2*
d^2 - 2*(b^3*e - 3*a*b^2)*c*d*f + (b^3*(e^2 - 1) - 6*a*b^2*e)*d^2 - 2*((3*a
^2*b - b^3)*c*d*f - (3*a^2*b*e - b^3*e + 3*a*b^2)*d^2)*(f*x + e))*sin(4*f*x
 + 4*e) - 2*(b^3*c^2*f^2 - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - 2*(b^3*e - 3*a
*b^2)*c*d*f + (b^3*(e^2 - 1) - 6*a*b^2*e)*d^2 - 2*((3*a^2*b - b^3)*c*d*f -
(3*a^2*b*e - b^3*e + 3*a*b^2)*d^2)*(f*x + e))*sin(2*f*x + 2*e))*log(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1) - 3*((-3*I*a^2*b
 + I*b^3)*d^2*cos(4*f*x + 4*e) + 2*(-3*I*a^2*b + I*b^3)*d^2*cos(2*f*x + 2*e
) + (3*a^2*b - b^3)*d^2*sin(4*f*x + 4*e) + 2*(3*a^2*b - b^3)*d^2*sin(2*f*x
+ 2*e) + (-3*I*a^2*b + I*b^3)*d^2)*polylog(3, -e^(2*I*f*x + 2*I*e)) - 2*((-
3*I*a^2*b + 3*a*b^2 + I*b^3)*(f*x + e)^3*d^2 + 3*((-3*I*a^2*b + 3*a*b^2 + I
*b^3)*c*d*f - (3*a*b^2*(e - 2*I) - 3*I*a^2*b*e + I*b^3*e)*d^2)*(f*x + e)^2
+ 3*((3*a*b^2 + I*b^3)*c^2*f^2 - 2*(3*a*b^2*(e - 2*I) + I*b^3*e)*c*d*f + (3
*a*b^2*(e^2 - 4*I*e) + b^3*(I*e^2 - 2*I))*d^2)*(f*x + e))*sin(4*f*x + 4*e)
- 4*((-3*I*a^2*b + 3*a*b^2 + I*b^3)*(f*x + e)^3*d^2 + 3*(-3*I*a*b^2 - b^3)*
c^2*f^2 + 3*(b^3*(2*e + I) + 6*I*a*b^2*e)*c*d*f + 3*((-3*I*a^2*b + 3*a*b^2
+ I*b^3)*c*d*f - (3*a*b^2*(e - I) - b^3*(-I*e - 1) - 3*I*a^2*b*e)*d^2)*(f*x
 + e)^2 - 3*(b^3*(e^2 + I*e) + 3*I*a*b^2*e^2)*d...
```

Fricas [**A**]
time = 0.41, size = 704, normalized size = 1.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(a^3 - 3*a*b^2)*d^2*f^3*x^3 - 3*(3*a^2*b - b^3)*d^2*polylog(3, (tan
(f*x + e)^2 + 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) - 3*(3*a^2*b - b^
```

```
3)*d^2*polylog(3, (tan(f*x + e)^2 - 2*I*tan(f*x + e) - 1)/(tan(f*x + e)^2 +
 1)) + 6*(b^3*d^2*f^2 + 2*(a^3 - 3*a*b^2)*c*d*f^3)*x^2 + 6*(b^3*d^2*f^2*x^2
 + 2*b^3*c*d*f^2*x + b^3*c^2*f^2)*tan(f*x + e)^2 + 12*(b^3*c*d*f^2 + (a^3 -
 3*a*b^2)*c^2*f^3)*x - 6*(-3*I*a*b^2*d^2 + I*(3*a^2*b - b^3)*d^2*f*x + I*(3
*a^2*b - b^3)*c*d*f)*dilog(2*(I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1) + 1)
 - 6*(3*I*a*b^2*d^2 - I*(3*a^2*b - b^3)*d^2*f*x - I*(3*a^2*b - b^3)*c*d*f)*
dilog(2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1) + 1) - 6*((3*a^2*b - b^3
)*d^2*f^2*x^2 - 6*a*b^2*c*d*f + b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*
b^2*d^2*f - (3*a^2*b - b^3)*c*d*f^2)*x)*log(-2*(I*tan(f*x + e) - 1)/(tan(f*
x + e)^2 + 1)) - 6*((3*a^2*b - b^3)*d^2*f^2*x^2 - 6*a*b^2*c*d*f + b^3*d^2 +
 (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*b^2*d^2*f - (3*a^2*b - b^3)*c*d*f^2)*x)*l
og(-2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) + 12*(3*a*b^2*d^2*f^2*x^2
 + 3*a*b^2*c^2*f^2 - b^3*c*d*f + (6*a*b^2*c*d*f^2 - b^3*d^2*f)*x)*tan(f*x +
 e))/f^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(e + fx))^3 (c + dx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(b*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\tan(e + f\,x))^3\,(c + d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*x)^2,x)
```

```
[Out] int((a + b*tan(e + f*x))^3*(c + d*x)^2, x)
```

3.51 $\int (c+dx)(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=277

$$-3ab^2cx+\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}+\frac{3ia^2b(c+dx)^2}{2d}-\frac{ib^3(c+dx)^2}{2d}-\frac{3a^2b(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{b^3}{}$$

```
[Out] -3*a*b^2*c*x+1/2*b^3*d*x/f-3/2*a*b^2*d*x^2+1/2*a^3*(d*x+c)^2/d+3/2*I*a^2*b*
(d*x+c)^2/d-1/2*I*b^3*(d*x+c)^2/d-3*a^2*b*(d*x+c)*ln(1+exp(2*I*(f*x+e)))/f+
b^3*(d*x+c)*ln(1+exp(2*I*(f*x+e)))/f+3*a*b^2*d*ln(cos(f*x+e))/f^2+3/2*I*a^2
*b*d*polylog(2,-exp(2*I*(f*x+e)))/f^2-1/2*I*b^3*d*polylog(2,-exp(2*I*(f*x+e
)))/f^2-1/2*b^3*d*tan(f*x+e)/f^2+3*a*b^2*(d*x+c)*tan(f*x+e)/f+1/2*b^3*(d*x+
c)*tan(f*x+e)^2/f
```

Rubi [A]
time = 0.23, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3803, 3800, 2221, 2317, 2438, 3801, 3556, 3554, 8}

$$\frac{a^3(c+dx)^2}{2d}-\frac{3a^2b(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{3ia^2b(c+dx)^2}{2d}+\frac{3ia^2bd\text{Li}_2\left(-e^{2i(e+fx)}\right)}{2f^2}+\frac{3ab^2(c+dx)\tan(e+fx)}{f}-3ab^2cx+\frac{3ab^2d\log(\cos(e+fx))}{f^2}-\frac{3}{2}ab^2dx^2+\frac{b^3(c+dx)\log\left(1+e^{2i(e+fx)}\right)}{f}+\frac{b^3(c+dx)\tan^2(e+fx)}{2f}-\frac{ib^3(c+dx)^2}{2d}-\frac{ib^3d\text{Li}_2\left(-e^{2i(e+fx)}\right)}{2f^2}-\frac{b^3d\tan(e+fx)}{2f^2}+\frac{b^3dx}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -3*a*b^2*c*x + (b^3*d*x)/(2*f) - (3*a*b^2*d*x^2)/2 + (a^3*(c + d*x)^2)/(2*d
) + (((3*I)/2)*a^2*b*(c + d*x)^2)/d - ((I/2)*b^3*(c + d*x)^2)/d - (3*a^2*b*
(c + d*x)*Log[1 + E^((2*I)*(e + f*x))])/f + (b^3*(c + d*x)*Log[1 + E^((2*I)
*(e + f*x))])/f + (3*a*b^2*d*Log[Cos[e + f*x]])/f^2 + (((3*I)/2)*a^2*b*d*Po
lyLog[2, -E^((2*I)*(e + f*x))])/f^2 - ((I/2)*b^3*d*PolyLog[2, -E^((2*I)*(e
+ f*x))])/f^2 - (b^3*d*Tan[e + f*x])/(2*f^2) + (3*a*b^2*(c + d*x)*Tan[e + f
*x])/f + (b^3*(c + d*x)*Tan[e + f*x]^2)/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)(a+b\tan(e+fx))^3\,dx &= \int \left(a^3(c+dx)+3a^2b(c+dx)\tan(e+fx)+3ab^2(c+dx)\tan^2(e+fx\right. \\
&= \frac{a^3(c+dx)^2}{2d}+\left(3a^2b\right)\int (c+dx)\tan(e+fx)\,dx+\left(3ab^2\right)\int (c+dx)\,\mathrm{ta} \\
&= \frac{a^3(c+dx)^2}{2d}+\frac{3ia^2b(c+dx)^2}{2d}+\frac{3ab^2(c+dx)\tan(e+fx)}{f}+\frac{b^3(c+dx}{} \\
&= -3ab^2cx-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}+\frac{3ia^2b(c+dx)^2}{2d}-\frac{ib^3(c+dx)^2}{2d}- \\
&= -3ab^2cx+\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}+\frac{3ia^2b(c+dx)^2}{2d}-\frac{ib^3(c+}{2d} \\
&= -3ab^2cx+\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}+\frac{3ia^2b(c+dx)^2}{2d}-\frac{ib^3(c+}{2d} \\
&= -3ab^2cx+\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}+\frac{3ia^2b(c+dx)^2}{2d}-\frac{ib^3(c+}{2d}
\end{aligned}
$$

Mathematica [A]
time = 3.51, size = 277, normalized size = 1.00

$$
\frac{\cos(e+fx)\left(\cos^2(e+fx)\left(-((e+fx)(-3ia^2bd(e+fx)+ib^3d(e+fx)+3ab^2(-de+2cf+dfx)+a^3(-2cf+d(e-fx))))+2b(-3a^2+b^2)\,d(e+fx)\log\left(1+e^{2i(e+fx)}\right)+2b(3abd+3a^2(de-cf)+b^2(-de+cf))\log(\cos(e+fx))\right)-ib(-3a^2+b^2)\,d\cos^2(e+fx)\mathrm{PolyLog}\left(2,-e^{2i(e+fx)}\right)+\frac{1}{2}b^2(2bf(c+dx)+(-bd+6af(c+dx))\sin(2(e+fx)))\right)(a+b\tan(e+fx))^3}{2f^2(a\cos(e+fx)+b\sin(e+fx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]*(Cos[e + f*x]^2*(-((e + f*x)*((-3*I)*a^2*b*d*(e + f*x) + I*b^
3*d*(e + f*x) + 3*a*b^2*(-(d*e) + 2*c*f + d*f*x) + a^3*(-2*c*f + d*(e - f*x
)))) + 2*b*(-3*a^2 + b^2)*d*(e + f*x)*Log[1 + E^((2*I)*(e + f*x))] + 2*b*(3
*a*b*d + 3*a^2*(d*e - c*f) + b^2*(-(d*e) + c*f))*Log[Cos[e + f*x]]) - I*b*(
-3*a^2 + b^2)*d*Cos[e + f*x]^2*PolyLog[2, -E^((2*I)*(e + f*x))] + (b^2*(2*b
*f*(c + d*x) + (-(b*d) + 6*a*f*(c + d*x))*Sin[2*(e + f*x)]))/2)*(a + b*Tan[
e + f*x])^3)/(2*f^2*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [A]
time = 0.26, size = 493, normalized size = 1.78

method	result
risch	$-\frac{ib^3d\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(fx+e)}\right)}{2f^2}+\frac{3ia^2bd\,x^2}{2}-3ia^2bcx+\frac{a^3d\,x^2}{2}+a^3cx-\frac{3b\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)a^2dx}{f}-\frac{6b\,a^2de\ln\left(\mathrm{e}^{i(fx+e}\right.}{f^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*I*a^2*b*d*x^2-3*I*a^2*b*c*x+1/f*b^3*ln(exp(2*I*(f*x+e))+1)*d*x+3/f^2*b^
2*a*d*ln(exp(2*I*(f*x+e))+1)-6/f^2*b^2*a*d*ln(exp(I*(f*x+e)))-3/f*b*a^2*c*l
n(exp(2*I*(f*x+e))+1)+1/f*b^3*c*ln(exp(2*I*(f*x+e))+1)-2/f*b^3*c*ln(exp(I*(
f*x+e)))-1/2*I*b^3*d*x^2+b^2*(6*I*a*d*f*x*exp(2*I*(f*x+e))+6*I*a*c*f*exp(2*
I*(f*x+e))+2*b*d*f*x*exp(2*I*(f*x+e))+6*I*a*d*f*x-I*b*d*exp(2*I*(f*x+e))+2*
b*c*f*exp(2*I*(f*x+e))+6*I*a*c*f-I*b*d)/f^2/(exp(2*I*(f*x+e))+1)^2+6/f*b*a^
2*c*ln(exp(I*(f*x+e)))+2/f^2*b^3*d*e*ln(exp(I*(f*x+e)))-I/f^2*b^3*d*e^2-3/f
*b*ln(exp(2*I*(f*x+e))+1)*a^2*d*x-6/f^2*b*a^2*d*e*ln(exp(I*(f*x+e)))-2*I/f*
b^3*d*e*x+3*I/f^2*b*a^2*d*e^2+1/2*a^3*d*x^2+a^3*c*x+3/2*I*a^2*b*d*polylog(2
,-exp(2*I*(f*x+e)))/f^2+I*b^3*c*x+6*I/f*b*a^2*d*e*x-3*a*b^2*c*x-3/2*a*b^2*d
*x^2-1/2*I*b^3*d*polylog(2,-exp(2*I*(f*x+e)))/f^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1384 vs. 2(249) = 498.
time = 0.88, size = 1384, normalized size = 5.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a^3*c + (f*x + e)^2*a^3*d/f - 2*(f*x + e)*a^3*d*e/f + 6*a^
2*b*c*log(sec(f*x + e)) - 6*a^2*b*d*e*log(sec(f*x + e))/f + 2*(12*a*b^2*c*f
 + (3*a^2*b + 3*I*a*b^2 - b^3)*(f*x + e)^2*d - 2*((-3*I*a*b^2 + b^3)*c*f +
(3*I*a*b^2*e - b^3*e)*d)*(f*x + e) - 2*(6*a*b^2*e + b^3)*d + 2*(b^3*c*f - (
3*a^2*b - b^3)*(f*x + e)*d - (b^3*e - 3*a*b^2)*d + (b^3*c*f - (3*a^2*b - b^
3)*(f*x + e)*d - (b^3*e - 3*a*b^2)*d)*cos(4*f*x + 4*e) + 2*(b^3*c*f - (3*a^
2*b - b^3)*(f*x + e)*d - (b^3*e - 3*a*b^2)*d)*cos(2*f*x + 2*e) - (-I*b^3*c*
f + (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b^3*e - 3*I*a*b^2)*d)*sin(4*f*x +
4*e) - 2*(-I*b^3*c*f + (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b^3*e - 3*I*a*b
^2)*d)*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) +
((3*a^2*b + 3*I*a*b^2 - b^3)*(f*x + e)^2*d - 2*((-3*I*a*b^2 + b^3)*c*f + (3
*a*b^2*(I*e + 2) - b^3*e)*d)*(f*x + e))*cos(4*f*x + 4*e) + 2*((3*a^2*b + 3*
I*a*b^2 - b^3)*(f*x + e)^2*d + 2*(3*a*b^2 - I*b^3)*c*f - 2*((-3*I*a*b^2 + b
^3)*c*f - (b^3*(e - I) - 3*a*b^2*(I*e + 1))*d)*(f*x + e) - (b^3*(-2*I*e + 1
) + 6*a*b^2*e)*d)*cos(2*f*x + 2*e) + ((3*a^2*b - b^3)*d*cos(4*f*x + 4*e) +
2*(3*a^2*b - b^3)*d*cos(2*f*x + 2*e) + (3*I*a^2*b - I*b^3)*d*sin(4*f*x + 4*
e) - 2*(-3*I*a^2*b + I*b^3)*d*sin(2*f*x + 2*e) + (3*a^2*b - b^3)*d)*dilog(-
e^(2*I*f*x + 2*I*e)) + (-I*b^3*c*f + (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b
^3*e - 3*I*a*b^2)*d + (-I*b^3*c*f + (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b^
3*e - 3*I*a*b^2)*d)*cos(4*f*x + 4*e) - 2*(I*b^3*c*f + (-3*I*a^2*b + I*b^3)*
(f*x + e)*d + (-I*b^3*e + 3*I*a*b^2)*d)*cos(2*f*x + 2*e) + (b^3*c*f - (3*a^
2*b - b^3)*(f*x + e)*d - (b^3*e - 3*a*b^2)*d)*sin(4*f*x + 4*e) + 2*(b^3*c*f
 - (3*a^2*b - b^3)*(f*x + e)*d - (b^3*e - 3*a*b^2)*d)*sin(2*f*x + 2*e))*log
```

```
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1) + ((3*I*
a^2*b - 3*a*b^2 - I*b^3)*(f*x + e)^2*d - 2*((3*a*b^2 + I*b^3)*c*f - (3*a*b^
2*(e - 2*I) + I*b^3*e)*d)*(f*x + e))*sin(4*f*x + 4*e) - 2*((-3*I*a^2*b + 3*
a*b^2 + I*b^3)*(f*x + e)^2*d + 2*(-3*I*a*b^2 - b^3)*c*f + 2*((3*a*b^2 + I*b
^3)*c*f - (3*a*b^2*(e - I) - b^3*(-I*e - 1))*d)*(f*x + e) + (b^3*(2*e + I)
+ 6*I*a*b^2*e)*d)*sin(2*f*x + 2*e))/(-2*I*f*cos(4*f*x + 4*e) - 4*I*f*cos(2*
f*x + 2*e) + 2*f*sin(4*f*x + 4*e) + 4*f*sin(2*f*x + 2*e) - 2*I*f))/f
```

Fricas [**A**]
time = 0.37, size = 337, normalized size = 1.22

$$\frac{2\,(a^3-3\,ab^2)df^2x^2-i\,(3\,a^2b-b^3)d\mathrm{Li}_2\left(\frac{2\,(i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}+1\right)+i\,(3\,a^2b-b^3)d\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}+1\right)+2\,(b^3dfx+b^3cf)\tan\left(fx+e\right)^2+2\,(b^3df+2\,(a^3-3\,ab^2)cf^2)x+2\,(3\,ab^2d-(3\,a^2b-b^3)dfx-(3\,a^2b-b^3)cf)\log\left(-\frac{2\,(i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}\right)+2\,(3\,ab^2d-(3\,a^2b-b^3)dfx-(3\,a^2b-b^3)cf)\log\left(-\frac{2\,(-i\,\tan(fx+e)-1)}{\tan(fx+e)^2+1}\right)+2\,(6\,ab^2dfx+6\,ab^2cf-b^3d)\tan\left(fx+e\right)}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^3 - 3*a*b^2)*d*f^2*x^2 - I*(3*a^2*b - b^3)*d*dilog(2*(I*tan(f*x +
 e) - 1)/(tan(f*x + e)^2 + 1) + 1) + I*(3*a^2*b - b^3)*d*dilog(2*(-I*tan(f*
x + e) - 1)/(tan(f*x + e)^2 + 1) + 1) + 2*(b^3*d*f*x + b^3*c*f)*tan(f*x + e
)^2 + 2*(b^3*d*f + 2*(a^3 - 3*a*b^2)*c*f^2)*x + 2*(3*a*b^2*d - (3*a^2*b - b
^3)*d*f*x - (3*a^2*b - b^3)*c*f)*log(-2*(I*tan(f*x + e) - 1)/(tan(f*x + e)^
2 + 1)) + 2*(3*a*b^2*d - (3*a^2*b - b^3)*d*f*x - (3*a^2*b - b^3)*c*f)*log(-
2*(-I*tan(f*x + e) - 1)/(tan(f*x + e)^2 + 1)) + 2*(6*a*b^2*d*f*x + 6*a*b^2*
c*f - b^3*d)*tan(f*x + e))/f^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(e+fx\right)\right)^3\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^3\,(c+d\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*x),x)
[Out] int((a + b*tan(e + f*x))^3*(c + d*x), x)
```

3.52 $\int \frac{(a+b\tan(e+fx))^3}{c+dx} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\tan(e+fx))^3}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*tan(f*x+e))^3/(d*x+c),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\tan(e+fx))^3}{c+dx} dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Tan[e + f*x])^3/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*Tan[e + f*x])^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{c+dx} dx = \int \frac{(a+b\tan(e+fx))^3}{c+dx} dx$$

Mathematica [A]
time = 14.89, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^3}{c+dx} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(c + d*x),x]
```

```
[Out] Integrate[(a + b*Tan[e + f*x])^3/(c + d*x), x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*x+c),x)
```

```
[Out] int((a+b*tan(f*x+e))^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^
2)*c^2*f^2)*cos(4*f*x + 4*e)^2*log(d*x + c) + ((a^3 - 3*a*b^2)*d^2*f^2*x^2
+ 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c)*sin(4
*f*x + 4*e)^2 + 4*(b^3*d^2*f*x + b^3*c*d*f + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 +
 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*cos(2
*f*x + 2*e)^2 + 4*(b^3*d^2*f*x + b^3*c*d*f + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 +
 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*sin(2
*f*x + 2*e)^2 + (2*(b^3*d^2*f*x + b^3*c*d*f + 2*((a^3 - 3*a*b^2)*d^2*f^2*x^
2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*co
s(2*f*x + 2*e) + 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2
*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c) - (6*a*b^2*d^2*f*x + 6*a*b^2*c*d
*f + b^3*d^2)*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(b^3*d^2*f*x + b^3*c*d
*f + 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 -
3*a*b^2)*c^2*f^2)*log(d*x + c))*cos(2*f*x + 2*e) - (d^3*f^2*x^2 + 2*c*d^2*f
^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(4*f*x + 4*
e)^2 + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e)^2 + (d^
3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(4*f*x + 4*e)^2 + 4*(d^3*f^2*x^2
+ 2*c*d^2*f^2*x + c^2*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d^3*f^2
*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(2*f*x + 2*e)^2 + 2*(d^3*f^2*x^2 + 2*c
*d^2*f^2*x + c^2*d*f^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*
f*x + 2*e))*cos(4*f*x + 4*e) + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*
cos(2*f*x + 2*e))*integrate(-2*((3*a^2*b - b^3)*d^2*f^2*x^2 + 3*a*b^2*c*d*f
 + b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 + (3*a*b^2*d^2*f + 2*(3*a^2*b - b^3)*c
*d*f^2)*x)*sin(2*f*x + 2*e)/(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x
+ c^3*f^2 + (d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2)*cos(2
*f*x + 2*e)^2 + (d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2)*s
in(2*f*x + 2*e)^2 + 2*(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*
f^2)*cos(2*f*x + 2*e)), x) + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^
2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c) + (6*a*b^2*d^2*f*x + 6
*a*b^2*c*d*f + b^3*d^2 + (6*a*b^2*d^2*f*x + 6*a*b^2*c*d*f + b^3*d^2)*cos(2*
f*x + 2*e) + 2*(b^3*d^2*f*x + b^3*c*d*f + 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 +
2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*sin(2*
f*x + 2*e))*sin(4*f*x + 4*e) + (6*a*b^2*d^2*f*x + 6*a*b^2*c*d*f + b^3*d^2)*
sin(2*f*x + 2*e))/(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 +
```

```
2*c*d^2*f^2*x + c^2*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^3*f^2*x^2 + 2*c*d^2*f
^2*x + c^2*d*f^2)*cos(2*f*x + 2*e)^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d
*f^2)*sin(4*f*x + 4*e)^2 + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(
4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)
*sin(2*f*x + 2*e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*(d^3*f
^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*
(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*x+c),x, algorithm="fricas")
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \tan{\left(e + f x \right)}\right)^{3}}{c + d x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*x+c),x)
[Out] Integral((a + b*tan(e + f*x))**3/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*x+c),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^3}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(c + d*x),x)
[Out] int((a + b*tan(e + f*x))^3/(c + d*x), x)
```

3.53 $\int \frac{(a+b\tan(e+fx))^3}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{(a+b\tan(e+fx))^3}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*tan(f*x+e))^3/(d*x+c)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\tan(e+fx))^3}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Tan[e + f*x])^3/(c + d*x)^2,x]
[Out] Defer[Int][(a + b*Tan[e + f*x])^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(a+b\tan(e+fx))^3}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 20.66, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^3}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(c + d*x)^2,x]
[Out] Integrate[(a + b*Tan[e + f*x])^3/(c + d*x)^2, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*x+c)^2,x)
```

```
[Out] int((a+b*tan(f*x+e))^3/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^
2)*c^2*f^2 + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (
a^3 - 3*a*b^2)*c^2*f^2)*cos(4*f*x + 4*e)^2 + 4*((a^3 - 3*a*b^2)*d^2*f^2*x^2
 - b^3*c*d*f + (a^3 - 3*a*b^2)*c^2*f^2 - (b^3*d^2*f - 2*(a^3 - 3*a*b^2)*c*d
*f^2)*x)*cos(2*f*x + 2*e)^2 + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b
^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*sin(4*f*x + 4*e)^2 + 4*((a^3 - 3*a
*b^2)*d^2*f^2*x^2 - b^3*c*d*f + (a^3 - 3*a*b^2)*c^2*f^2 - (b^3*d^2*f - 2*(a
^3 - 3*a*b^2)*c*d*f^2)*x)*sin(2*f*x + 2*e)^2 + 2*((a^3 - 3*a*b^2)*d^2*f^2*x
^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2 + (2*(a^3 - 3*a*
b^2)*d^2*f^2*x^2 - b^3*c*d*f + 2*(a^3 - 3*a*b^2)*c^2*f^2 - (b^3*d^2*f - 4*(
a^3 - 3*a*b^2)*c*d*f^2)*x)*cos(2*f*x + 2*e) + (3*a*b^2*d^2*f*x + 3*a*b^2*c*
d*f + b^3*d^2)*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(2*(a^3 - 3*a*b^2)*d^
2*f^2*x^2 - b^3*c*d*f + 2*(a^3 - 3*a*b^2)*c^2*f^2 - (b^3*d^2*f - 4*(a^3 - 3
*a*b^2)*c*d*f^2)*x)*cos(2*f*x + 2*e) + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c
^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x
 + c^3*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2
*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e)^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2
 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)^2 + 4*(d^4*f^2*x^3 + 3*c*d
^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(2*f*
x + 2*e)^2 + 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2
 + 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*
x + 2*e))*cos(4*f*x + 4*e) + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f
^2*x + c^3*d*f^2)*cos(2*f*x + 2*e))*integrate(-2*((3*a^2*b - b^3)*d^2*f^2*x
^2 + 6*a*b^2*c*d*f + 3*b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 + 2*(3*a*b^2*d^2*f
 + (3*a^2*b - b^3)*c*d*f^2)*x)*sin(2*f*x + 2*e)/(d^4*f^2*x^4 + 4*c*d^3*f^2*
x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f^2*x^4 + 4*c*d^3*
f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2)*cos(2*f*x + 2*e)^2 +
 (d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f
^2)*sin(2*f*x + 2*e)^2 + 2*(d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x
^2 + 4*c^3*d*f^2*x + c^4*f^2)*cos(2*f*x + 2*e)), x) - 2*(3*a*b^2*d^2*f*x +
3*a*b^2*c*d*f + b^3*d^2 + (3*a*b^2*d^2*f*x + 3*a*b^2*c*d*f + b^3*d^2)*cos(2
*f*x + 2*e) - (2*(a^3 - 3*a*b^2)*d^2*f^2*x^2 - b^3*c*d*f + 2*(a^3 - 3*a*b^2
)*c^2*f^2 - (b^3*d^2*f - 4*(a^3 - 3*a*b^2)*c*d*f^2)*x)*sin(2*f*x + 2*e))*si
```

```
n(4*f*x + 4*e) - 2*(3*a*b^2*d^2*f*x + 3*a*b^2*c*d*f + b^3*d^2)*sin(2*f*x +
2*e))/(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f
^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(4*f*x + 4*e)^2
+ 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x
 + 2*e)^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*s
in(4*f*x + 4*e)^2 + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^
3*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x
^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(2*f*x + 2*e)^2 + 2*(d^4*f^2*x^3 + 3*c
*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 + 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x
^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(d
^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e
))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan{(e + fx)})^3}{(c + dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\tan(e+f\,x))^3}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(c + d*x)^2,x)
[Out] int((a + b*tan(e + f*x))^3/(c + d*x)^2, x)
```

3.54 $\int \frac{(c+dx)^3}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=243

$$\frac{(c+dx)^4}{4(a+ib)d}+\frac{b(c+dx)^3\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f}-\frac{3ibd(c+dx)^2\text{PolyLog}\left(2,-\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{2\,(a^2+b^2)\,f^2}+\frac{3bd^2(c+dx)\text{P}}{}$$

```
[Out] 1/4*(d*x+c)^4/(a+I*b)/d+b*(d*x+c)^3*ln(1+(a^2+b^2)*exp(2*I*(f*x+e))/(a+I*b)
^2)/(a^2+b^2)/f-3/2*I*b*d*(d*x+c)^2*polylog(2,-(a^2+b^2)*exp(2*I*(f*x+e))/(
a+I*b)^2)/(a^2+b^2)/f^2+3/2*b*d^2*(d*x+c)*polylog(3,-(a^2+b^2)*exp(2*I*(f*x
+e))/(a+I*b)^2)/(a^2+b^2)/f^3+3/4*I*b*d^3*polylog(4,-(a^2+b^2)*exp(2*I*(f*x
+e))/(a+I*b)^2)/(a^2+b^2)/f^4
```

Rubi [A]

time = 0.24, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3813, 2221, 2611, 6744, 2320, 6724}

$$\frac{3bd^2(c+dx)\text{Li}_3\left(-\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{2f^3\,(a^2+b^2)}-\frac{3ibd(c+dx)^2\text{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{2f^2\,(a^2+b^2)}+\frac{b(c+dx)^3\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{f\,(a^2+b^2)}+\frac{3ibd^3\text{Li}_4\left(-\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{4f^4\,(a^2+b^2)}+\frac{(c+dx)^4}{4d(a+ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Tan[e + f*x]),x]
```

```
[Out] (c + d*x)^4/(4*(a + I*b)*d) + (b*(c + d*x)^3*Log[1 + ((a^2 + b^2)*E^((2*I)*
(e + f*x)))/(a + I*b)^2])/((a^2 + b^2)*f) - (((3*I)/2)*b*d*(c + d*x)^2*Poly
Log[2, -(((a^2 + b^2)*E^((2*I)*(e + f*x)))/(a + I*b)^2)])/((a^2 + b^2)*f^2)
 + (3*b*d^2*(c + d*x)*PolyLog[3, -(((a^2 + b^2)*E^((2*I)*(e + f*x)))/(a + I
*b)^2)])/(2*(a^2 + b^2)*f^3) + (((3*I)/4)*b*d^3*PolyLog[4, -(((a^2 + b^2)*E
^((2*I)*(e + f*x)))/(a + I*b)^2)])/((a^2 + b^2)*f^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3813

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int
[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2*
I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+b\tan(e+fx)}\,dx = \frac{(c+dx)^4}{4(a+ib)d} + (2ib)\int \frac{e^{2i(e+fx)}(c+dx)^3}{(a+ib)^2+(a^2+b^2)\,e^{2i(e+fx)}}\,dx$$

$$= \frac{(c+dx)^4}{4(a+ib)d} + \frac{b(c+dx)^3\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{(3bd)\int(c+dx)^2\log\left(1+\frac{(a^2+}{}\right.}{(a^2+b^2)\,f}$$

$$= \frac{(c+dx)^4}{4(a+ib)d} + \frac{b(c+dx)^3\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e}}{(a+ib)^2}\right.}{2\,(a^2+b^2)\,f^2}$$

$$= \frac{(c+dx)^4}{4(a+ib)d} + \frac{b(c+dx)^3\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e}}{(a+ib)^2}\right.}{2\,(a^2+b^2)\,f^2}$$

$$= \frac{(c+dx)^4}{4(a+ib)d} + \frac{b(c+dx)^3\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e}}{(a+ib)^2}\right.}{2\,(a^2+b^2)\,f^2}$$

$$= \frac{(c+dx)^4}{4(a+ib)d} + \frac{b(c+dx)^3\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e}}{(a+ib)^2}\right.}{2\,(a^2+b^2)\,f^2}$$

Mathematica [**A**]
time = 2.27, size = 418, normalized size = 1.72

$$\frac{4ac^3f^4x-4ibc^3f^4x+6ac^2df^4x^2-6ibc^2df^4x^2+4acd^2f^4x^3-4ibcd^2f^4x^3+ad^3f^4x^4-ibd^3f^4x^4+4bc^3f^3\log\left(1+\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)+12bc^2df^3x\log\left(1+\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)+12bcd^2f^3x^2\log\left(1+\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)+4bd^3f^3x^3\log\left(1+\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)-6ibdf^2(c+dx)^2\mathrm{PolyLog}\left(2,-\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)+6bd^2f(c+dx)\mathrm{PolyLog}\left(3,-\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)+3ibd^3\mathrm{PolyLog}\left(4,-\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)}{4\,(a^2+b^2)\,f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Tan[e + f*x]),x]
```

```
[Out] (4*a*c^3*f^4*x - (4*I)*b*c^3*f^4*x + 6*a*c^2*d*f^4*x^2 - (6*I)*b*c^2*d*f^4*
x^2 + 4*a*c*d^2*f^4*x^3 - (4*I)*b*c*d^2*f^4*x^3 + a*d^3*f^4*x^4 - I*b*d^3*f
^4*x^4 + 4*b*c^3*f^3*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + 1
2*b*c^2*d*f^3*x*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + 12*b*c
*d^2*f^3*x^2*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + 4*b*d^3*f
^3*x^3*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] - (6*I)*b*d*f^2*(
c + d*x)^2*PolyLog[2, -(((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b))] + 6*b*d
^2*f*(c + d*x)*PolyLog[3, -(((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b))] + (
3*I)*b*d^3*PolyLog[4, -(((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b))])/(4*(a^
2 + b^2)*f^4)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1467 vs. 2(220) = 440.
time = 0.54, size = 1468, normalized size = 6.04

method	result	size
risch	Expression too large to display	1468

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] I/f/(I*a+b)*b*c^3/(a+I*b)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)
-3*I/f/(I*a+b)*b*c*d^2/(-I*b-a)*ln(1-(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*x^2
-3*I/f/(I*a+b)*b*c^2*d/(-I*b-a)*ln(1-(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*x-3
*I/f^2/(I*a+b)*b*c^2*d/(-I*b-a)*ln(1-(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*e+3
*I/f^3/(I*a+b)*b*c*d^2*e^2/(-I*b-a)*ln(1-(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))
-1/4*d^3/(I*b-a)*x^4-1/(I*b-a)*c^3*x-1/4/d/(I*b-a)*c^4-1/2/(I*a+b)*b*d^3/(-
I*b-a)*x^4-6/f/(I*a+b)*b*c^2*d/(-I*b-a)*e*x-d^2/(I*b-a)*c*x^3-3/2*d/(I*b-a)
*c^2*x^2-I/f/(I*a+b)*b*d^3/(-I*b-a)*ln(1-(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))
*x^3-I/f^4/(I*a+b)*b*d^3/(-I*b-a)*ln(1-(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*e
^3+6/f^2/(I*a+b)*b*c*d^2/(-I*b-a)*e^2*x-3/f^2/(I*a+b)*b*c*d^2/(-I*b-a)*poly
log(2,(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*x-3/2*I/f^3/(I*a+b)*b*c*d^2/(-I*b-
a)*polylog(3,(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))+2*I/f^4/(I*a+b)*b*d^3*e^3/(
a+I*b)*ln(exp(I*(f*x+e)))-I/f^4/(I*a+b)*b*d^3*e^3/(a+I*b)*ln(I*exp(2*I*(f*x
+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-3/2*I/f^3/(I*a+b)*b*d^3/(-I*b-a)*polylog(3
,(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*x-3/f^2/(I*a+b)*b*c^2*d/(-I*b-a)*e^2-3/
2/f^2/(I*a+b)*b*c^2*d/(-I*b-a)*polylog(2,(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))
+4/f^3/(I*a+b)*b*c*d^2/(-I*b-a)*e^3-2/f^3/(I*a+b)*b*d^3*e^3/(-I*b-a)*x-3/2/
f^2/(I*a+b)*b*d^3/(-I*b-a)*polylog(2,(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*x^2
-2*I/f/(I*a+b)*b*c^3/(a+I*b)*ln(exp(I*(f*x+e)))-3*I/f^2/(I*a+b)*b*c^2*d*e/(
a+I*b)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)+3*I/f^3/(I*a+b)*b*
c*d^2*e^2/(a+I*b)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-6*I/f^3
/(I*a+b)*b*c*d^2*e^2/(a+I*b)*ln(exp(I*(f*x+e)))+6*I/f^2/(I*a+b)*b*c^2*d*e/(
a+I*b)*ln(exp(I*(f*x+e)))-2/(I*a+b)*b*c*d^2/(-I*b-a)*x^3-3/(I*a+b)*b*c^2*d/
(-I*b-a)*x^2+3/4/f^4/(I*a+b)*b*d^3/(-I*b-a)*polylog(4,(a-I*b)*exp(2*I*(f*x+
e))/(-I*b-a))-3/2/f^4/(I*a+b)*b*d^3*e^4/(-I*b-a)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(216) = 432$.
time = 0.75, size = 1038, normalized size = 4.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*(18*c^2*d*(2*(f*x + e)*a/((a^2 + b^2)*f) + 2*b*log(b*tan(f*x + e) + a
)/((a^2 + b^2)*f) - b*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*f))*e - 6*(2*(f*
x + e)*a/(a^2 + b^2) + 2*b*log(b*tan(f*x + e) + a)/(a^2 + b^2) - b*log(tan(
```

```
f*x + e)^2 + 1)/(a^2 + b^2))*c^3 - (3*(f*x + e)^4*(a - I*b)*d^3 + 12*I*b*d^
3*polylog(4, (I*a + b)*e^(2*I*f*x + 2*I*e)/(-I*a + b)) + 12*((a - I*b)*c*d^
2*f - (a*e - I*b*e)*d^3)*(f*x + e)^3 + 18*((a - I*b)*c^2*d*f^2 - 2*(a*e - I
*b*e)*c*d^2*f + (a*e^2 - I*b*e^2)*d^3)*(f*x + e)^2 + 12*(3*(a*e^2 - I*b*e^2
)*c*d^2*f - (a*e^3 - I*b*e^3)*d^3)*(f*x + e) - 12*(-3*I*b*c*d^2*f*e^2 + I*b
*d^3*e^3)*arctan2(-b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*cos(2*f*x
 + 2*e) + b*sin(2*f*x + 2*e) + a) - 4*(4*I*(f*x + e)^3*b*d^3 + 9*(I*b*c*d^2
*f - I*b*d^3*e)*(f*x + e)^2 + 9*(I*b*c^2*d*f^2 - 2*I*b*c*d^2*f*e + I*b*d^3*
e^2)*(f*x + e))*arctan2((2*a*b*cos(2*f*x + 2*e) - (a^2 - b^2)*sin(2*f*x + 2
*e))/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 + (a^2 - b^2)*cos(2*f
*x + 2*e))/(a^2 + b^2)) - 6*(4*I*(f*x + e)^2*b*d^3 + 3*I*b*c^2*d*f^2 - 6*I*
b*c*d^2*f*e + 3*I*b*d^3*e^2 + 6*(I*b*c*d^2*f - I*b*d^3*e)*(f*x + e))*dilog(
(I*a + b)*e^(2*I*f*x + 2*I*e)/(-I*a + b)) + 6*(3*b*c*d^2*f*e^2 - b*d^3*e^3)
*log((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*
sin(2*f*x + 2*e)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*f*x + 2*e)) + 2*(4*(f*
x + e)^3*b*d^3 + 9*(b*c*d^2*f - b*d^3*e)*(f*x + e)^2 + 9*(b*c^2*d*f^2 - 2*b
*c*d^2*f*e + b*d^3*e^2)*(f*x + e))*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*
a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 + 2*(a^2
- b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) + 6*(4*(f*x + e)*b*d^3 + 3*b*c*d^2*f
- 3*b*d^3*e)*polylog(3, (I*a + b)*e^(2*I*f*x + 2*I*e)/(-I*a + b)))/((a^2 +
b^2)*f^3))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1209 vs. $2(216) = 432$.
time = 0.47, size = 1209, normalized size = 4.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*d^3*f^4*x^4 + 8*a*c*d^2*f^4*x^3 + 12*a*c^2*d*f^4*x^2 + 8*a*c^3*f^4
*x - 3*I*b*d^3*polylog(4, ((a^2 + 2*I*a*b - b^2)*tan(f*x + e)^2 - a^2 - 2*I
*a*b + b^2 - 2*(-I*a^2 + 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x
+ e)^2 + a^2 + b^2)) + 3*I*b*d^3*polylog(4, ((a^2 - 2*I*a*b - b^2)*tan(f*x
+ e)^2 - a^2 + 2*I*a*b + b^2 - 2*(I*a^2 + 2*a*b - I*b^2)*tan(f*x + e))/((a^
2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) - 6*(-I*b*d^3*f^2*x^2 - 2*I*b*c*d^2*f
^2*x - I*b*c^2*d*f^2)*dilog(2*((I*a*b - b^2)*tan(f*x + e)^2 - a^2 - I*a*b +
 (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 +
b^2) + 1) - 6*(I*b*d^3*f^2*x^2 + 2*I*b*c*d^2*f^2*x + I*b*c^2*d*f^2)*dilog(2
*((-I*a*b - b^2)*tan(f*x + e)^2 - a^2 + I*a*b + (-I*a^2 - 2*a*b + I*b^2)*ta
n(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2) + 1) + 4*(b*d^3*f^3*x^
3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*c^2*d*f^2*e - 3*b*c*d^2*f*e^2
 + b*d^3*e^3)*log(-2*((I*a*b - b^2)*tan(f*x + e)^2 - a^2 - I*a*b + (I*a^2 -
 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + 4
```

```
*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*c^2*d*f^2*e - 3
*b*c*d^2*f*e^2 + b*d^3*e^3)*log(-2*((-I*a*b - b^2)*tan(f*x + e)^2 - a^2 + I
*a*b + (-I*a^2 - 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 +
 a^2 + b^2)) + 4*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f*e^2 - b*d^3*e^3
)*log(((I*a*b + b^2)*tan(f*x + e)^2 - a^2 + I*a*b + (I*a^2 + I*b^2)*tan(f*x
 + e))/(tan(f*x + e)^2 + 1)) + 4*(b*c^3*f^3 - 3*b*c^2*d*f^2*e + 3*b*c*d^2*f
*e^2 - b*d^3*e^3)*log(((I*a*b - b^2)*tan(f*x + e)^2 + a^2 + I*a*b + (I*a^2
+ I*b^2)*tan(f*x + e))/(tan(f*x + e)^2 + 1)) + 6*(b*d^3*f*x + b*c*d^2*f)*po
lylog(3, ((a^2 + 2*I*a*b - b^2)*tan(f*x + e)^2 - a^2 - 2*I*a*b + b^2 - 2*(-
I*a^2 + 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^
2)) + 6*(b*d^3*f*x + b*c*d^2*f)*polylog(3, ((a^2 - 2*I*a*b - b^2)*tan(f*x +
 e)^2 - a^2 + 2*I*a*b + b^2 - 2*(I*a^2 + 2*a*b - I*b^2)*tan(f*x + e))/((a^2
 + b^2)*tan(f*x + e)^2 + a^2 + b^2)))/((a^2 + b^2)*f^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{a+b\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*tan(f*x+e)),x)
[Out] Integral((c + d*x)**3/(a + b*tan(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*x + c)^3/(b*tan(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{a+b\tan(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*tan(e + f*x)),x)
[Out] int((c + d*x)^3/(a + b*tan(e + f*x)), x)
```

3.55 $\int \frac{(c+dx)^2}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=181

$$\frac{(c+dx)^3}{3(a+ib)d}+\frac{b(c+dx)^2\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f}-\frac{ibd(c+dx)\text{PolyLog}\left(2,-\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f^2}+\frac{bd^2\text{PolyLog}\left(3,-\right.}{2\,(a^2+}$$

```
[Out] 1/3*(d*x+c)^3/(a+I*b)/d+b*(d*x+c)^2*ln(1+(a^2+b^2)*exp(2*I*(f*x+e))/(a+I*b)
^2)/(a^2+b^2)/f-I*b*d*(d*x+c)*polylog(2,-(a^2+b^2)*exp(2*I*(f*x+e))/(a+I*b)
^2)/(a^2+b^2)/f^2+1/2*b*d^2*polylog(3,-(a^2+b^2)*exp(2*I*(f*x+e))/(a+I*b)^2
)/(a^2+b^2)/f^3
```

Rubi [A]
time = 0.20, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3813, 2221, 2611, 2320, 6724}

$$-\frac{ibd(c+dx)\text{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{f^2\,(a^2+b^2)}+\frac{b(c+dx)^2\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{f\,(a^2+b^2)}+\frac{bd^2\text{Li}_3\left(-\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{2f^3\,(a^2+b^2)}+\frac{(c+dx)^3}{3d(a+ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Tan[e + f*x]),x]
```

```
[Out] (c + d*x)^3/(3*(a + I*b)*d) + (b*(c + d*x)^2*Log[1 + ((a^2 + b^2)*E^((2*I)*
(e + f*x)))/(a + I*b)^2])/((a^2 + b^2)*f) - (I*b*d*(c + d*x)*PolyLog[2, -((
(a^2 + b^2)*E^((2*I)*(e + f*x)))/(a + I*b)^2)])/((a^2 + b^2)*f^2) + (b*d^2*
PolyLog[3, -(((a^2 + b^2)*E^((2*I)*(e + f*x)))/(a + I*b)^2)])/(2*(a^2 + b^2
)*f^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3813

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int
[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2*
I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+b\tan(e+fx)}\,dx = \frac{(c+dx)^3}{3(a+ib)d} + (2ib)\int \frac{e^{2i(e+fx)}(c+dx)^2}{(a+ib)^2+(a^2+b^2)\,e^{2i(e+fx)}}\,dx$$

$$= \frac{(c+dx)^3}{3(a+ib)d} + \frac{b(c+dx)^2\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{(2bd)\int(c+dx)\log\left(1+\frac{(a^2+\cdots\right.}{(a^2+b^2)\,f}$$

$$= \frac{(c+dx)^3}{3(a+ib)d} + \frac{b(c+dx)^2\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{ibd(c+dx)\mathrm{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e+\cdots}}{(a+ib)^2}\right.}{(a^2+b^2)\,f^2}$$

$$= \frac{(c+dx)^3}{3(a+ib)d} + \frac{b(c+dx)^2\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{ibd(c+dx)\mathrm{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e+\cdots}}{(a+ib)^2}\right.}{(a^2+b^2)\,f^2}$$

$$= \frac{(c+dx)^3}{3(a+ib)d} + \frac{b(c+dx)^2\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{ibd(c+dx)\mathrm{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e+\cdots}}{(a+ib)^2}\right.}{(a^2+b^2)\,f^2}$$

Mathematica [A]

time = 1.70, size = 321, normalized size = 1.77

$$\frac{b\left(2f^2\left(-2i(a-ib)e^{2ie}fx(3c^2+3cdx+d^2x^2)+3(-ib(-1+e^{2ie})+a(1+e^{2ie}))\,(c+dx)^2\log\left(1+\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)\right)+6d(b-be^{2ie}-ia(1+e^{2ie}))\,f(c+dx)\mathrm{PolyLog}\left(2,-\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)+3d^2(-ib(-1+e^{2ie})+a(1+e^{2ie}))\,\mathrm{PolyLog}\left(3,-\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)\right)}{6\left(a^2+b^2\right)\left(-ib\left(-1+e^{2ie}\right)+a\left(1+e^{2ie}\right)\right)f^3}+\frac{x(3c^2+3cdx+d^2x^2)\cos(e)}{3(a\cos(e)+b\sin(e))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Tan[e + f*x]),x]
```

```
[Out] (b*(2*f^2*((-2*I)*(a - I*b)*E^((2*I)*e)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 3
*((-I)*b*(-1 + E^((2*I)*e)) + a*(1 + E^((2*I)*e)))*(c + d*x)^2*Log[1 + ((a
- I*b)*E^((2*I)*(e + f*x)))/(a + I*b)]) + 6*d*(b - b*E^((2*I)*e) - I*a*(1 +
 E^((2*I)*e)))*f*(c + d*x)*PolyLog[2, -(((a - I*b)*E^((2*I)*(e + f*x)))/(a
+ I*b))] + 3*d^2*((-I)*b*(-1 + E^((2*I)*e)) + a*(1 + E^((2*I)*e)))*PolyLog[
3, -(((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b))]))/(6*(a^2 + b^2)*((-I)*b*(
-1 + E^((2*I)*e)) + a*(1 + E^((2*I)*e)))*f^3) + (x*(3*c^2 + 3*c*d*x + d^2*x
^2)*Cos[e])/(3*(a*Cos[e] + b*Sin[e]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(166) = 332$.
time = 0.39, size = 940, normalized size = 5.19

method	result
risch	$-\frac{d^{2} x^{3}}{3(ib-a)}-\frac{d c\,x^{2}}{ib-a}-\frac{c^{2} x}{ib-a}-\frac{c^{3}}{3d(ib-a)}+\frac{ib\,d^{2} e^{2} \ln\left(1-\frac{(-ib+a)\,\mathrm{e}^{2i(fx+e)}}{-ib-a}\right)}{f^{3}(ia+b)(-ib-a)}-\frac{b\,d^{2}\,\mathrm{polylog}\left(2,\frac{(-ib+a)\,\mathrm{e}^{2i(fx+e)}}{-ib-a}\right)x}{f^{2}(ia+b)(-ib-a)}+\frac{ib\,d^{2} e^{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*d^2/(I*b-a)*x^3-d/(I*b-a)*c*x^2-1/(I*b-a)*c^2*x-1/3/d/(I*b-a)*c^3+I/f^
3/(I*a+b)*b*d^2*e^2/(-I*b-a)*ln(1-(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))-1/f^2/
(I*a+b)*b*d^2/(-I*b-a)*polylog(2,(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*x+I/f^3
/(I*a+b)*b*d^2*e^2/(a+I*b)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a
)-1/2*I/f^3/(I*a+b)*b*d^2/(-I*b-a)*polylog(3,(a-I*b)*exp(2*I*(f*x+e))/(-I*b
-a))-2*I/f^3/(I*a+b)*b*d^2*e^2/(a+I*b)*ln(exp(I*(f*x+e)))-2*I/f^2/(I*a+b)*b
*c*d*e/(a+I*b)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)+I/f/(I*a+b
)*b*c^2/(a+I*b)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-2*I/f^2/(
I*a+b)*b*c*d/(-I*b-a)*ln(1-(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*e-2*I/f/(I*a+
b)*b*c^2/(a+I*b)*ln(exp(I*(f*x+e)))-2*I/f/(I*a+b)*b*c*d/(-I*b-a)*ln(1-(a-I*
b)*exp(2*I*(f*x+e))/(-I*b-a))*x-2/3/(I*a+b)*b*d^2/(-I*b-a)*x^3+2/f^2/(I*a+b
)*b*d^2/(-I*b-a)*e^2*x+4/3/f^3/(I*a+b)*b*d^2/(-I*b-a)*e^3+4*I/f^2/(I*a+b)*b
*c*d*e/(a+I*b)*ln(exp(I*(f*x+e)))-I/f/(I*a+b)*b*d^2/(-I*b-a)*ln(1-(a-I*b)*e
xp(2*I*(f*x+e))/(-I*b-a))*x^2-2/(I*a+b)*b*c*d/(-I*b-a)*x^2-4/f/(I*a+b)*b*c*
d/(-I*b-a)*e*x-2/f^2/(I*a+b)*b*c*d/(-I*b-a)*e^2-1/f^2/(I*a+b)*b*c*d/(-I*b-a
)*polylog(2,(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(163) = 326$.
time = 0.65, size = 755, normalized size = 4.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*c*d*(2*(f*x + e)*a/((a^2 + b^2)*f) + 2*b*log(b*tan(f*x + e) + a)/((
a^2 + b^2)*f) - b*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*f))*e - 3*(2*(f*x +
e)*a/(a^2 + b^2) + 2*b*log(b*tan(f*x + e) + a)/(a^2 + b^2) - b*log(tan(f*x
+ e)^2 + 1)/(a^2 + b^2))*c^2 - (2*(f*x + e)^3*(a - I*b)*d^2 + 6*I*b*d^2*arc
tan2(-b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*cos(2*f*x + 2*e) + b*s
in(2*f*x + 2*e) + a)*e^2 + 3*b*d^2*e^2*log((a^2 + b^2)*cos(2*f*x + 2*e)^2 +
 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 + 2*(a
^2 - b^2)*cos(2*f*x + 2*e)) + 6*(f*x + e)*(a*e^2 - I*b*e^2)*d^2 + 3*b*d^2*p
olylog(3, (I*a + b)*e^(2*I*f*x + 2*I*e)/(-I*a + b)) + 6*((a - I*b)*c*d*f -
(a*e - I*b*e)*d^2)*(f*x + e)^2 - 6*(I*(f*x + e)^2*b*d^2 + 2*(I*b*c*d*f - I*
b*d^2*e)*(f*x + e))*arctan2((2*a*b*cos(2*f*x + 2*e) - (a^2 - b^2)*sin(2*f*x
 + 2*e))/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 + (a^2 - b^2)*cos
(2*f*x + 2*e))/(a^2 + b^2)) - 6*(I*(f*x + e)*b*d^2 + I*b*c*d*f - I*b*d^2*e)
*dilog((I*a + b)*e^(2*I*f*x + 2*I*e)/(-I*a + b)) + 3*((f*x + e)^2*b*d^2 + 2
*(b*c*d*f - b*d^2*e)*(f*x + e))*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b
*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 + 2*(a^2 - b
^2)*cos(2*f*x + 2*e))/(a^2 + b^2)))/((a^2 + b^2)*f^2))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(163) = 326$.
time = 0.39, size = 858, normalized size = 4.74

$$\frac{4ad^2f^3x^3+12acdf^3x^2+12ac^2f^3x+3bd^2\operatorname{polylog}\left(3,\frac{(a^2+2iab-b^2)\tan(fx+e)^2-a^2-2iab+b^2-2(-ia^2+2ab+ib^2)\tan(fx+e)}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}\right)+3bd^2\operatorname{polylog}\left(3,\frac{(a^2-2iab-b^2)\tan(fx+e)^2-a^2+2iab+b^2-2(ia^2+2ab-ib^2)\tan(fx+e)}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}\right)-6(-ibd^2fx-ibcdf)\operatorname{Li}_2\left(\frac{2((iab-b^2)\tan(fx+e)^2-a^2-iab+(ia^2-2ab-ib^2)\tan(fx+e))}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}+1\right)-6(ibd^2fx+ibcdf)\operatorname{Li}_2\left(\frac{2((-iab-b^2)\tan(fx+e)^2-a^2+iab+(-ia^2-2ab+ib^2)\tan(fx+e))}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}+1\right)+6(bd^2f^2x^2+2bcdf^2x+2bcdfe-bd^2e^2)\log\left(-\frac{2((iab-b^2)\tan(fx+e)^2-a^2-iab+(ia^2-2ab-ib^2)\tan(fx+e))}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}\right)+6(bd^2f^2x^2+2bcdf^2x+2bcdfe-bd^2e^2)\log\left(-\frac{2((-iab-b^2)\tan(fx+e)^2-a^2+iab+(-ia^2-2ab+ib^2)\tan(fx+e))}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}\right)+6(bc^2f^2-2bcdfe+bd^2e^2)\log\left(\frac{(iab+b^2)\tan(fx+e)^2-a^2+iab+(ia^2+ib^2)\tan(fx+e)}{\tan(fx+e)^2+1}\right)+6(bc^2f^2-2bcdfe+bd^2e^2)\log\left(\frac{(iab-b^2)\tan(fx+e)^2+a^2+iab+(ia^2+ib^2)\tan(fx+e)}{\tan(fx+e)^2+1}\right)}{12(a^2+b^2)f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*d^2*f^3*x^3 + 12*a*c*d*f^3*x^2 + 12*a*c^2*f^3*x + 3*b*d^2*polylog
(3, ((a^2 + 2*I*a*b - b^2)*tan(f*x + e)^2 - a^2 - 2*I*a*b + b^2 - 2*(-I*a^2
 + 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) +
 3*b*d^2*polylog(3, ((a^2 - 2*I*a*b - b^2)*tan(f*x + e)^2 - a^2 + 2*I*a*b +
 b^2 - 2*(I*a^2 + 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2
+ a^2 + b^2)) - 6*(-I*b*d^2*f*x - I*b*c*d*f)*dilog(2*((I*a*b - b^2)*tan(f*x
 + e)^2 - a^2 - I*a*b + (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*
tan(f*x + e)^2 + a^2 + b^2) + 1) - 6*(I*b*d^2*f*x + I*b*c*d*f)*dilog(2*((-I
*a*b - b^2)*tan(f*x + e)^2 - a^2 + I*a*b + (-I*a^2 - 2*a*b + I*b^2)*tan(f*x
 + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2) + 1) + 6*(b*d^2*f^2*x^2 + 2
*b*c*d*f^2*x + 2*b*c*d*f*e - b*d^2*e^2)*log(-2*((I*a*b - b^2)*tan(f*x + e)^
2 - a^2 - I*a*b + (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*
x + e)^2 + a^2 + b^2)) + 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f*e - b
*d^2*e^2)*log(-2*((-I*a*b - b^2)*tan(f*x + e)^2 - a^2 + I*a*b + (-I*a^2 - 2
*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + 6*(
```

```
b*c^2*f^2 - 2*b*c*d*f*e + b*d^2*e^2)*log(((I*a*b + b^2)*tan(f*x + e)^2 - a^
2 + I*a*b + (I*a^2 + I*b^2)*tan(f*x + e))/(tan(f*x + e)^2 + 1)) + 6*(b*c^2*
f^2 - 2*b*c*d*f*e + b*d^2*e^2)*log(((I*a*b - b^2)*tan(f*x + e)^2 + a^2 + I*
a*b + (I*a^2 + I*b^2)*tan(f*x + e))/(tan(f*x + e)^2 + 1)))/((a^2 + b^2)*f^3
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{a+b\tan{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**2/(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^2}{a+b\,\mathrm{tan}\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*tan(e + f*x)),x)
```

```
[Out] int((c + d*x)^2/(a + b*tan(e + f*x)), x)
```

3.56 $\int \frac{c+dx}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=125

$$\frac{(c+dx)^2}{2(a+ib)d}+\frac{b(c+dx)\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f}-\frac{ibd\mathrm{PolyLog}\left(2,-\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{2\,(a^2+b^2)\,f^2}$$

```
[Out] 1/2*(d*x+c)^2/(a+I*b)/d+b*(d*x+c)*ln(1+(a^2+b^2)*exp(2*I*(f*x+e))/(a+I*b)^2
)/(a^2+b^2)/f-1/2*I*b*d*polylog(2,-(a^2+b^2)*exp(2*I*(f*x+e))/(a+I*b)^2)/(a
^2+b^2)/f^2
```

Rubi [A]
time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {3813, 2221, 2317, 2438}

$$\frac{b(c+dx)\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{f\,(a^2+b^2)}-\frac{ibd\mathrm{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{2f^2\,(a^2+b^2)}+\frac{(c+dx)^2}{2d(a+ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (c + d*x)^2/(2*(a + I*b)*d) + (b*(c + d*x)*Log[1 + ((a^2 + b^2)*E^((2*I)*(e
 + f*x)))/(a + I*b)^2])/((a^2 + b^2)*f) - ((I/2)*b*d*PolyLog[2, -(((a^2 + b
^2)*E^((2*I)*(e + f*x)))/(a + I*b)^2)])/((a^2 + b^2)*f^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3813

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int
[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2*
I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+b\tan(e+fx)}\,dx = \frac{(c+dx)^2}{2(a+ib)d} + (2ib)\int \frac{e^{2i(e+fx)}(c+dx)}{(a+ib)^2+(a^2+b^2)\,e^{2i(e+fx)}}\,dx$$

$$= \frac{(c+dx)^2}{2(a+ib)d} + \frac{b(c+dx)\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{(bd)\int \log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f}$$

$$= \frac{(c+dx)^2}{2(a+ib)d} + \frac{b(c+dx)\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} + \frac{(ibd)\text{Subst}\left(\int \frac{\log\left(1+\frac{(a^2+b^2)x}{(a+ib)^2}\right)}{x}\,d\right.}{2\,(a^2+b^2)\,f^2}$$

$$= \frac{(c+dx)^2}{2(a+ib)d} + \frac{b(c+dx)\log\left(1+\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{(a^2+b^2)\,f} - \frac{ibd\text{Li}_2\left(-\frac{(a^2+b^2)e^{2i(e+fx)}}{(a+ib)^2}\right)}{2\,(a^2+b^2)\,f^2}$$

Mathematica [A]
time = 1.02, size = 113, normalized size = 0.90

$$\frac{f\left((a-ib)fx(2c+dx)+2b(c+dx)\log\left(1+\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)\right)-ibd\text{PolyLog}\left(2,-\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)}{2\,(a^2+b^2)\,f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (f*((a - I*b)*f*x*(2*c + d*x) + 2*b*(c + d*x)*Log[1 + ((a - I*b)*E^((2*I)*(
e + f*x)))/(a + I*b)]) - I*b*d*PolyLog[2, -(((a - I*b)*E^((2*I)*(e + f*x)))
/(a + I*b))])/(2*(a^2 + b^2)*f^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(113) = 226$.
time = 0.39, size = 462, normalized size = 3.70

method	result

risch	$-\frac{d x^2}{2(ib-a)}-\frac{cx}{ib-a}+\frac{bc\ln\left(i\mathrm{e}^{2i(fx+e)}b-a\,\mathrm{e}^{2i(fx+e)}-ib-a\right)}{f(-ib+a)(ib+a)}-\frac{2bc\ln\left(\mathrm{e}^{i(fx+e)}\right)}{f(-ib+a)(ib+a)}-\frac{bd\ln\left(1-\frac{(-ib+a)\mathrm{e}^{2i(fx+e)}}{-ib-a}\right)x}{f(-ib+a)(-ib-a)}-\frac{bd\ln\left(1-\right.}{f^2(-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(I*b-a)*d*x^2-1/(I*b-a)*c*x+1/f*b/(a-I*b)*c/(a+I*b)*ln(I*exp(2*I*(f*x+
e))*b-a*exp(2*I*(f*x+e))-I*b-a)-2/f*b/(a-I*b)*c/(a+I*b)*ln(exp(I*(f*x+e)))-
1/f*b/(a-I*b)*d/(-I*b-a)*ln(1-(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*x-1/f^2*b/
(a-I*b)*d/(-I*b-a)*ln(1-(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))*e+I*b/(a-I*b)*d/
(-I*b-a)*x^2+2*I/f*b/(a-I*b)*d/(-I*b-a)*e*x+I/f^2*b/(a-I*b)*d/(-I*b-a)*e^2+
1/2*I/f^2*b/(a-I*b)*d/(-I*b-a)*polylog(2,(a-I*b)*exp(2*I*(f*x+e))/(-I*b-a))
-1/f^2*b/(a-I*b)*d*e/(a+I*b)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b
-a)+2/f^2*b/(a-I*b)*d*e/(a+I*b)*ln(exp(I*(f*x+e)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(110) = 220$.
time = 0.62, size = 422, normalized size = 3.38

$$\frac{(a-ib)df^2x^2+2(a-ib)cf^2x-2ibdfx\arctan\left(\frac{2ab\cos(2fx+2e)-(a^2-b^2)\sin(2fx+2e)}{a^2+b^2},\frac{2ab\sin(2fx+2e)+a^2+b^2+(a^2-b^2)\cos(2fx+2e)}{a^2+b^2}\right)+bdfx\log\left(\frac{(a^2+b^2)\cos(2fx+2e)^2+4ab\sin(2fx+2e)+(a^2+b^2)\sin(2fx+2e)^2+a^2+b^2+2(a^2-b^2)\cos(2fx+2e)}{a^2+b^2}\right)+2ibcf\arctan\left(-b\cos(2fx+2e)+a\sin(2fx+2e)+b,a\cos(2fx+2e)+b\sin(2fx+2e)+a\right)+bcf\log\left((a^2+b^2)\cos(2fx+2e)^2+4ab\sin(2fx+2e)+(a^2+b^2)\sin(2fx+2e)^2+a^2+b^2+2(a^2-b^2)\cos(2fx+2e)\right)-ibd\mathrm{Li}_2\left(\frac{(iae^{(2ie)}+be^{(2ie)})e^{(2ifx)}}{-ia+b}\right)}{2(a^2+b^2)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a - I*b)*d*f^2*x^2 + 2*(a - I*b)*c*f^2*x - 2*I*b*d*f*x*arctan2((2*a*b
*cos(2*f*x + 2*e) - (a^2 - b^2)*sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*sin(2
*f*x + 2*e) + a^2 + b^2 + (a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) + b*d*
f*x*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b
^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 +
 b^2)) + 2*I*b*c*f*arctan2(-b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*
cos(2*f*x + 2*e) + b*sin(2*f*x + 2*e) + a) + b*c*f*log((a^2 + b^2)*cos(2*f*
x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2
+ b^2 + 2*(a^2 - b^2)*cos(2*f*x + 2*e)) - I*b*d*dilog((I*a*e^(2*I*e) + b*e^
(2*I*e))*e^(2*I*f*x)/(-I*a + b)))/((a^2 + b^2)*f^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(110) = 220$.
time = 0.40, size = 549, normalized size = 4.39

$$\frac{2adf^2x^2+4acf^2x+ibd\mathrm{Li}_2\left(\frac{2\left((iab-b^2)\tan(fx+e)^2-a^2-iab+(ia^2-2ab-ib^2)\tan(fx+e)\right)}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}+1\right)-ibd\mathrm{Li}_2\left(\frac{2\left((-iab-b^2)\tan(fx+e)^2-a^2+iab+(-ia^2-2ab+ib^2)\tan(fx+e)\right)}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}+1\right)+2(bdfx+bde)\log\left(-\frac{2\left((iab-b^2)\tan(fx+e)^2-a^2-iab+(ia^2-2ab-ib^2)\tan(fx+e)\right)}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}\right)+2(bdfx+bde)\log\left(-\frac{2\left((-iab-b^2)\tan(fx+e)^2-a^2+iab+(-ia^2-2ab+ib^2)\tan(fx+e)\right)}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}\right)+2(bcf-bde)\log\left(\frac{(iab+b^2)\tan(fx+e)^2-a^2+iab+(ia^2+ib^2)\tan(fx+e)}{\tan(fx+e)^2+1}\right)+2(bcf-bde)\log\left(\frac{(iab-b^2)\tan(fx+e)^2+a^2+iab+(ia^2+ib^2)\tan(fx+e)}{\tan(fx+e)^2+1}\right)}{4(a^2+b^2)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d*f^2*x^2 + 4*a*c*f^2*x + I*b*d*dilog(2*((I*a*b - b^2)*tan(f*x + e
)^2 - a^2 - I*a*b + (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(
f*x + e)^2 + a^2 + b^2) + 1) - I*b*d*dilog(2*((-I*a*b - b^2)*tan(f*x + e)^2
 - a^2 + I*a*b + (-I*a^2 - 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*
x + e)^2 + a^2 + b^2) + 1) + 2*(b*d*f*x + b*d*e)*log(-2*((I*a*b - b^2)*tan(
f*x + e)^2 - a^2 - I*a*b + (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^
2)*tan(f*x + e)^2 + a^2 + b^2)) + 2*(b*d*f*x + b*d*e)*log(-2*((-I*a*b - b^2
)*tan(f*x + e)^2 - a^2 + I*a*b + (-I*a^2 - 2*a*b + I*b^2)*tan(f*x + e))/((a
^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + 2*(b*c*f - b*d*e)*log(((I*a*b + b^
2)*tan(f*x + e)^2 - a^2 + I*a*b + (I*a^2 + I*b^2)*tan(f*x + e))/(tan(f*x +
e)^2 + 1)) + 2*(b*c*f - b*d*e)*log(((I*a*b - b^2)*tan(f*x + e)^2 + a^2 + I*
a*b + (I*a^2 + I*b^2)*tan(f*x + e))/(tan(f*x + e)^2 + 1)))/((a^2 + b^2)*f^2
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx}{a + b\tan(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((c + d*x)/(a + b*tan(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*tan(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + d\,x}{a + b\tan\left(e + f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*tan(e + f*x)),x)
```

```
[Out] int((c + d*x)/(a + b*tan(e + f*x)), x)
```

3.57 $\int \frac{1}{(c+dx)(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\tan(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*tan(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\tan(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Tan[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)*(a + b*Tan[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\tan(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\tan(e+fx))}\,dx$$

Mathematica [A]
time = 1.73, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\tan(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Tan[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a + b*Tan[e + f*x])), x]
```

Maple [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a+b\tan(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*tan(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+b*tan(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] (2*(a^2*b + b^3)*d*integrate((2*a*b*cos(2*f*x + 2*e) - (a^2 - b^2)*sin(2*f*
x + 2*e))/((a^4 + 2*a^2*b^2 + b^4)*d*x + ((a^4 + 2*a^2*b^2 + b^4)*d*x + (a^
4 + 2*a^2*b^2 + b^4)*c)*cos(2*f*x + 2*e)^2 + ((a^4 + 2*a^2*b^2 + b^4)*d*x +
 (a^4 + 2*a^2*b^2 + b^4)*c)*sin(2*f*x + 2*e)^2 + (a^4 + 2*a^2*b^2 + b^4)*c
+ 2*((a^4 - b^4)*d*x + (a^4 - b^4)*c)*cos(2*f*x + 2*e) + 4*((a^3*b + a*b^3)
*d*x + (a^3*b + a*b^3)*c)*sin(2*f*x + 2*e)), x) + a*log(d*x + c))/((a^2 + b
^2)*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*tan(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\tan(e + fx))(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))*(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*tan(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\tan\left(e+f\,x\right)\right)\,\left(c+d\,x\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a + b*tan(e + f*x))*(c + d*x)), x)
```

3.58 $\int \frac{1}{(c+dx)^2(a+b\tan(e+fx))} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\tan(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*tan(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\tan(e+fx))} dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Tan[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Tan[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\tan(e+fx))} dx = \int \frac{1}{(c+dx)^2(a+b\tan(e+fx))} dx$$

Mathematica [A]
time = 4.40, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\tan(e+fx))} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Tan[e + f*x])),x]
[Out] Integrate[1/((c + d*x)^2*(a + b*Tan[e + f*x])), x]
```

Maple [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(a+b\tan\left(fx+e\right)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*tan(f*x+e)),x)
[Out] int(1/(d*x+c)^2/(a+b*tan(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] (2*((a^2*b + b^3)*d^2*x + (a^2*b + b^3)*c*d)*integrate((2*a*b*cos(2*f*x + 2
*e) - (a^2 - b^2)*sin(2*f*x + 2*e))/((a^4 + 2*a^2*b^2 + b^4)*d^2*x^2 + 2*(a
^4 + 2*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b^4)*c^2 + ((a^4 + 2*a^2*b
^2 + b^4)*d^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b^
4)*c^2)*cos(2*f*x + 2*e)^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^2*x^2 + 2*(a^4 + 2*
a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b^4)*c^2)*sin(2*f*x + 2*e)^2 + 2*
((a^4 - b^4)*d^2*x^2 + 2*(a^4 - b^4)*c*d*x + (a^4 - b^4)*c^2)*cos(2*f*x + 2
*e) + 4*((a^3*b + a*b^3)*d^2*x^2 + 2*(a^3*b + a*b^3)*c*d*x + (a^3*b + a*b^3
)*c^2)*sin(2*f*x + 2*e)), x) - a)/((a^2 + b^2)*d^2*x + (a^2 + b^2)*c*d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*tan(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan\left(e + fx\right)\right)\left(c + dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*tan(f*x+e)),x)
[Out] Integral(1/((a + b*tan(e + f*x))*(c + d*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*tan(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\tan\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))*(c + d*x)^2),x)
```

```
[Out] int(1/((a + b*tan(e + f*x))*(c + d*x)^2), x)
```

3.59 $\int \frac{(c+dx)^3}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=848

$$-\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2 f}+\frac{2b^2(c+dx)^3}{(a+ib)(ia+b)^2\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f}+\frac{(c+dx)^4}{4(a-ib)^2d}+\frac{b(c+dx)^4}{(ia-b)(a-ib)^2d}-\frac{b^2(c+d}{(a^2+b^2)}$$

```
[Out] -2*I*b^2*(d*x+c)^3/(a^2+b^2)^2/f+2*b^2*(d*x+c)^3/(a+I*b)/(I*a+b)^2/(I*a-b+(
I*a+b)*exp(2*I*e+2*I*f*x))/f+1/4*(d*x+c)^4/(a-I*b)^2/d+b*(d*x+c)^4/(I*a-b)/
(a-I*b)^2/d-b^2*(d*x+c)^4/(a^2+b^2)^2/d+3*b^2*d*(d*x+c)^2*ln(1+(a-I*b)*exp(
2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)^2/f^2+2*b*(d*x+c)^3*ln(1+(a-I*b)*exp(2*I*
e+2*I*f*x)/(a+I*b))/(a-I*b)^2/(a+I*b)/f-3*I*b^2*d^2*(d*x+c)*polylog(2,-(a-I
*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)^2/f^3-3*I*b^2*d^2*(d*x+c)*polylog
(3,-(a-I*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)^2/f^3+3*b*d*(d*x+c)^2*pol
ylog(2,-(a-I*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(I*a-b)/(a-I*b)^2/f^2-3*b^2*d*(
d*x+c)^2*polylog(2,-(a-I*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)^2/f^2+3/2
*b^2*d^3*polylog(3,-(a-I*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)^2/f^4+3*b
*d^2*(d*x+c)*polylog(3,-(a-I*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a-I*b)^2/(a+I*
b)/f^3-2*I*b^2*(d*x+c)^3*ln(1+(a-I*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)
^2/f-3/2*b*d^3*polylog(4,-(a-I*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(I*a-b)/(a-I*
b)^2/f^4+3/2*b^2*d^3*polylog(4,-(a-I*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^
2)^2/f^4
```

Rubi [A]
time = 1.38, antiderivative size = 848, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3815, 2216, 2215, 2221, 2611, 6744, 2320, 6724, 2222}

$$\frac{b(c+dx)^4}{(ia-b)(a-ib)^2d}+\frac{(c+dx)^4}{4(a-ib)^2d}-\frac{b^2(c+dx)^4}{(a^2+b^2)^2d}+\frac{2b\log\left(\frac{e^{2ie+2ifx}(a-ib)}{a+ib}+1\right)(c+dx)^3}{(a-ib)^2(a+ib)f}-\frac{2ib^2\log\left(\frac{e^{2ie+2ifx}(a-ib)}{a+ib}+1\right)(c+dx)^3}{(a^2+b^2)^2f}+\frac{2b^2(c+dx)^3}{(a+ib)(ia+b)^2\left(ia+(ia+b)e^{2ie+2ifx}-b\right)f}-\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2f}+\frac{3b^2d\log\left(\frac{e^{2ie+2ifx}(a-ib)}{a+ib}+1\right)(c+dx)^2}{(a^2+b^2)^2f^2}+\frac{3bd\text{Li}_2\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)(c+dx)^2}{(ia-b)(a-ib)^2f^2}-\frac{3b^2d\text{Li}_2\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)(c+dx)^2}{(a^2+b^2)^2f^2}-\frac{3ib^2d^2\text{Li}_2\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)(c+dx)}{(a^2+b^2)^2f^3}+\frac{3bd^2\text{Li}_3\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)(c+dx)}{(a-ib)^2(a+ib)f^3}-\frac{3ib^2d^2\text{Li}_3\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)(c+dx)}{(a^2+b^2)^2f^3}+\frac{3b^2d^3\text{Li}_3\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{2(a^2+b^2)^2f^4}-\frac{3bd^3\text{Li}_4\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{2(ia-b)(a-ib)^2f^4}+\frac{3b^2d^3\text{Li}_4\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{2(a^2+b^2)^2f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((-2*I)*b^2*(c + d*x)^3)/((a^2 + b^2)^2*f) + (2*b^2*(c + d*x)^3)/((a + I*b)
*(I*a + b)^2*(I*a - b + (I*a + b)*E^((2*I)*e + (2*I)*f*x))*f) + (c + d*x)^4
/(4*(a - I*b)^2*d) + (b*(c + d*x)^4)/((I*a - b)*(a - I*b)^2*d) - (b^2*(c +
d*x)^4)/((a^2 + b^2)^2*d) + (3*b^2*d*(c + d*x)^2*Log[1 + ((a - I*b)*E^((2*I
)*e + (2*I)*f*x))/(a + I*b)])/((a^2 + b^2)^2*f^2) + (2*b*(c + d*x)^3*Log[1
+ ((a - I*b)*E^((2*I)*e + (2*I)*f*x))/(a + I*b)])/((a - I*b)^2*(a + I*b)*f)
 - ((2*I)*b^2*(c + d*x)^3*Log[1 + ((a - I*b)*E^((2*I)*e + (2*I)*f*x))/(a +
I*b)])/((a^2 + b^2)^2*f) - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[2, -(((a - I*b)
*E^((2*I)*e + (2*I)*f*x))/(a + I*b))])/((a^2 + b^2)^2*f^3) + (3*b*d*(c + d*
x)^2*PolyLog[2, -(((a - I*b)*E^((2*I)*e + (2*I)*f*x))/(a + I*b))])/((I*a -
b)*(a - I*b)^2*f^2) - (3*b^2*d*(c + d*x)^2*PolyLog[2, -(((a - I*b)*E^((2*I)
```

```
*e + (2*I)*f*x))/(a + I*b))])/((a^2 + b^2)^2*f^2) + (3*b^2*d^3*PolyLog[3, -
(((a - I*b)*E^((2*I)*e + (2*I)*f*x))/(a + I*b))])/(2*(a^2 + b^2)^2*f^4) + (
3*b*d^2*(c + d*x)*PolyLog[3, -(((a - I*b)*E^((2*I)*e + (2*I)*f*x))/(a + I*b
))])/((a - I*b)^2*(a + I*b)*f^3) - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[3, -(((
a - I*b)*E^((2*I)*e + (2*I)*f*x))/(a + I*b))])/((a^2 + b^2)^2*f^3) - (3*b*d
^3*PolyLog[4, -(((a - I*b)*E^((2*I)*e + (2*I)*f*x))/(a + I*b))])/(2*(I*a -
b)*(a - I*b)^2*f^4) + (3*b^2*d^3*PolyLog[4, -(((a - I*b)*E^((2*I)*e + (2*I)
*f*x))/(a + I*b))])/(2*(a^2 + b^2)^2*f^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3815

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
 b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+b\tan(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^3}{(a-ib)^2} - \frac{4b^2(c+dx)^3}{(ia+b)^2\left(ia\left(1+\frac{ib}{a}\right)+ia\left(1-\frac{ib}{a}\right)e^{2ie+2ifx}\right)^2} + \frac{\quad}{(a-ib)^2\left(ia\right.}\right.$$

$$= \frac{(c+dx)^4}{4(a-ib)^2d} + \frac{(4b)\int \frac{(c+dx)^3}{ia\left(1+\frac{ib}{a}\right)+ia\left(1-\frac{ib}{a}\right)e^{2ie+2ifx}}dx}{(a-ib)^2} - \frac{(4b^2)\int \frac{(c+dx)^3}{\left(ia\left(1+\frac{ib}{a}\right)+ia\left(1-\frac{ib}{a}\right)e^{2ie\cdots}\right.}}{(ia+b)^2}$$

$$= \frac{(c+dx)^4}{4(a-ib)^2d} + \frac{b(c+dx)^4}{(ia-b)(a-ib)^2d} + \frac{(4b^2)\int \frac{(c+dx)^3}{ia\left(1+\frac{ib}{a}\right)+ia\left(1-\frac{ib}{a}\right)e^{2ie+2ifx}}dx}{(ia-b)(a-ib)^2} - \frac{(4b)\int}{}$$

$$= -\frac{2b^2(c+dx)^3}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a-ib)^2d} + \frac{b(c+dx)^4}{(ia-b)(a-i\cdots}$$

$$= -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2f} - \frac{2b^2(c+dx)^3}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a-ib)^2d}$$

$$= -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2f} - \frac{2b^2(c+dx)^3}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a-ib)^2d}$$

$$= -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2f} - \frac{2b^2(c+dx)^3}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a-ib)^2d}$$

$$= -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2f} - \frac{2b^2(c+dx)^3}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a-ib)^2d}$$

$$= -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2f} - \frac{2b^2(c+dx)^3}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a-ib)^2d}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2857 vs. $2(848) = 1696$.
time = 11.03, size = 2857, normalized size = 3.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (b*(12*a*b*c^2*d*E^((2*I)*e)*f^3*x - (12*I)*b^2*c^2*d*E^((2*I)*e)*f^3*x + 8
*a^2*c^3*E^((2*I)*e)*f^4*x - (8*I)*a*b*c^3*E^((2*I)*e)*f^4*x + 12*a*b*c*d^2
```

*E^((2*I)*e)*f^3*x^2 - (12*I)*b^2*c*d^2*E^((2*I)*e)*f^3*x^2 + 12*a^2*c^2*d*
E^((2*I)*e)*f^4*x^2 - (12*I)*a*b*c^2*d*E^((2*I)*e)*f^4*x^2 + 4*a*b*d^3*E^((
2*I)*e)*f^3*x^3 - (4*I)*b^2*d^3*E^((2*I)*e)*f^3*x^3 + 8*a^2*c*d^2*E^((2*I)*
e)*f^4*x^3 - (8*I)*a*b*c*d^2*E^((2*I)*e)*f^4*x^3 + 2*a^2*d^3*E^((2*I)*e)*f^
4*x^4 - (2*I)*a*b*d^3*E^((2*I)*e)*f^4*x^4 + (12*I)*a*b*c*d^2*f^2*x*Log[1 +
((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] - 12*b^2*c*d^2*f^2*x*Log[1 + ((a
 - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + (12*I)*a*b*c*d^2*E^((2*I)*e)*f^2*
x*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + 12*b^2*c*d^2*E^((2*I
)*e)*f^2*x*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + (12*I)*a^2*
c^2*d*f^3*x*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] - 12*a*b*c^2
*d*f^3*x*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + (12*I)*a^2*c^
2*d*E^((2*I)*e)*f^3*x*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] +
12*a*b*c^2*d*E^((2*I)*e)*f^3*x*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a +
 I*b)] + (6*I)*a*b*d^3*f^2*x^2*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a +
 I*b)] - 6*b^2*d^3*f^2*x^2*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b
)] + (6*I)*a*b*d^3*E^((2*I)*e)*f^2*x^2*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x
)))/(a + I*b)] + 6*b^2*d^3*E^((2*I)*e)*f^2*x^2*Log[1 + ((a - I*b)*E^((2*I)*
(e + f*x)))/(a + I*b)] + (12*I)*a^2*c*d^2*f^3*x^2*Log[1 + ((a - I*b)*E^((2*
I)*(e + f*x)))/(a + I*b)] - 12*a*b*c*d^2*f^3*x^2*Log[1 + ((a - I*b)*E^((2*I
)*(e + f*x)))/(a + I*b)] + (12*I)*a^2*c*d^2*E^((2*I)*e)*f^3*x^2*Log[1 + ((a
 - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + 12*a*b*c*d^2*E^((2*I)*e)*f^3*x^2*
Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + (4*I)*a^2*d^3*f^3*x^3*
Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] - 4*a*b*d^3*f^3*x^3*Log[
1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + (4*I)*a^2*d^3*E^((2*I)*e)*
f^3*x^3*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + 4*a*b*d^3*E^((
2*I)*e)*f^3*x^3*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)] + (6*I)*
a*b*c^2*d*f^2*Log[I*a - b + (I*a + b)*E^((2*I)*(e + f*x))] - 6*b^2*c^2*d*f^
2*Log[I*a - b + (I*a + b)*E^((2*I)*(e + f*x))] + (6*I)*a*b*c^2*d*E^((2*I)*e
)*f^2*Log[I*a - b + (I*a + b)*E^((2*I)*(e + f*x))] + 6*b^2*c^2*d*E^((2*I)*e
)*f^2*Log[I*a - b + (I*a + b)*E^((2*I)*(e + f*x))] + (4*I)*a^2*c^3*f^3*Log[
I*a - b + (I*a + b)*E^((2*I)*(e + f*x))] - 4*a*b*c^3*f^3*Log[I*a - b + (I*a
 + b)*E^((2*I)*(e + f*x))] + (4*I)*a^2*c^3*E^((2*I)*e)*f^3*Log[I*a - b + (I
*a + b)*E^((2*I)*(e + f*x))] + 4*a*b*c^3*E^((2*I)*e)*f^3*Log[I*a - b + (I*a
 + b)*E^((2*I)*(e + f*x))] + 6*d*((-I)*b*(-1 + E^((2*I)*e)) + a*(1 + E^((2*
I)*e)))*f*(c + d*x)*(b*d + a*f*(c + d*x))*PolyLog[2, -(((a - I*b)*E^((2*I)*
(e + f*x)))/(a + I*b))] + 3*d^2*(b*(-1 + E^((2*I)*e)) + I*a*(1 + E^((2*I)*e
)))*(b*d + 2*a*f*(c + d*x))*PolyLog[3, -(((a - I*b)*E^((2*I)*(e + f*x)))/(a
 + I*b))] - 3*a^2*d^3*PolyLog[4, -(((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b
))] - (3*I)*a*b*d^3*PolyLog[4, -(((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b))
] - 3*a^2*d^3*E^((2*I)*e)*PolyLog[4, -(((a - I*b)*E^((2*I)*(e + f*x)))/(a +
 I*b))] + (3*I)*a*b*d^3*E^((2*I)*e)*PolyLog[4, -(((a - I*b)*E^((2*I)*(e + f
*x)))/(a + I*b))]))/(2*(a^2 + b^2)^2*(b*(-1 + E^((2*I)*e)) + I*a*(1 + E^((2
*I)*e)))*f^4) + (3*x^2*(a*c^2*d - I*b*c^2*d + a*c^2*d*Cos[2*e] + I*b*c^2*d*
Cos[2*e] + I*a*c^2*d*Sin[2*e] - b*c^2*d*Sin[2*e]))/(2*(a - I*b)*(a + I*b)*(
a + I*b + a*Cos[2*e] - I*b*Cos[2*e] + I*a*Sin[2*e] + b*Sin[2*e])) + (x^3*(a

```
*c*d^2 - I*b*c*d^2 + a*c*d^2*Cos[2*e] + I*b*c*d^2*Cos[2*e] + I*a*c*d^2*Sin[
2*e] - b*c*d^2*Sin[2*e]))/((a - I*b)*(a + I*b)*(a + I*b + a*Cos[2*e] - I*b*
Cos[2*e] + I*a*Sin[2*e] + b*Sin[2*e])) + (x^4*(a*d^3 - I*b*d^3 + a*d^3*Cos[
2*e] + I*b*d^3*Cos[2*e] + I*a*d^3*Sin[2*e] - b*d^3*Sin[2*e]))/(4*(a - I*b)*
(a + I*b)*(a + I*b + a*Cos[2*e] - I*b*Cos[2*e] + I*a*Sin[2*e] + b*Sin[2*e])
) + x*(c^3/(a^2 + (2*I)*a*b - b^2 + a^2*Cos[4*e] - (2*I)*a*b*Cos[4*e] - b^2
*Cos[4*e] + I*a^2*Sin[4*e] + 2*a*b*Sin[4*e] - I*b^2*Sin[4*e]) + ((-a - I*b
+ a*Cos[2*e] - I*b*Cos[2*e] + I*a*Sin[2*e] + b*Sin[2*e])*((-4*I)*a*b*c^3*Co
s[2*e] + 4*a*b*c^3*Sin[2*e]))/((a - I*b)*(a + I*b)*(a + I*b + a*Cos[2*e] -
I*b*Cos[2*e] + I*a*Sin[2*e] + b*Sin[2*e])*(a^2 + (2*I)*a*b - b^2 + a^2*Cos[
4*e] - (2*I)*a*b*Cos[4*e] - b^2*Cos[4*e] + I*a^2*Sin[4*e] + 2*a*b*Sin[4*e]
- I*b^2*Sin[4*e])) + (c^3*Cos[4*e] + I*c^3*Sin[4*e])/(a^2 + (2*I)*a*b - b^2
 + a^2*Cos[4*e] - (2*I)*a*b*Cos[4*e] - b^2*Cos[4*e] + I*a^2*Sin[4*e] + 2*a*
b*Sin[4*e] - I*b^2*Sin[4*e])) + (b^2*c^3*Sin[f*x] + 3*b^2*c^2*d*x*Sin[f*x]
+ 3*b^2*c*d^2*x^2*Sin[f*x] + b^2*d^3*x^3*Sin[f*x])/((a - I*b)*(a + I*b)*f*(
a*Cos[e] + b*Sin[e])*(a*Cos[e + f*x] + b*Sin[e + f*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3511 vs. $2(763) = 1526$.
time = 0.66, size = 3512, normalized size = 4.14

method	result	size
risch	Expression too large to display	3512

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*b^2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(b-I*a)/f/(I*a+b)^2/(b*exp(2*I
*(f*x+e))+I*a*exp(2*I*(f*x+e))-b+I*a)+6/(I*a+b)^2/f^2/(b-I*a)*b*a*c^2*d/(a+
I*b)*e^2+3/(I*a+b)^2/f^2/(b-I*a)*b*a*c^2*d/(a+I*b)*polylog(2,(I*b-a)*exp(2*
I*(f*x+e))/(a+I*b))-3/(I*a+b)^2/f^2/(b-I*a)*b^3*c^2*d/(a+I*b)/(I*b-a)*ln(I*
exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-6*I/(I*a+b)^2/f^4/(b-I*a)*b^2*
d^3*e^2/(a+I*b)*ln(exp(I*(f*x+e)))+3*I/(I*a+b)^2/f^2/(b-I*a)*b^2*d^3/(a+I*b
)*ln(1-(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))*x^2-6*I/(I*a+b)^2/f^2/(b-I*a)*b^2*
c^2*d/(a+I*b)*ln(exp(I*(f*x+e)))-4*I/(I*a+b)^2/f/(b-I*a)*b*a*c^3/(a+I*b)*ln
(exp(I*(f*x+e)))-1/4*d^3/(2*I*a*b-a^2+b^2)*x^4-1/(2*I*a*b-a^2+b^2)*c^3*x-1/
4/d/(2*I*a*b-a^2+b^2)*c^4-3*I/(I*a+b)^2/f^4/(b-I*a)*b^2*d^3*e^2/(a+I*b)*ln(
1-(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))+4/(I*a+b)^2/(b-I*a)*b*a*c*d^2/(a+I*b)*x
^3+6/(I*a+b)^2/(b-I*a)*b*a*c^2*d/(a+I*b)*x^2+3/(I*a+b)^2/f^3/(b-I*a)*b^2*d^
3/(a+I*b)*polylog(2,(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))*x+3/2*I/(I*a+b)^2/f^4
/(b-I*a)*b^2*d^3/(a+I*b)*polylog(3,(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))-6/(I*a
+b)^2/f^3/(b-I*a)*b^2*d^3/(a+I*b)*e^2*x+6/(I*a+b)^2/f/(b-I*a)*b^2*c*d^2/(a+
I*b)*x^2+6/(I*a+b)^2/f^3/(b-I*a)*b^2*c*d^2/(a+I*b)*e^2+3/(I*a+b)^2/f^4/(b-I
*a)*b*a*d^3*e^4/(a+I*b)-3/2/(I*a+b)^2/f^4/(b-I*a)*b*a*d^3/(a+I*b)*polylog(4
,(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))+3/(I*a+b)^2/f^3/(b-I*a)*b^2*c*d^2/(a+I*b
```

```
)*polylog(2,(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))-12/(I*a+b)^2/f^2/(b-I*a)*b*a*
c*d^2*e^2/(a+I*b)*x+12/(I*a+b)^2/f/(b-I*a)*b*a*c^2*d/(a+I*b)*e*x+2/(I*a+b)^
2/f^4/(b-I*a)*b^2*a*d^3*e^3/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2
*I*(f*x+e))-I*b-a)+6/(I*a+b)^2/f^3/(b-I*a)*b^3*c*d^2*e/(a+I*b)/(I*b-a)*ln(I
*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-6/(I*a+b)^2/f^3/(b-I*a)*b^2*a
*c*d^2*e^2/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a
)+6/(I*a+b)^2/f^2/(b-I*a)*b^2*a*c^2*d*e/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e
))*b-a*exp(2*I*(f*x+e))-I*b-a)-6*I/(I*a+b)^2/f^3/(b-I*a)*b*a^2*c*d^2*e^2/(a
+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)+6*I/(I*a+b)
^2/f^2/(b-I*a)*b*a^2*c^2*d*e/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(
2*I*(f*x+e))-I*b-a)+6*I/(I*a+b)^2/f^3/(b-I*a)*b^2*c*d^2*e/(a+I*b)/(I*b-a)*l
n(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)*a-3/(I*a+b)^2/f^4/(b-I*a)*
b^3*d^3*e^2/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-
a)-2/(I*a+b)^2/f/(b-I*a)*b^2*a*c^3/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-
a*exp(2*I*(f*x+e))-I*b-a)+4/(I*a+b)^2/f^3/(b-I*a)*b*a*d^3*e^3/(a+I*b)*x+12/
(I*a+b)^2/f^2/(b-I*a)*b^2*c*d^2/(a+I*b)*e*x-8/(I*a+b)^2/f^3/(b-I*a)*b*a*c*d
^2*e^3/(a+I*b)+3/(I*a+b)^2/f^2/(b-I*a)*b*a*d^3/(a+I*b)*polylog(2,(I*b-a)*ex
p(2*I*(f*x+e))/(a+I*b))*x^2+1/(I*a+b)^2/(b-I*a)*b*a*d^3/(a+I*b)*x^4+2/(I*a+
b)^2/f/(b-I*a)*b^2*d^3/(a+I*b)*x^3-4/(I*a+b)^2/f^4/(b-I*a)*b^2*d^3/(a+I*b)*
e^3-6*I/(I*a+b)^2/f^3/(b-I*a)*b*a*c*d^2*e^2/(a+I*b)*ln(1-(I*b-a)*exp(2*I*(f
*x+e))/(a+I*b))+12*I/(I*a+b)^2/f^2/(b-I*a)*b*a*c^2*d*e/(a+I*b)*ln(exp(I*(f*
x+e)))+6*I/(I*a+b)^2/f/(b-I*a)*b*a*c^2*d/(a+I*b)*ln(1-(I*b-a)*exp(2*I*(f*x+
e))/(a+I*b))*x+6*I/(I*a+b)^2/f/(b-I*a)*b*a*c*d^2/(a+I*b)*ln(1-(I*b-a)*exp(2
*I*(f*x+e))/(a+I*b))*x^2+6*I/(I*a+b)^2/f^2/(b-I*a)*b*a*c^2*d/(a+I*b)*ln(1-(
I*b-a)*exp(2*I*(f*x+e))/(a+I*b))*e-3*I/(I*a+b)^2/f^4/(b-I*a)*b^2*d^3*e^2/(a
+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)*a-3*I/(I*a+
b)^2/f^2/(b-I*a)*b^2*c^2*d/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*
I*(f*x+e))-I*b-a)*a+2*I/(I*a+b)^2/f^4/(b-I*a)*b*a^2*d^3*e^3/(a+I*b)/(I*b-a)
*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-12*I/(I*a+b)^2/f^3/(b-I*
a)*b*a*c*d^2*e^2/(a+I*b)*ln(exp(I*(f*x+e)))+6/(I*a+b)^2/f^2/(b-I*a)*b*a*c*d
^2/(a+I*b)*polylog(2,(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))*x+6*I/(I*a+b)^2/f^2/
(b-I*a)*b^2*c*d^2/(a+I*b)*ln(1-(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))*x+6*I/(I*a
+b)^2/f^3/(b-I*a)*b^2*c*d^2/(a+I*b)*ln(1-(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))*
e+4*I/(I*a+b)^2/f^4/(b-I*a)*b*a*d^3*e^3/(a+I*b)*ln(exp(I*(f*x+e)))+3*I/(I*a
+b)^2/f^3/(b-I*a)*b*a*c*d^2/(a+I*b)*polylog(3,(I*b-a)*exp(2*I*(f*x+e))/(a+I
*b))-2*I/(I*a+b)^2/f/(b-I*a)*b*a^2*c^3/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e)
)*b-a*exp(2*I*(f*x+e))-I*b-a)+12*I/(I*a+b)^2/f^3/(b-I*a)*b^2*c*d^2*e/(a+I*b
)*ln(exp(I*(f*x+e)))+3*I/(I*a+b)^2/f^3/(b-I*a)*b*a*d^3/(a+I*b)*polylog(3,(I
*b-a)*exp(2*I*(f*x+e))/(a+I*b))*x+2*I/(I*a+b)^2/f/(b-I*a)*b*a*d^3/(a+I*b)*l
n(1-(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))*x^3+2*I/(I*a+b)^2/f^4/(b-I*a)*b*a*d^3
*e^3/(a+I*b)*ln(1-(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))-d^2/(2*I*a*b-a^2+b^2)*c
*x^3-3/2*d/(2*I*a*b-a^2+b^2)*c^2*x^2
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4521 vs. $2(707) = 1414$.

time = 2.81, size = 4521, normalized size = 5.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(36*c^2*d*(2*a*b*log(b*tan(f*x + e) + a)/((a^4 + 2*a^2*b^2 + b^4)*f)
- a*b*log(tan(f*x + e)^2 + 1)/((a^4 + 2*a^2*b^2 + b^4)*f) - b/((a^2*b + b^3
)*f*tan(f*x + e) + (a^3 + a*b^2)*f) + (a^2 - b^2)*(f*x + e)/((a^4 + 2*a^2*b
^2 + b^4)*f))*e - 12*(2*a*b*log(b*tan(f*x + e) + a)/(a^4 + 2*a^2*b^2 + b^4)
 - a*b*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(f*x +
 e)/(a^4 + 2*a^2*b^2 + b^4) - b/(a^3 + a*b^2 + (a^2*b + b^3)*tan(f*x + e)))
*c^3 - (3*(a^3 - I*a^2*b + a*b^2 - I*b^3)*(f*x + e)^4*d^3 - 72*(-I*a*b^2*e^
2 + b^3*e^2)*c*d^2*f + 12*((a^3 - I*a^2*b + a*b^2 - I*b^3)*c*d^2*f - (a^3*e
 - I*a^2*b*e + a*b^2*e - I*b^3*e)*d^3)*(f*x + e)^3 - 24*(I*a*b^2*e^3 - b^3*
e^3)*d^3 + 18*((a^3 - I*a^2*b + a*b^2 - I*b^3)*c^2*d*f^2 - 2*(a^3*e - I*a^2
*b*e + a*b^2*e - I*b^3*e)*c*d^2*f + (a^3*e^2 - I*a^2*b*e^2 + a*b^2*e^2 - I*
b^3*e^2)*d^3)*(f*x + e)^2 + 12*(3*(a^3*e^2 - I*a^2*b*e^2 + a*b^2*e^2 - I*b^
3*e^2)*c*d^2*f - (a^3*e^3 - I*a^2*b*e^3 + a*b^2*e^3 - I*b^3*e^3)*d^3)*(f*x
+ e) - 12*(3*(-I*a*b^2 + b^3)*c^2*d*f^2 + 6*(a*b^2*(e^2 + I*e) - I*a^2*b*e^
2 - b^3*e)*c*d^2*f - (a*b^2*(2*e^3 + 3*I*e^2) - 2*I*a^2*b*e^3 - 3*b^3*e^2)*
d^3 + (3*(-I*a*b^2 - b^3)*c^2*d*f^2 - 6*(a*b^2*(e^2 - I*e) + I*a^2*b*e^2 -
b^3*e)*c*d^2*f + (a*b^2*(2*e^3 - 3*I*e^2) + 2*I*a^2*b*e^3 - 3*b^3*e^2)*d^3)
*cos(2*f*x + 2*e) + (3*(a*b^2 - I*b^3)*c^2*d*f^2 + 6*(a*b^2*(-I*e^2 - e) +
a^2*b*e^2 + I*b^3*e)*c*d^2*f + (a*b^2*(2*I*e^3 + 3*e^2) - 2*a^2*b*e^3 - 3*I
*b^3*e^2)*d^3)*sin(2*f*x + 2*e))*arctan2(-b*cos(2*f*x + 2*e) + a*sin(2*f*x
+ 2*e) + b, a*cos(2*f*x + 2*e) + b*sin(2*f*x + 2*e) + a) - 4*(8*(I*a^2*b -
a*b^2)*(f*x + e)^3*d^3 + 9*(2*(I*a^2*b - a*b^2)*c*d^2*f + (a*b^2*(2*e + I)
- 2*I*a^2*b*e - b^3)*d^3)*(f*x + e)^2 + 18*((I*a^2*b - a*b^2)*c^2*d*f^2 + (
a*b^2*(2*e + I) - 2*I*a^2*b*e - b^3)*c*d^2*f - (a*b^2*(e^2 + I*e) - I*a^2*b
*e^2 - b^3*e)*d^3)*(f*x + e) + (8*(I*a^2*b + a*b^2)*(f*x + e)^3*d^3 + 9*(2*
(I*a^2*b + a*b^2)*c*d^2*f - (a*b^2*(2*e - I) + 2*I*a^2*b*e - b^3)*d^3)*(f*x
 + e)^2 + 18*((I*a^2*b + a*b^2)*c^2*d*f^2 - (a*b^2*(2*e - I) + 2*I*a^2*b*e
- b^3)*c*d^2*f + (a*b^2*(e^2 - I*e) + I*a^2*b*e^2 - b^3*e)*d^3)*(f*x + e))*
cos(2*f*x + 2*e) - (8*(a^2*b - I*a*b^2)*(f*x + e)^3*d^3 + 9*(2*(a^2*b - I*a
*b^2)*c*d^2*f - (a*b^2*(-2*I*e - 1) + 2*a^2*b*e + I*b^3)*d^3)*(f*x + e)^2 +
 18*((a^2*b - I*a*b^2)*c^2*d*f^2 - (a*b^2*(-2*I*e - 1) + 2*a^2*b*e + I*b^3)
*c*d^2*f - (a*b^2*(I*e^2 + e) - a^2*b*e^2 - I*b^3*e)*d^3)*(f*x + e))*sin(2*
f*x + 2*e))*arctan2((2*a*b*cos(2*f*x + 2*e) - (a^2 - b^2)*sin(2*f*x + 2*e))
/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 + (a^2 - b^2)*cos(2*f*x +
 2*e))/(a^2 + b^2)) + 3*((a^3 - 3*I*a^2*b - 3*a*b^2 + I*b^3)*(f*x + e)^4*d^
3 + 4*((a^3 - 3*I*a^2*b - 3*a*b^2 + I*b^3)*c*d^2*f + (a*b^2*(3*e - 2*I) - b
^3*(I*e + 2) - a^3*e + 3*I*a^2*b*e)*d^3)*(f*x + e)^3 + 6*((a^3 - 3*I*a^2*b
- 3*a*b^2 + I*b^3)*c^2*d*f^2 + 2*(a*b^2*(3*e - 2*I) - b^3*(I*e + 2) - a^3*e
```

```
 + 3*I*a^2*b*e)*c*d^2*f - (a*b^2*(3*e^2 - 4*I*e) + b^3*(-I*e^2 - 4*e) - a^3
*e^2 + 3*I*a^2*b*e^2)*d^3)*(f*x + e)^2 - 4*(6*(I*a*b^2 + b^3)*c^2*d*f^2 + 3
*(a*b^2*(3*e^2 - 4*I*e) + b^3*(-I*e^2 - 4*e) - a^3*e^2 + 3*I*a^2*b*e^2)*c*d
^2*f - (3*a*b^2*(e^3 - 2*I*e^2) - b^3*(I*e^3 + 6*e^2) - a^3*e^3 + 3*I*a^2*b
*e^3)*d^3)*(f*x + e))*cos(2*f*x + 2*e) - 12*(4*(I*a^2*b - a*b^2)*(f*x + e)^
2*d^3 + 3*(I*a^2*b - a*b^2)*c^2*d*f^2 + 3*(a*b^2*(2*e + I) - 2*I*a^2*b*e -
b^3)*c*d^2*f - 3*(a*b^2*(e^2 + I*e) - I*a^2*b*e^2 - b^3*e)*d^3 + 3*(2*(I*a^
2*b - a*b^2)*c*d^2*f + (a*b^2*(2*e + I) - 2*I*a^2*b*e - b^3)*d^3)*(f*x + e)
 + (4*(I*a^2*b + a*b^2)*(f*x + e)^2*d^3 + 3*(I*a^2*b + a*b^2)*c^2*d*f^2 - 3
*(a*b^2*(2*e - I) + 2*I*a^2*b*e - b^3)*c*d^2*f + 3*(a*b^2*(e^2 - I*e) + I*a
^2*b*e^2 - b^3*e)*d^3 + 3*(2*(I*a^2*b + a*b^2)*c*d^2*f - (a*b^2*(2*e - I) +
 2*I*a^2*b*e - b^3)*d^3)*(f*x + e))*cos(2*f*x + 2*e) - (4*(a^2*b - I*a*b^2)
*(f*x + e)^2*d^3 + 3*(a^2*b - I*a*b^2)*c^2*d*f^2 - 3*(a*b^2*(-2*I*e - 1) +
2*a^2*b*e + I*b^3)*c*d^2*f - 3*(a*b^2*(I*e^2 + e) - a^2*b*e^2 - I*b^3*e)*d^
3 + 3*(2*(a^2*b - I*a*b^2)*c*d^2*f - (a*b^2*(-2*I*e - 1) + 2*a^2*b*e + I*b^
3)*d^3)*(f*x + e))*sin(2*f*x + 2*e))*dilog((I*a + b)*e^(2*I*f*x + 2*I*e)/(-
I*a + b)) + 6*(3*(a*b^2 + I*b^3)*c^2*d*f^2 - 6*(a*b^2*(-I*e^2 + e) - a^2*b*
e^2 + I*b^3*e)*c*d^2*f - (a*b^2*(2*I*e^3 - 3*e^2) + 2*a^2*b*e^3 - 3*I*b^3*e
^2)*d^3 + (3*(a*b^2 - I*b^3)*c^2*d*f^2 - 6*(a*b^2*(I*e^2 + e) - a^2*b*e^2 -
 I*b^3*e)*c*d^2*f - (a*b^2*(-2*I*e^3 - 3*e^2) + 2*a^2*b*e^3 + 3*I*b^3*e^2)*
d^3)*cos(2*f*x + 2*e) - (3*(-I*a*b^2 - b^3)*c^2*d*f^2 - 6*(a*b^2*(e^2 - I*e
) + I*a^2*b*e^2 - b^3*e)*c*d^2*f + (a*b^2*(2*e^3 - 3*I*e^2) + 2*I*a^2*b*e^3
 - 3*b^3*e^2)*d^3)*sin(2*f*x + 2*e))*log((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4
*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 + 2*(a^2
 - b^2)*cos(2*f*x + 2*e)) + 2*(8*(a^2*b + I*a*b^2)*(f*x + e)^3*d^3 + 9*(2*(
a^2*b + I*a*b^2)*c*d^2*f - (a*b^2*(2*I*e - 1) + 2*a^2*b*e - I*b^3)*d^3)*(f*
x + e)^2 + 18*((a^2*b + I*a*b^2)*c^2*d*f^2 - (a*b^2*(2*I*e - 1) + 2*a^2*b*e
 - I*b^3)*c*d^2*f - (a*b^2*(-I*e^2 + e) - a^2*b*e^2 + I*b^3*e)*d^3)*(f*x +
e) + (8*(a^2*b - I*a*b^2)*(f*x + e)^3*d^3 + 9*(...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2555 vs. $2(707) = 1414$.
time = 0.51, size = 2555, normalized size = 3.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^3 - a*b^2)*d^3*f^4*x^4 - 4*b^3*c^3*f^3 - 4*(b^3*d^3*f^3 - (a^3 - a*
b^2)*c*d^2*f^4)*x^3 - 6*(2*b^3*c*d^2*f^3 - (a^3 - a*b^2)*c^2*d*f^4)*x^2 - 4
*(3*b^3*c^2*d*f^3 - (a^3 - a*b^2)*c^3*f^4)*x - 6*(-I*a^2*b*d^3*f^2*x^2 - I*
a^2*b*c^2*d*f^2 - I*a*b^2*c*d^2*f - I*(2*a^2*b*c*d^2*f^2 + a*b^2*d^3*f)*x +
 (-I*a*b^2*d^3*f^2*x^2 - I*a*b^2*c^2*d*f^2 - I*b^3*c*d^2*f - I*(2*a*b^2*c*d
^2*f^2 + b^3*d^3*f)*x)*tan(f*x + e))*dilog(2*((I*a*b - b^2)*tan(f*x + e)^2
```

- a^2 - I*a*b + (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2) + 1) - 6*(I*a^2*b*d^3*f^2*x^2 + I*a^2*b*c^2*d*f^2 + I*a*b^2*c*d^2*f + I*(2*a^2*b*c*d^2*f^2 + a*b^2*d^3*f)*x + (I*a*b^2*d^3*f^2*x^2 + I*a*b^2*c^2*d*f^2 + I*b^3*c*d^2*f + I*(2*a*b^2*c*d^2*f^2 + b^3*d^3*f)*x)*tan(f*x + e))*dilog(2*((-I*a*b - b^2)*tan(f*x + e)^2 - a^2 + I*a*b + (-I*a^2 - 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2) + 1) + 2*(2*a^2*b*d^3*f^3*x^3 + 2*a^2*b*d^3*e^3 + 3*(2*a^2*b*c*d^2*f^3 + a*b^2*d^3*f^2)*x^2 + 6*(a^2*b*c^2*d*f^3 + a*b^2*c*d^2*f^2)*x - 3*(2*a^2*b*c*d^2*f + a*b^2*d^3)*e^2 + 6*(a^2*b*c^2*d*f^2 + a*b^2*c*d^2*f)*e + (2*a*b^2*d^3*f^3*x^3 + 2*a*b^2*d^3*e^3 + 3*(2*a*b^2*c*d^2*f^3 + b^3*d^3*f^2)*x^2 + 6*(a*b^2*c^2*d*f^3 + b^3*c*d^2*f^2)*x - 3*(2*a*b^2*c*d^2*f + b^3*d^3)*e^2 + 6*(a*b^2*c^2*d*f^2 + b^3*c*d^2*f)*e)*tan(f*x + e))*log(-2*((I*a*b - b^2)*tan(f*x + e)^2 - a^2 - I*a*b + (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + 2*(2*a^2*b*d^3*f^3*x^3 + 2*a^2*b*d^3*e^3 + 3*(2*a^2*b*c*d^2*f^3 + a*b^2*d^3*f^2)*x^2 + 6*(a^2*b*c^2*d*f^3 + a*b^2*c*d^2*f^2)*x - 3*(2*a^2*b*c*d^2*f + a*b^2*d^3)*e^2 + 6*(a^2*b*c^2*d*f^2 + a*b^2*c*d^2*f)*e + (2*a*b^2*d^3*f^3*x^3 + 2*a*b^2*d^3*e^3 + 3*(2*a*b^2*c*d^2*f^3 + b^3*d^3*f^2)*x^2 + 6*(a*b^2*c^2*d*f^3 + b^3*c*d^2*f^2)*x - 3*(2*a*b^2*c*d^2*f + b^3*d^3)*e^2 + 6*(a*b^2*c^2*d*f^2 + b^3*c*d^2*f)*e)*tan(f*x + e))*log(-2*((-I*a*b - b^2)*tan(f*x + e)^2 - a^2 + I*a*b + (-I*a^2 - 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + 2*(2*a^2*b*c^3*f^3 + 3*a*b^2*c^2*d*f^2 - 2*a^2*b*d^3*e^3 + 3*(2*a^2*b*c*d^2*f + a*b^2*d^3)*e^2 - 6*(a^2*b*c^2*d*f^2 + a*b^2*c*d^2*f)*e + (2*a*b^2*c^3*f^3 + 3*b^3*c^2*d*f^2 - 2*a*b^2*d^3*e^3 + 3*(2*a*b^2*c*d^2*f + b^3*d^3)*e^2 - 6*(a*b^2*c^2*d*f^2 + b^3*c*d^2*f)*e)*tan(f*x + e))*log(((I*a*b + b^2)*tan(f*x + e)^2 - a^2 + I*a*b + (I*a^2 + I*b^2)*tan(f*x + e))/(tan(f*x + e)^2 + 1)) + 2*(2*a^2*b*c^3*f^3 + 3*a*b^2*c^2*d*f^2 - 2*a^2*b*d^3*e^3 + 3*(2*a^2*b*c*d^2*f + a*b^2*d^3)*e^2 - 6*(a^2*b*c^2*d*f^2 + a*b^2*c*d^2*f)*e + (2*a*b^2*c^3*f^3 + 3*b^3*c^2*d*f^2 - 2*a*b^2*d^3*e^3 + 3*(2*a*b^2*c*d^2*f + b^3*d^3)*e^2 - 6*(a*b^2*c^2*d*f^2 + b^3*c*d^2*f)*e)*tan(f*x + e))*log(((I*a*b - b^2)*tan(f*x + e)^2 + a^2 + I*a*b + (I*a^2 + I*b^2)*tan(f*x + e))/(tan(f*x + e)^2 + 1)) - 3*(I*a*b^2*d^3*tan(f*x + e) + I*a^2*b*d^3)*polylog(4, ((a^2 + 2*I*a*b - b^2)*tan(f*x + e)^2 - a^2 - 2*I*a*b + b^2 - 2*(-I*a^2 + 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) - 3*(-I*a*b^2*d^3*tan(f*x + e) - I*a^2*b*d^3)*polylog(4, ((a^2 - 2*I*a*b - b^2)*tan(f*x + e)^2 - a^2 + 2*I*a*b + b^2 - 2*(I*a^2 + 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + 3*(2*a^2*b*d^3*f*x + 2*a^2*b*c*d^2*f + a*b^2*d^3 + (2*a*b^2*d^3*f*x + 2*a*b^2*c*d^2*f + b^3*d^3)*tan(f*x + e))*polylog(3, ((a^2 + 2*I*a*b - b^2)*tan(f*x + e)^2 - a^2 - 2*I*a*b + b^2 - 2*(-I*a^2 + 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + 3*(2*a^2*b*d^3*f*x + 2*a^2*b*c*d^2*f + a*b^2*d^3 + (2*a*b^2*d^3*f*x + 2*a*b^2*c*d^2*f + b^3*d^3)*tan(f*x + e))*polylog(3, ((a^2 - 2*I*a*b - b^2)*tan(f*x + e)^2 - a^2 + 2*I*a*b + b^2 - 2*(I*a^2 + 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + ((a^2*b - b^3)*d^3*f^4*x^4 + 4*a*b^2*c^3*f^3 + 4*(a*b^2*d^3*f^3 + (a^2*b - b^3)*c*d^2*f^4)*x^3 + 6*(2*a*b^2

```
*c*d^2*f^3 + (a^2*b - b^3)*c^2*d*f^4)*x^2 + 4*(3*a*b^2*c^2*d*f^3 + (a^2*b -
 b^3)*c^3*f^4)*x)*tan(f*x + e))/((a^4*b + 2*a^2*b^3 + b^5)*f^4*tan(f*x + e)
 + (a^5 + 2*a^3*b^2 + a*b^4)*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+b\tan(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**3/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{(a+b\tan(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*tan(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^3/(a + b*tan(e + f*x))^2, x)
```

3.60 $\int \frac{(c+dx)^2}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=654

$$-\frac{2ib^2(c+dx)^2}{(a^2+b^2)^2 f}+\frac{2b^2(c+dx)^2}{(a+ib)(ia+b)^2\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f}+\frac{(c+dx)^3}{3(a-ib)^2d}+\frac{4b(c+dx)^3}{3(ia-b)(a-ib)^2d}-\frac{4b^2(c+}{3\left(a^2+b\right.}$$

```
[Out] -2*I*b^2*(d*x+c)^2/(a^2+b^2)^2/f+2*b^2*(d*x+c)^2/(a+I*b)/(I*a+b)^2/(I*a-b+(
I*a+b)*exp(2*I*e+2*I*f*x))/f+1/3*(d*x+c)^3/(a-I*b)^2/d+4/3*b*(d*x+c)^3/(I*a
-b)/(a-I*b)^2/d-4/3*b^2*(d*x+c)^3/(a^2+b^2)^2/d+2*b^2*d*(d*x+c)*ln(1+(a-I*b
)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)^2/f^2+2*b*(d*x+c)^2*ln(1+(a-I*b)*ex
p(2*I*e+2*I*f*x)/(a+I*b))/(a-I*b)^2/(a+I*b)/f-2*I*b^2*(d*x+c)^2*ln(1+(a-I*b
)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)^2/f-I*b^2*d^2*polylog(2,-(a-I*b)*ex
p(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)^2/f^3+2*b*d*(d*x+c)*polylog(2,-(a-I*b)*
exp(2*I*e+2*I*f*x)/(a+I*b))/(I*a-b)/(a-I*b)^2/f^2-2*b^2*d*(d*x+c)*polylog(2
,-(a-I*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)^2/f^2+b*d^2*polylog(3,-(a-I
*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a-I*b)^2/(a+I*b)/f^3-I*b^2*d^2*polylog(3,-
(a-I*b)*exp(2*I*e+2*I*f*x)/(a+I*b))/(a^2+b^2)^2/f^3
```

Rubi [A]
time = 1.02, antiderivative size = 654, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3815, 2216, 2215, 2221, 2611, 2320, 6724, 2222, 2317, 2438}

$$-\frac{2b^2d(c+dx)\text{Li}_2\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{f^2\left(a^2+b^2\right)^2}+\frac{2b^2d(c+dx)\log\left(1+\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{f^2\left(a^2+b^2\right)^2}-\frac{2ib^2(c+dx)^2\log\left(1+\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{f\left(a^2+b^2\right)^2}-\frac{2ib^2(c+dx)^2}{f\left(a^2+b^2\right)^2}-\frac{4b^2(c+dx)^3}{3d\left(a^2+b^2\right)^2}-\frac{ib^2d^2\text{Li}_2\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{f^3\left(a^2+b^2\right)^2}-\frac{ib^2d^2\text{Li}_3\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{f^3\left(a^2+b^2\right)^2}+\frac{2b^2(c+dx)^2}{f(a+ib)(b+ia)^2\left((b+ia)e^{2ie+2ifx}+ia-b\right)}+\frac{2bd(c+dx)\text{Li}_2\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{f^2(-b+ia)(a-ib)^2}+\frac{2b(c+dx)^2\log\left(1+\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{f(a-ib)^2(a+ib)}+\frac{4b(c+dx)^3}{3d(-b+ia)(a-ib)^2}+\frac{(c+dx)^3}{3d(a-ib)^2}+\frac{bd^2\text{Li}_3\left(-\frac{(a-ib)e^{2ie+2ifx}}{a+ib}\right)}{f^3(a-ib)^2(a+ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((-2*I)*b^2*(c + d*x)^2)/((a^2 + b^2)^2*f) + (2*b^2*(c + d*x)^2)/((a + I*b)
*(I*a + b)^2*(I*a - b + (I*a + b)*E^((2*I)*e + (2*I)*f*x))*f) + (c + d*x)^3
/(3*(a - I*b)^2*d) + (4*b*(c + d*x)^3)/(3*(I*a - b)*(a - I*b)^2*d) - (4*b^2
*(c + d*x)^3)/(3*(a^2 + b^2)^2*d) + (2*b^2*d*(c + d*x)*Log[1 + ((a - I*b)*E
^((2*I)*e + (2*I)*f*x))/(a + I*b)])/((a^2 + b^2)^2*f^2) + (2*b*(c + d*x)^2*
Log[1 + ((a - I*b)*E^((2*I)*e + (2*I)*f*x))/(a + I*b)])/((a - I*b)^2*(a + I
*b)*f) - ((2*I)*b^2*(c + d*x)^2*Log[1 + ((a - I*b)*E^((2*I)*e + (2*I)*f*x))
/(a + I*b)])/((a^2 + b^2)^2*f) - (I*b^2*d^2*PolyLog[2, -(((a - I*b)*E^((2*I
)*e + (2*I)*f*x))/(a + I*b))])/((a^2 + b^2)^2*f^3) + (2*b*d*(c + d*x)*PolyL
og[2, -(((a - I*b)*E^((2*I)*e + (2*I)*f*x))/(a + I*b))])/((I*a - b)*(a - I*
b)^2*f^2) - (2*b^2*d*(c + d*x)*PolyLog[2, -(((a - I*b)*E^((2*I)*e + (2*I)*f
*x))/(a + I*b))])/((a^2 + b^2)^2*f^2) + (b*d^2*PolyLog[3, -(((a - I*b)*E^((
2*I)*e + (2*I)*f*x))/(a + I*b))])/((a - I*b)^2*(a + I*b)*f^3) - (I*b^2*d^2*
PolyLog[3, -(((a - I*b)*E^((2*I)*e + (2*I)*f*x))/(a + I*b))])/((a^2 + b^2)^
2*f^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3815

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
 b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+b\tan(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^2}{(a-ib)^2} - \frac{4b^2(c+dx)^2}{(ia+b)^2\left(ia\left(1+\frac{ib}{a}\right)+ia\left(1-\frac{ib}{a}\right)e^{2ie+2ifx}\right)^2} + \frac{\cdots}{(a-ib)^2\,(ia\cdots}\right.$$

$$= \frac{(c+dx)^3}{3(a-ib)^2d} + \frac{(4b)\int \frac{(c+dx)^2}{ia\left(1+\frac{ib}{a}\right)+ia\left(1-\frac{ib}{a}\right)e^{2ie+2ifx}}\,dx}{(a-ib)^2} - \frac{(4b^2)\int \frac{(c+dx)^2}{\left(ia\left(1+\frac{ib}{a}\right)+ia\left(1-\frac{ib}{a}\right)e^{2\cdots}\right.}}{(ia+b)^2}$$

$$= \frac{(c+dx)^3}{3(a-ib)^2d} + \frac{4b(c+dx)^3}{3(ia-b)(a-ib)^2d} + \frac{(4b^2)\int \frac{(c+dx)^2}{ia\left(1+\frac{ib}{a}\right)+ia\left(1-\frac{ib}{a}\right)e^{2ie+2ifx}}\,dx}{(ia-b)(a-ib)^2} - \frac{(4b\cdots}{\cdots}$$

$$= -\frac{2b^2(c+dx)^2}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a-ib)^2d} + \frac{4b(c+d\cdots}{3(ia-b)(a\cdots}$$

$$= -\frac{2ib^2(c+dx)^2}{(a^2+b^2)^2f} - \frac{2b^2(c+dx)^2}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a-ib)^2\cdots}$$

$$= -\frac{2ib^2(c+dx)^2}{(a^2+b^2)^2f} - \frac{2b^2(c+dx)^2}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a-ib)^2\cdots}$$

$$= -\frac{2ib^2(c+dx)^2}{(a^2+b^2)^2f} - \frac{2b^2(c+dx)^2}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a-ib)^2\cdots}$$

$$= -\frac{2ib^2(c+dx)^2}{(a^2+b^2)^2f} - \frac{2b^2(c+dx)^2}{(a-ib)^2(a+ib)\left(ia-b+(ia+b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a-ib)^2\cdots}$$

<u>Mathematica</u> [A]
time = 7.76, size = 535, normalized size = 0.82

$$\frac{\frac{2b\left(-2f\left((a-ib)e^{2ie}fx(3bd(2c+dx)+2af(3c^2+3cdx+d^2x^2))+3d(b(-1+e^{2ie})+ia(1+e^{2ie}))x(bd+af(2c+dx))\log\left(1+\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)+3c(b(-1+e^{2ie})+ia(1+e^{2ie}))(bd+acf)\log(ia-b+(ia+b)e^{2i(e+fx)})\right)-3d(-ib(-1+e^{2ie})+a(1+e^{2ie}))(bd+2af(c+dx))\mathrm{PolyLog}\left(2,-\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)+3ad^2(b-be^{2ie}-ia(1+e^{2ie}))\mathrm{PolyLog}\left(3,-\frac{(a-ib)e^{2i(e+fx)}}{a+ib}\right)\right)}{(a^2+b^2)(b-be^{2ie}-ia(1+e^{2ie}))}+\frac{f^2((a^2-b^2)fx(3c^2+3cdx+d^2x^2)\cos(fx)+(a^2+b^2)fx(3c^2+3cdx+d^2x^2)\cos(2e+fx)+2b(3b(c+dx)^2+afx(3c^2+3cdx+d^2x^2))\sin(fx))}{(a\cos(e)+b\sin(e))(a\cos(e+fx)+b\sin(e+fx))}}{6(a^2+b^2)f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((2*b*(-2*f*((a - I*b)*E^((2*I)*e)*f*x*(3*b*d*(2*c + d*x) + 2*a*f*(3*c^2 +
3*c*d*x + d^2*x^2)) + 3*d*(b*(-1 + E^((2*I)*e)) + I*a*(1 + E^((2*I)*e)))*x*
(b*d + a*f*(2*c + d*x))*Log[1 + ((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b)]
+ 3*c*(b*(-1 + E^((2*I)*e)) + I*a*(1 + E^((2*I)*e)))*(b*d + a*c*f)*Log[I*a
- b + (I*a + b)*E^((2*I)*(e + f*x))]) - 3*d*((-I)*b*(-1 + E^((2*I)*e)) + a*
```

```
(1 + E^((2*I)*e)))*(b*d + 2*a*f*(c + d*x))*PolyLog[2, -(((a - I*b)*E^((2*I)
*(e + f*x)))/(a + I*b))] + 3*a*d^2*(b - b*E^((2*I)*e) - I*a*(1 + E^((2*I)*e
)))*PolyLog[3, -(((a - I*b)*E^((2*I)*(e + f*x)))/(a + I*b))]))/((a^2 + b^2)
*(b - b*E^((2*I)*e) - I*a*(1 + E^((2*I)*e)))) + (f^2*((a^2 - b^2)*f*x*(3*c^
2 + 3*c*d*x + d^2*x^2)*Cos[f*x] + (a^2 + b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^
2)*Cos[2*e + f*x] + 2*b*(3*b*(c + d*x)^2 + a*f*x*(3*c^2 + 3*c*d*x + d^2*x^2
))*Sin[f*x]))/((a*Cos[e] + b*Sin[e])*(a*Cos[e + f*x] + b*Sin[e + f*x])))/(6
*(a^2 + b^2)*f^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2183 vs. $2(588) = 1176$.
time = 0.56, size = 2184, normalized size = 3.34

method	result	size
risch	Expression too large to display	2184

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*I/(I*a+b)^2/f^2/(b-I*a)*b*a^2*c*d*e/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))
*b-a*exp(2*I*(f*x+e))-I*b-a)+4*I/(I*a+b)^2/f^3/(b-I*a)*b^2*d^2*e/(a+I*b)*ln
(exp(I*(f*x+e)))+2*I/(I*a+b)^2/f^2/(b-I*a)*b^2*d^2/(a+I*b)*ln(1-(I*b-a)*exp
(2*I*(f*x+e))/(a+I*b))*x+2*I/(I*a+b)^2/f^3/(b-I*a)*b^2*d^2/(a+I*b)*ln(1-(I*
b-a)*exp(2*I*(f*x+e))/(a+I*b))*e-4*I/(I*a+b)^2/f^2/(b-I*a)*b^2*c*d/(a+I*b)*
ln(exp(I*(f*x+e)))-4*I/(I*a+b)^2/f/(b-I*a)*b*a*c^2/(a+I*b)*ln(exp(I*(f*x+e)
))-2*I*b^2*(d^2*x^2+2*c*d*x+c^2)/(b-I*a)/f/(I*a+b)^2/(b*exp(2*I*(f*x+e))+I*
a*exp(2*I*(f*x+e))-b+I*a)+4/(I*a+b)^2/f^2/(b-I*a)*b^2*a*c*d*e/(a+I*b)/(I*b-
a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)+8*I/(I*a+b)^2/f^2/(b-I
*a)*b*a*c*d*e/(a+I*b)*ln(exp(I*(f*x+e)))+2/(I*a+b)^2/f^2/(b-I*a)*b*a*c*d/(a
+I*b)*polylog(2,(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))-2/(I*a+b)^2/f/(b-I*a)*b^2
*a*c^2/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)+2/
(I*a+b)^2/f^3/(b-I*a)*b^3*d^2*e/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*e
xp(2*I*(f*x+e))-I*b-a)+4/(I*a+b)^2/f^2/(b-I*a)*b*a*c*d/(a+I*b)*e^2-4/(I*a+b
)^2/f^2/(b-I*a)*b*a*d^2/(a+I*b)*e^2*x-2/(I*a+b)^2/f^2/(b-I*a)*b^3*c*d/(a+I*
b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)+I/(I*a+b)^2/f^
3/(b-I*a)*b*a*d^2/(a+I*b)*polylog(3,(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))+2/(I*
a+b)^2/f^2/(b-I*a)*b*a*d^2/(a+I*b)*polylog(2,(I*b-a)*exp(2*I*(f*x+e))/(a+I*
b))*x+2/(I*a+b)^2/f/(b-I*a)*b^2*d^2/(a+I*b)*x^2+2/(I*a+b)^2/f^3/(b-I*a)*b^2
*d^2/(a+I*b)*e^2+1/(I*a+b)^2/f^3/(b-I*a)*b^2*d^2/(a+I*b)*polylog(2,(I*b-a)*
exp(2*I*(f*x+e))/(a+I*b))+4/3/(I*a+b)^2/(b-I*a)*b*a*d^2/(a+I*b)*x^3+4*I/(I*
a+b)^2/f^2/(b-I*a)*b*a*c*d/(a+I*b)*ln(1-(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))*e
+2*I/(I*a+b)^2/f^3/(b-I*a)*b^2*d^2*e/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*
b-a*exp(2*I*(f*x+e))-I*b-a)*a+4*I/(I*a+b)^2/f/(b-I*a)*b*a*c*d/(a+I*b)*ln(1-
(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))*x-2*I/(I*a+b)^2/f^3/(b-I*a)*b*a^2*d^2*e^2
/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-2*I/(I*a
```

```
+b)^2/f^2/(b-I*a)*b^2*c*d/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I
*(f*x+e))-I*b-a)*a+4/(I*a+b)^2/f^2/(b-I*a)*b^2*d^2/(a+I*b)*e*x-8/3/(I*a+b)^
2/f^3/(b-I*a)*b*a*d^2/(a+I*b)*e^3+4/(I*a+b)^2/(b-I*a)*b*a*c*d/(a+I*b)*x^2+8
/(I*a+b)^2/f/(b-I*a)*b*a*c*d/(a+I*b)*e*x-2/(I*a+b)^2/f^3/(b-I*a)*b^2*a*d^2*
e^2/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-2*I/(
I*a+b)^2/f/(b-I*a)*b*a^2*c^2/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(
2*I*(f*x+e))-I*b-a)-4*I/(I*a+b)^2/f^3/(b-I*a)*b*a*d^2*e^2/(a+I*b)*ln(exp(I*
(f*x+e)))-1/3*d^2/(2*I*a*b-a^2+b^2)*x^3-1/(2*I*a*b-a^2+b^2)*c^2*x-1/3/d/(2*
I*a*b-a^2+b^2)*c^3-d/(2*I*a*b-a^2+b^2)*c*x^2+2*I/(I*a+b)^2/f/(b-I*a)*b*a*d^
2/(a+I*b)*ln(1-(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))*x^2-2*I/(I*a+b)^2/f^3/(b-I
*a)*b*a*d^2*e^2/(a+I*b)*ln(1-(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2556 vs. $2(543) = 1086$.
time = 1.24, size = 2556, normalized size = 3.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(6*c*d*(2*a*b*log(b*tan(f*x + e) + a)/((a^4 + 2*a^2*b^2 + b^4)*f) - a*
b*log(tan(f*x + e)^2 + 1)/((a^4 + 2*a^2*b^2 + b^4)*f) - b/((a^2*b + b^3)*f*
tan(f*x + e) + (a^3 + a*b^2)*f) + (a^2 - b^2)*(f*x + e)/((a^4 + 2*a^2*b^2 +
 b^4)*f))*e - 3*(2*a*b*log(b*tan(f*x + e) + a)/(a^4 + 2*a^2*b^2 + b^4) - a*
b*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(f*x + e)/(
a^4 + 2*a^2*b^2 + b^4) - b/(a^3 + a*b^2 + (a^2*b + b^3)*tan(f*x + e)))*c^2
- ((a^3 - I*a^2*b + a*b^2 - I*b^3)*(f*x + e)^3*d^2 + 3*(a^3*e^2 - I*a^2*b*e
^2 + a*b^2*e^2 - I*b^3*e^2)*(f*x + e)*d^2 + 3*((a^3 - I*a^2*b + a*b^2 - I*b
^3)*c*d*f - (a^3*e - I*a^2*b*e + a*b^2*e - I*b^3*e)*d^2)*(f*x + e)^2 - 6*(-
I*a*b^2*e^2 + b^3*e^2)*d^2 - 6*((-I*a*b^2 + b^3)*c*d*f + (a*b^2*(e^2 + I*e)
 - I*a^2*b*e^2 - b^3*e)*d^2 + ((-I*a*b^2 - b^3)*c*d*f - (a*b^2*(e^2 - I*e)
+ I*a^2*b*e^2 - b^3*e)*d^2)*cos(2*f*x + 2*e) + ((a*b^2 - I*b^3)*c*d*f + (a*
b^2*(-I*e^2 - e) + a^2*b*e^2 + I*b^3*e)*d^2)*sin(2*f*x + 2*e))*arctan2(-b*c
os(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*cos(2*f*x + 2*e) + b*sin(2*f*x
+ 2*e) + a) - 6*((I*a^2*b - a*b^2)*(f*x + e)^2*d^2 + (2*(I*a^2*b - a*b^2)*c
*d*f + (a*b^2*(2*e + I) - 2*I*a^2*b*e - b^3)*d^2)*(f*x + e) + ((I*a^2*b + a
*b^2)*(f*x + e)^2*d^2 + (2*(I*a^2*b + a*b^2)*c*d*f - (a*b^2*(2*e - I) + 2*I
*a^2*b*e - b^3)*d^2)*(f*x + e))*cos(2*f*x + 2*e) - ((a^2*b - I*a*b^2)*(f*x
+ e)^2*d^2 + (2*(a^2*b - I*a*b^2)*c*d*f - (a*b^2*(-2*I*e - 1) + 2*a^2*b*e +
 I*b^3)*d^2)*(f*x + e))*sin(2*f*x + 2*e))*arctan2((2*a*b*cos(2*f*x + 2*e) -
 (a^2 - b^2)*sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 +
 b^2 + (a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) + ((a^3 - 3*I*a^2*b - 3*a
*b^2 + I*b^3)*(f*x + e)^3*d^2 + 3*((a^3 - 3*I*a^2*b - 3*a*b^2 + I*b^3)*c*d*
f + (a*b^2*(3*e - 2*I) - b^3*(I*e + 2) - a^3*e + 3*I*a^2*b*e)*d^2)*(f*x + e
```

```
)^2 - 3*(4*(I*a*b^2 + b^3)*c*d*f + (a*b^2*(3*e^2 - 4*I*e) + b^3*(-I*e^2 - 4
*e) - a^3*e^2 + 3*I*a^2*b*e^2)*d^2)*(f*x + e))*cos(2*f*x + 2*e) - 3*(2*(I*a
^2*b - a*b^2)*(f*x + e)*d^2 + 2*(I*a^2*b - a*b^2)*c*d*f + (a*b^2*(2*e + I)
- 2*I*a^2*b*e - b^3)*d^2 + (2*(I*a^2*b + a*b^2)*(f*x + e)*d^2 + 2*(I*a^2*b
+ a*b^2)*c*d*f - (a*b^2*(2*e - I) + 2*I*a^2*b*e - b^3)*d^2)*cos(2*f*x + 2*e
) - (2*(a^2*b - I*a*b^2)*(f*x + e)*d^2 + 2*(a^2*b - I*a*b^2)*c*d*f - (a*b^2
*(-2*I*e - 1) + 2*a^2*b*e + I*b^3)*d^2)*sin(2*f*x + 2*e))*dilog((I*a + b)*e
^(2*I*f*x + 2*I*e)/(-I*a + b)) + 3*((a*b^2 + I*b^3)*c*d*f - (a*b^2*(-I*e^2
+ e) - a^2*b*e^2 + I*b^3*e)*d^2 + ((a*b^2 - I*b^3)*c*d*f - (a*b^2*(I*e^2 +
e) - a^2*b*e^2 - I*b^3*e)*d^2)*cos(2*f*x + 2*e) - ((-I*a*b^2 - b^3)*c*d*f -
 (a*b^2*(e^2 - I*e) + I*a^2*b*e^2 - b^3*e)*d^2)*sin(2*f*x + 2*e))*log((a^2
+ b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x
+ 2*e)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*f*x + 2*e)) + 3*((a^2*b + I*a*b^
2)*(f*x + e)^2*d^2 + (2*(a^2*b + I*a*b^2)*c*d*f - (a*b^2*(2*I*e - 1) + 2*a^
2*b*e - I*b^3)*d^2)*(f*x + e) + ((a^2*b - I*a*b^2)*(f*x + e)^2*d^2 + (2*(a^
2*b - I*a*b^2)*c*d*f - (a*b^2*(-2*I*e - 1) + 2*a^2*b*e + I*b^3)*d^2)*(f*x +
 e))*cos(2*f*x + 2*e) - ((-I*a^2*b - a*b^2)*(f*x + e)^2*d^2 + (2*(-I*a^2*b
- a*b^2)*c*d*f + (a*b^2*(2*e - I) + 2*I*a^2*b*e - b^3)*d^2)*(f*x + e))*sin(
2*f*x + 2*e))*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e)
+ (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*f*x + 2*
e))/(a^2 + b^2)) + 3*((a^2*b - I*a*b^2)*d^2*cos(2*f*x + 2*e) - (-I*a^2*b -
a*b^2)*d^2*sin(2*f*x + 2*e) + (a^2*b + I*a*b^2)*d^2)*polylog(3, (I*a + b)*e
^(2*I*f*x + 2*I*e)/(-I*a + b)) + ((I*a^3 + 3*a^2*b - 3*I*a*b^2 - b^3)*(f*x
+ e)^3*d^2 - 3*((-I*a^3 - 3*a^2*b + 3*I*a*b^2 + b^3)*c*d*f - (b^3*(e - 2*I)
 - a*b^2*(-3*I*e - 2) - I*a^3*e - 3*a^2*b*e)*d^2)*(f*x + e)^2 + 3*(4*(a*b^2
 - I*b^3)*c*d*f - (b^3*(e^2 - 4*I*e) + a*b^2*(3*I*e^2 + 4*e) - I*a^3*e^2 -
3*a^2*b*e^2)*d^2)*(f*x + e))*sin(2*f*x + 2*e))/((a^5 - I*a^4*b + 2*a^3*b^2
- 2*I*a^2*b^3 + a*b^4 - I*b^5)*f^2*cos(2*f*x + 2*e) - (-I*a^5 - a^4*b - 2*I
*a^3*b^2 - 2*a^2*b^3 - I*a*b^4 - b^5)*f^2*sin(2*f*x + 2*e) + (a^5 + I*a^4*b
 + 2*a^3*b^2 + 2*I*a^2*b^3 + a*b^4 + I*b^5)*f^2))/f
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1606 vs. $2(543) = 1086$.
time = 0.43, size = 1606, normalized size = 2.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^3 - a*b^2)*d^2*f^3*x^3 - 6*b^3*c^2*f^2 - 6*(b^3*d^2*f^2 - (a^3 -
a*b^2)*c*d*f^3)*x^2 - 6*(2*b^3*c*d*f^2 - (a^3 - a*b^2)*c^2*f^3)*x - 3*(-2*I
*a^2*b*d^2*f*x - 2*I*a^2*b*c*d*f - I*a*b^2*d^2 + (-2*I*a*b^2*d^2*f*x - 2*I*
a*b^2*c*d*f - I*b^3*d^2)*tan(f*x + e))*dilog(2*((I*a*b - b^2)*tan(f*x + e)^
2 - a^2 - I*a*b + (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*
```

```
x + e)^2 + a^2 + b^2) + 1) - 3*(2*I*a^2*b*d^2*f*x + 2*I*a^2*b*c*d*f + I*a*b
^2*d^2 + (2*I*a*b^2*d^2*f*x + 2*I*a*b^2*c*d*f + I*b^3*d^2)*tan(f*x + e))*di
log(2*((-I*a*b - b^2)*tan(f*x + e)^2 - a^2 + I*a*b + (-I*a^2 - 2*a*b + I*b^
2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2) + 1) + 6*(a^2*b*d
^2*f^2*x^2 - a^2*b*d^2*e^2 + (2*a^2*b*c*d*f^2 + a*b^2*d^2*f)*x + (2*a^2*b*c
*d*f + a*b^2*d^2)*e + (a*b^2*d^2*f^2*x^2 - a*b^2*d^2*e^2 + (2*a*b^2*c*d*f^2
 + b^3*d^2*f)*x + (2*a*b^2*c*d*f + b^3*d^2)*e)*tan(f*x + e))*log(-2*((I*a*b
 - b^2)*tan(f*x + e)^2 - a^2 - I*a*b + (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e)
)/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + 6*(a^2*b*d^2*f^2*x^2 - a^2*b*
d^2*e^2 + (2*a^2*b*c*d*f^2 + a*b^2*d^2*f)*x + (2*a^2*b*c*d*f + a*b^2*d^2)*e
 + (a*b^2*d^2*f^2*x^2 - a*b^2*d^2*e^2 + (2*a*b^2*c*d*f^2 + b^3*d^2*f)*x + (
2*a*b^2*c*d*f + b^3*d^2)*e)*tan(f*x + e))*log(-2*((-I*a*b - b^2)*tan(f*x +
e)^2 - a^2 + I*a*b + (-I*a^2 - 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*ta
n(f*x + e)^2 + a^2 + b^2)) + 6*(a^2*b*c^2*f^2 + a*b^2*c*d*f + a^2*b*d^2*e^2
 - (2*a^2*b*c*d*f + a*b^2*d^2)*e + (a*b^2*c^2*f^2 + b^3*c*d*f + a*b^2*d^2*e
^2 - (2*a*b^2*c*d*f + b^3*d^2)*e)*tan(f*x + e))*log(((I*a*b + b^2)*tan(f*x
+ e)^2 - a^2 + I*a*b + (I*a^2 + I*b^2)*tan(f*x + e))/(tan(f*x + e)^2 + 1))
+ 6*(a^2*b*c^2*f^2 + a*b^2*c*d*f + a^2*b*d^2*e^2 - (2*a^2*b*c*d*f + a*b^2*d
^2)*e + (a*b^2*c^2*f^2 + b^3*c*d*f + a*b^2*d^2*e^2 - (2*a*b^2*c*d*f + b^3*d
^2)*e)*tan(f*x + e))*log(((I*a*b - b^2)*tan(f*x + e)^2 + a^2 + I*a*b + (I*a
^2 + I*b^2)*tan(f*x + e))/(tan(f*x + e)^2 + 1)) + 3*(a*b^2*d^2*tan(f*x + e)
 + a^2*b*d^2)*polylog(3, ((a^2 + 2*I*a*b - b^2)*tan(f*x + e)^2 - a^2 - 2*I*
a*b + b^2 - 2*(-I*a^2 + 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x +
 e)^2 + a^2 + b^2)) + 3*(a*b^2*d^2*tan(f*x + e) + a^2*b*d^2)*polylog(3, ((a
^2 - 2*I*a*b - b^2)*tan(f*x + e)^2 - a^2 + 2*I*a*b + b^2 - 2*(I*a^2 + 2*a*b
 - I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + 2*((a^2
*b - b^3)*d^2*f^3*x^3 + 3*a*b^2*c^2*f^2 + 3*(a*b^2*d^2*f^2 + (a^2*b - b^3)*
c*d*f^3)*x^2 + 3*(2*a*b^2*c*d*f^2 + (a^2*b - b^3)*c^2*f^3)*x)*tan(f*x + e))
/((a^4*b + 2*a^2*b^3 + b^5)*f^3*tan(f*x + e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^
3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+b\tan(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**2/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{(a+b\tan(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*tan(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^2/(a + b*tan(e + f*x))^2, x)
```

3.61 $\int \frac{c+dx}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=214

$$-\frac{(c+dx)^2}{2\left(a^2+b^2\right)d}+\frac{(bd+2acf+2adfx)^2}{4a(a+ib)\left(a^2+b^2\right)df^2}+\frac{b(bd+2acf+2adfx)\log\left(1+\frac{\left(a^2+b^2\right)e^{2i(e+fx)}}{(a+ib)^2}\right)}{\left(a^2+b^2\right)^2f^2}-\frac{iabd\text{PolyLog}\Big(2,-}{\left(a^2+b\right.}$$

```
[Out] -1/2*(d*x+c)^2/(a^2+b^2)/d+1/4*(2*a*d*f*x+2*a*c*f+b*d)^2/a/(a+I*b)/(a^2+b^2
)/d/f^2+b*(2*a*d*f*x+2*a*c*f+b*d)*ln(1+(a^2+b^2)*exp(2*I*(f*x+e))/(a+I*b)^2
)/(a^2+b^2)^2/f^2-I*a*b*d*polylog(2,-(a^2+b^2)*exp(2*I*(f*x+e))/(a+I*b)^2)/
(a^2+b^2)^2/f^2-b*(d*x+c)/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.19, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3814, 3813, 2221, 2317, 2438}

$$\frac{b(2acf+2adfx+bd)\log\left(1+\frac{\left(a^2+b^2\right)e^{2i(e+fx)}}{(a+ib)^2}\right)}{f^2\left(a^2+b^2\right)^2}-\frac{b(c+dx)}{f\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{(2acf+2adfx+bd)^2}{4adf^2(a+ib)\left(a^2+b^2\right)}-\frac{(c+dx)^2}{2d\left(a^2+b^2\right)}-\frac{iabd\text{Li}_2\left(-\frac{\left(a^2+b^2\right)e^{2i(e+fx)}}{(a+ib)^2}\right)}{f^2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -1/2*(c + d*x)^2/((a^2 + b^2)*d) + (b*d + 2*a*c*f + 2*a*d*f*x)^2/(4*a*(a +
I*b)*(a^2 + b^2)*d*f^2) + (b*(b*d + 2*a*c*f + 2*a*d*f*x)*Log[1 + ((a^2 + b^
2)*E^((2*I)*(e + f*x)))/(a + I*b)^2])/((a^2 + b^2)^2*f^2) - (I*a*b*d*PolyLo
g[2, -(((a^2 + b^2)*E^((2*I)*(e + f*x)))/(a + I*b)^2)])/((a^2 + b^2)^2*f^2)
 - (b*(c + d*x))/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3813

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int
[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2*
I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && IGtQ[m, 0]
```

Rule 3814

```
Int[((c_.) + (d_.)*(x_))/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol
] :> Simp[-(c + d*x)^2/(2*d*(a^2 + b^2)), x] + (Dist[1/(f*(a^2 + b^2)), Int
[(b*d + 2*a*c*f + 2*a*d*f*x)/(a + b*Tan[e + f*x]), x], x] - Simp[b*((c + d*
x)/(f*(a^2 + b^2)*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+b\tan(e+fx))^2}\,dx = -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d} - \frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \frac{\int \frac{bd+2acf+2adfx}{a+b\tan(e+fx)}\,dx}{\left(a^2+b^2\right)f}$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d} + \frac{(bd+2acf+2adfx)^2}{4a(a+ib)\left(a^2+b^2\right)df^2} - \frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \frac{(2}{}$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d} + \frac{(bd+2acf+2adfx)^2}{4a(a+ib)\left(a^2+b^2\right)df^2} + \frac{b(bd+2acf+2adfx)\log\left(1+\frac{(a^2+}{}\right.}{\left(a^2+b^2\right)^2f^2}$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d} + \frac{(bd+2acf+2adfx)^2}{4a(a+ib)\left(a^2+b^2\right)df^2} + \frac{b(bd+2acf+2adfx)\log\left(1+\frac{(a^2+}{}\right.}{\left(a^2+b^2\right)^2f^2}$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d} + \frac{(bd+2acf+2adfx)^2}{4a(a+ib)\left(a^2+b^2\right)df^2} + \frac{b(bd+2acf+2adfx)\log\left(1+\frac{(a^2+}{}\right.}{\left(a^2+b^2\right)^2f^2}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 745 vs. 2(214) = 428.
time = 6.87, size = 745, normalized size = 3.48

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((e + f*x)*(-2*d*e + 2*c*f + d*(e + f*x))*Sec[e + f*x]^2*(a*Cos[e + f*x] +
b*Sin[e + f*x])^2)/(2*(a - I*b)*(a + I*b)*f^2*(a + b*Tan[e + f*x])^2) + (b^
2*d*(-(b*(e + f*x)) + a*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])*Sec[e + f*x]^
2*(a*Cos[e + f*x] + b*Sin[e + f*x])^2)/(a*(a - I*b)*(a + I*b)*(a^2 + b^2)*f
^2*(a + b*Tan[e + f*x])^2) - (2*b*d*e*(-(b*(e + f*x)) + a*Log[a*Cos[e + f*x
] + b*Sin[e + f*x]])*Sec[e + f*x]^2*(a*Cos[e + f*x] + b*Sin[e + f*x])^2)/((
a - I*b)*(a + I*b)*(a^2 + b^2)*f^2*(a + b*Tan[e + f*x])^2) + (2*b*c*(-(b*(e
 + f*x)) + a*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])*Sec[e + f*x]^2*(a*Cos[e
+ f*x] + b*Sin[e + f*x])^2)/((a - I*b)*(a + I*b)*(a^2 + b^2)*f*(a + b*Tan[e
 + f*x])^2) - (d*(E^(I*ArcTan[a/b])*(e + f*x)^2 + (a*(I*(e + f*x)*(-Pi + 2*
ArcTan[a/b]) - Pi*Log[1 + E^((-2*I)*(e + f*x))] - 2*(e + f*x + ArcTan[a/b])
*Log[1 - E^((2*I)*(e + f*x + ArcTan[a/b]))] + Pi*Log[Cos[e + f*x]] + 2*ArcT
an[a/b]*Log[Sin[e + f*x + ArcTan[a/b]]] + I*PolyLog[2, E^((2*I)*(e + f*x +
ArcTan[a/b]))]))/(Sqrt[1 + a^2/b^2]*b))*Sec[e + f*x]^2*(a*Cos[e + f*x] + b*
Sin[e + f*x])^2)/((a - I*b)*(a + I*b)*Sqrt[(a^2 + b^2)/b^2]*f^2*(a + b*Tan[
e + f*x])^2) + (Sec[e + f*x]^2*(a*Cos[e + f*x] + b*Sin[e + f*x])*(-(b^2*d*e
*Sin[e + f*x]) + b^2*c*f*Sin[e + f*x] + b^2*d*(e + f*x)*Sin[e + f*x]))/(a*(
a - I*b)*(a + I*b)*f^2*(a + b*Tan[e + f*x])^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 998 vs. 2(202) = 404.
time = 0.52, size = 999, normalized size = 4.67

method	result
risch	$-\frac{d\,x^{2}}{2(2iab-a^{2}+b^{2})}-\frac{cx}{2iab-a^{2}+b^{2}}-\frac{2ib^{2}d\ln\left(\mathrm{e}^{i(fx+e)}\right)}{(ia+b)^{2}f^{2}(-ia+b)(ib+a)}+\frac{4ibade\ln\left(\mathrm{e}^{i(fx+e)}\right)}{(ia+b)^{2}f^{2}(-ia+b)(ib+a)}+\frac{2ib\,a^{2}de\ln\left(i\mathrm{e}^{2i(fx+e)}b-a\,\mathrm{e}^{2i(f}\right.}{(ia+b)^{2}f^{2}(-ia+b)(ib+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(2*I*a*b-a^2+b^2)*d*x^2-1/(2*I*a*b-a^2+b^2)*c*x-2*I/(I*a+b)^2/f^2/(b-I
*a)*b^2*d/(a+I*b)*ln(exp(I*(f*x+e)))+4*I/(I*a+b)^2/f^2/(b-I*a)*b*a*d*e/(a+I
*b)*ln(exp(I*(f*x+e)))+2*I/(I*a+b)^2/f^2/(b-I*a)*b*a^2*d*e/(a+I*b)/(I*b-a)*
ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-1/(I*a+b)^2/f^2/(b-I*a)*b
^3*d/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-2*I/
(I*a+b)^2/f/(b-I*a)*b*a^2*c/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2
*I*(f*x+e))-I*b-a)-4*I/(I*a+b)^2/f/(b-I*a)*b*a*c/(a+I*b)*ln(exp(I*(f*x+e)))
-2/(I*a+b)^2/f/(b-I*a)*b^2*a*c/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*ex
p(2*I*(f*x+e))-I*b-a)+2*I/(I*a+b)^2/f/(b-I*a)*b*a*d/(a+I*b)*ln(1-(I*b-a)*ex
p(2*I*(f*x+e))/(a+I*b))*x-2*I*b^2*(d*x+c)/(b-I*a)/f/(I*a+b)^2/(b*exp(2*I*(f
*x+e))+I*a*exp(2*I*(f*x+e))-b+I*a)+2/(I*a+b)^2/f^2/(b-I*a)*b^2*a*d*e/(a+I*b
```

```
)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I*b-a)-I/(I*a+b)^2/f^2
/(b-I*a)*b^2*d/(a+I*b)/(I*b-a)*ln(I*exp(2*I*(f*x+e))*b-a*exp(2*I*(f*x+e))-I
*b-a)*a+2*I/(I*a+b)^2/f^2/(b-I*a)*b*a*d/(a+I*b)*ln(1-(I*b-a)*exp(2*I*(f*x+e
))/(a+I*b))*e+2/(I*a+b)^2/(b-I*a)*b*a*d/(a+I*b)*x^2+4/(I*a+b)^2/f/(b-I*a)*b
*a*d/(a+I*b)*e*x+2/(I*a+b)^2/f^2/(b-I*a)*b*a*d/(a+I*b)*e^2+1/(I*a+b)^2/f^2/
(b-I*a)*b*a*d/(a+I*b)*polylog(2,(I*b-a)*exp(2*I*(f*x+e))/(a+I*b))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(200) = 400$.
time = 0.88, size = 1193, normalized size = 5.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^3 - I*a^2*b + a*b^2 - I*b^3)*d*f^2*x^2 + 2*(a^3 - I*a^2*b + a*b^2 -
 I*b^3)*c*f^2*x - 4*(-I*a*b^2 + b^3)*c*f - 2*(2*(-I*a^2*b + a*b^2)*c*f + (-
I*a*b^2 + b^3)*d + (2*(-I*a^2*b - a*b^2)*c*f + (-I*a*b^2 - b^3)*d)*cos(2*f*
x + 2*e) + (2*(a^2*b - I*a*b^2)*c*f + (a*b^2 - I*b^3)*d)*sin(2*f*x + 2*e))*
arctan2(-b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*cos(2*f*x + 2*e) +
b*sin(2*f*x + 2*e) + a) - 4*((I*a^2*b + a*b^2)*d*f*x*cos(2*f*x + 2*e) - (a^
2*b - I*a*b^2)*d*f*x*sin(2*f*x + 2*e) + (I*a^2*b - a*b^2)*d*f*x)*arctan2((2
*a*b*cos(2*f*x + 2*e) - (a^2 - b^2)*sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*s
in(2*f*x + 2*e) + a^2 + b^2 + (a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) +
((a^3 - 3*I*a^2*b - 3*a*b^2 + I*b^3)*d*f^2*x^2 + 2*((a^3 - 3*I*a^2*b - 3*a*
b^2 + I*b^3)*c*f^2 - 2*(I*a*b^2 + b^3)*d*f)*x)*cos(2*f*x + 2*e) - 2*((I*a^2
*b + a*b^2)*d*cos(2*f*x + 2*e) - (a^2*b - I*a*b^2)*d*sin(2*f*x + 2*e) + (I*
a^2*b - a*b^2)*d)*dilog((I*a*e^(2*I*e) + b*e^(2*I*e))*e^(2*I*f*x)/(-I*a + b
)) + (2*(a^2*b + I*a*b^2)*c*f + (a*b^2 + I*b^3)*d + (2*(a^2*b - I*a*b^2)*c*
f + (a*b^2 - I*b^3)*d)*cos(2*f*x + 2*e) - (2*(-I*a^2*b - a*b^2)*c*f - (I*a*
b^2 + b^3)*d)*sin(2*f*x + 2*e))*log((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*
sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 + 2*(a^2 - b^
2)*cos(2*f*x + 2*e)) + 2*((a^2*b - I*a*b^2)*d*f*x*cos(2*f*x + 2*e) - (-I*a^
2*b - a*b^2)*d*f*x*sin(2*f*x + 2*e) + (a^2*b + I*a*b^2)*d*f*x)*log(((a^2 +
b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x +
2*e)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) + ((I*a^3
 + 3*a^2*b - 3*I*a*b^2 - b^3)*d*f^2*x^2 - 2*((-I*a^3 - 3*a^2*b + 3*I*a*b^2
+ b^3)*c*f^2 - 2*(a*b^2 - I*b^3)*d*f)*x)*sin(2*f*x + 2*e))/((a^5 - I*a^4*b
+ 2*a^3*b^2 - 2*I*a^2*b^3 + a*b^4 - I*b^5)*f^2*cos(2*f*x + 2*e) - (-I*a^5 -
 a^4*b - 2*I*a^3*b^2 - 2*a^2*b^3 - I*a*b^4 - b^5)*f^2*sin(2*f*x + 2*e) + (a
^5 + I*a^4*b + 2*a^3*b^2 + 2*I*a^2*b^3 + a*b^4 + I*b^5)*f^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(200) = 400$.

time = 0.44, size = 871, normalized size = 4.07

$$\frac{(a^3-ab^2)df^2x^2-2b^3cf-2(b^3df-(a^3-ab^2)cf^2)x+(i\,ab^2d\tan(fx+e)+i\,a^2bd)\operatorname{Li}_2\left(\frac{2((i\,ab-b^2)\tan(fx+e)^2-a^2-i\,ab+(i\,a^2-2\,ab-i\,b^2)\tan(fx+e))}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}+1\right)+(-i\,ab^2d\tan(fx+e)-i\,a^2bd)\operatorname{Li}_2\left(\frac{2((-i\,ab-b^2)\tan(fx+e)^2-a^2+i\,ab+(-i\,a^2-2\,ab+i\,b^2)\tan(fx+e))}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}+1\right)+2(a^2bdfx+a^2bde+(ab^2dfx+ab^2de)\tan(fx+e))\log\left(-\frac{2((i\,ab-b^2)\tan(fx+e)^2-a^2-i\,ab+(i\,a^2-2\,ab-i\,b^2)\tan(fx+e))}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}\right)+2(a^2bdfx+a^2bde+(ab^2dfx+ab^2de)\tan(fx+e))\log\left(-\frac{2((-i\,ab-b^2)\tan(fx+e)^2-a^2+i\,ab+(-i\,a^2-2\,ab+i\,b^2)\tan(fx+e))}{(a^2+b^2)\tan(fx+e)^2+a^2+b^2}\right)+(2\,a^2bcf-2\,a^2bde+ab^2d+(2\,ab^2cf-2\,ab^2de+b^3d)\tan(fx+e))\log\left(\frac{(i\,ab+b^2)\tan(fx+e)^2-a^2+i\,ab+(i\,a^2+i\,b^2)\tan(fx+e)}{\tan(fx+e)^2+1}\right)+(2\,a^2bcf-2\,a^2bde+ab^2d+(2\,ab^2cf-2\,ab^2de+b^3d)\tan(fx+e))\log\left(\frac{(i\,ab-b^2)\tan(fx+e)^2+a^2+i\,ab+(i\,a^2+i\,b^2)\tan(fx+e)}{\tan(fx+e)^2+1}\right)+((a^2b-b^3)df^2x^2+2\,ab^2cf+2(ab^2df+(a^2b-b^3)cf^2)x)\tan(fx+e)}{2((a^4b+2\,a^2b^3+b^5)f^2\tan(fx+e)+(a^5+2\,a^3b^2+ab^4)f^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^3 - a*b^2)*d*f^2*x^2 - 2*b^3*c*f - 2*(b^3*d*f - (a^3 - a*b^2)*c*f^2
)*x + (I*a*b^2*d*tan(f*x + e) + I*a^2*b*d)*dilog(2*((I*a*b - b^2)*tan(f*x +
 e)^2 - a^2 - I*a*b + (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e))/((a^2 + b^2)*ta
n(f*x + e)^2 + a^2 + b^2) + 1) + (-I*a*b^2*d*tan(f*x + e) - I*a^2*b*d)*dilo
g(2*((-I*a*b - b^2)*tan(f*x + e)^2 - a^2 + I*a*b + (-I*a^2 - 2*a*b + I*b^2)
*tan(f*x + e))/((a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2) + 1) + 2*(a^2*b*d*f
*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*tan(f*x + e))*log(-2*((I*a*b - b
^2)*tan(f*x + e)^2 - a^2 - I*a*b + (I*a^2 - 2*a*b - I*b^2)*tan(f*x + e))/((
a^2 + b^2)*tan(f*x + e)^2 + a^2 + b^2)) + 2*(a^2*b*d*f*x + a^2*b*d*e + (a*b
^2*d*f*x + a*b^2*d*e)*tan(f*x + e))*log(-2*((-I*a*b - b^2)*tan(f*x + e)^2 -
 a^2 + I*a*b + (-I*a^2 - 2*a*b + I*b^2)*tan(f*x + e))/((a^2 + b^2)*tan(f*x
+ e)^2 + a^2 + b^2)) + (2*a^2*b*c*f - 2*a^2*b*d*e + a*b^2*d + (2*a*b^2*c*f
- 2*a*b^2*d*e + b^3*d)*tan(f*x + e))*log(((I*a*b + b^2)*tan(f*x + e)^2 - a^
2 + I*a*b + (I*a^2 + I*b^2)*tan(f*x + e))/(tan(f*x + e)^2 + 1)) + (2*a^2*b*
c*f - 2*a^2*b*d*e + a*b^2*d + (2*a*b^2*c*f - 2*a*b^2*d*e + b^3*d)*tan(f*x +
 e))*log(((I*a*b - b^2)*tan(f*x + e)^2 + a^2 + I*a*b + (I*a^2 + I*b^2)*tan(
f*x + e))/(tan(f*x + e)^2 + 1)) + ((a^2*b - b^3)*d*f^2*x^2 + 2*a*b^2*c*f +
2*(a*b^2*d*f + (a^2*b - b^3)*c*f^2)*x)*tan(f*x + e))/((a^4*b + 2*a^2*b^3 +
b^5)*f^2*tan(f*x + e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(a+b\tan(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+d\,x}{{\left(a+b\,\tan\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*tan(e + f*x))^2,x)
[Out] int((c + d*x)/(a + b*tan(e + f*x))^2, x)
```

3.62 $\int \frac{1}{(c+dx)(a+b\tan(e+fx))^2} \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\tan(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*tan(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\tan(e+fx))^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Tan[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\tan(e+fx))^2} \, dx = \int \frac{1}{(c+dx)(a+b\tan(e+fx))^2} \, dx$$

Mathematica [A]
time = 16.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\tan(e+fx))^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Tan[e + f*x])^2), x]
```

Maple [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a+b\tan(fx+e))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+b*tan(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*cos(2*f*x + 2*e)^2*log(d*x + c) + ((
a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*log(d*x + c)*sin(2*f*x + 2*e)^2 - 2*(2*
a*b^3*d - ((a^4 - 2*a^2*b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*f)*log
(d*x + c))*cos(2*f*x + 2*e) + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x
+ (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f)*cos(2*f*x + 2*
e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*c*d*f)*sin(2*f*x + 2*e)^2 + 2*((a^6 + a^4*b^2 - a^2*b^4 - b^
6)*d^2*f*x + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d*f)*cos(2*f*x + 2*e) + 4*((
a^5*b + 2*a^3*b^3 + a*b^5)*d^2*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*d*f)*sin
(2*f*x + 2*e))*integrate(2*(2*(2*a^2*b^2*d*f*x + 2*a^2*b^2*c*f - a*b^3*d)*c
os(2*f*x + 2*e) - (2*(a^3*b - a*b^3)*d*f*x + 2*(a^3*b - a*b^3)*c*f - (a^2*b
^2 - b^4)*d)*sin(2*f*x + 2*e))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x
^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f*x + (a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*c^2*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x^2 + 2*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*c^2*f)*cos(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f
*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f*x + (a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*c^2*f)*sin(2*f*x + 2*e)^2 + 2*((a^6 + a^4*b^2 - a^2*b^4 - b
^6)*d^2*f*x^2 + 2*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d*f*x + (a^6 + a^4*b^2
- a^2*b^4 - b^6)*c^2*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a^3*b^3 + a*b^5)*d
^2*f*x^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d*f*x + (a^5*b + 2*a^3*b^3 + a*b
^5)*c^2*f)*sin(2*f*x + 2*e)), x) + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*lo
g(d*x + c) + 2*((a^2*b^2 - b^4)*d + 2*((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b
^3)*c*f)*log(d*x + c))*sin(2*f*x + 2*e))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f + ((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f)*cos
(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x + (a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f)*sin(2*f*x + 2*e)^2 + 2*((a^6 + a^4*b^2 - a
^2*b^4 - b^6)*d^2*f*x + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d*f)*cos(2*f*x +
2*e) + 4*((a^5*b + 2*a^3*b^3 + a*b^5)*d^2*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)
*c*d*f)*sin(2*f*x + 2*e))
```

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*tan(f*x + e)^2 + 2*(a*b*d*x
 + a*b*c)*tan(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\tan(e + fx))^2 (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))**2*(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + b\tan(e + f\,x))^2\,(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^2*(c + d*x)),x)
```

```
[Out] int(1/((a + b*tan(e + f*x))^2*(c + d*x)), x)
```

3.63 $\int \frac{1}{(c+dx)^2(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\tan(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*tan(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\tan(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Tan[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+b\tan(e+fx))^2}\,dx$$

Mathematica [A]
time = 16.81, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\tan(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Tan[e + f*x])^2), x]
```

Maple [A]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a+b\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*tan(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*
f)*cos(2*f*x + 2*e)^2 + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*sin(2*f*x + 2
*e)^2 + 2*(2*a*b^3*d + (a^4 - 2*a^2*b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 + b
^4)*c*f)*cos(2*f*x + 2*e) - ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^2
+ 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*c^2*d*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^2 + 2*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*c^2*d*f)*cos(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
d^3*f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*c^2*d*f)*sin(2*f*x + 2*e)^2 + 2*((a^6 + a^4*b^2 - a^
2*b^4 - b^6)*d^3*f*x^2 + 2*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^2*f*x + (a^6
 + a^4*b^2 - a^2*b^4 - b^6)*c^2*d*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a^3*b
^3 + a*b^5)*d^3*f*x^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d^2*f*x + (a^5*b +
2*a^3*b^3 + a*b^5)*c^2*d*f)*sin(2*f*x + 2*e))*integrate(4*(2*(a^2*b^2*d*f*x
 + a^2*b^2*c*f - a*b^3*d)*cos(2*f*x + 2*e) - ((a^3*b - a*b^3)*d*f*x + (a^3*
b - a*b^3)*c*f - (a^2*b^2 - b^4)*d)*sin(2*f*x + 2*e))/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*d^3*f*x^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x
^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d*f*x + (a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*c^3*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^3 + 3*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*c^2*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3*f)*cos(2*f
*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^3 + 3*(a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*c^2*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3*f)*sin(2*f*x + 2*e)^
2 + 2*((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^3*f*x^3 + 3*(a^6 + a^4*b^2 - a^2*b
^4 - b^6)*c*d^2*f*x^2 + 3*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^2*d*f*x + (a^6
+ a^4*b^2 - a^2*b^4 - b^6)*c^3*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a^3*b^3
+ a*b^5)*d^3*f*x^3 + 3*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d^2*f*x^2 + 3*(a^5*b +
 2*a^3*b^3 + a*b^5)*c^2*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c^3*f)*sin(2*f*
x + 2*e)), x) + 2*(2*(a^3*b - a*b^3)*d*f*x + 2*(a^3*b - a*b^3)*c*f - (a^2*b
^2 - b^4)*d)*sin(2*f*x + 2*e))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x
^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*c^2*d*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^2 +
2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*
```

```
b^4 + b^6)*c^2*d*f)*cos(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*d^3*f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d*f)*sin(2*f*x + 2*e)^2 + 2*((a^6 + a^4*b^2 -
 a^2*b^4 - b^6)*d^3*f*x^2 + 2*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^2*f*x + (
a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^2*d*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a^
3*b^3 + a*b^5)*d^3*f*x^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d^2*f*x + (a^5*b
 + 2*a^3*b^3 + a*b^5)*c^2*d*f)*sin(2*f*x + 2*e))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*tan(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*tan(f
*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \tan{\left(e + f x \right)}\right)^{2} \left(c + d x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/((a + b*tan(e + f*x))**2*(c + d*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*tan(f*x + e) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^2\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tan(e + f*x))^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + b*tan(e + f*x))^2*(c + d*x)^2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
           'sin','cos','tan','cot','sec','csc',
           'arcsin','arccos','arctan','arccot','arcsec','arccsc',
           'sinh','cosh','tanh','coth','sech','csch',
           'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
       'arctan2','floor','abs'
       ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```